Formulæ For Math 114

x-coordinate of the vertex of the parabola $y = ax^2 + bx + c$: $\frac{-b}{2a}$

Simple interest: $I = Prt$, $A = P(1 + rt)$

Compound interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Effective annual interest rate for APR r when interest is compounded monthly: $\left(1 + \frac{r}{12}\right)^{12} - 1$

Present value of a single future payment: $\frac{A}{(1+r)^n}$

Loan formula: $PMT = \frac{A\left(\frac{r}{n}\right)}{1-\left(1+\frac{r}{n}\right)^{-nt}}$

Practice Final

Math 114 Fall 2011

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1. Write each number in scientific notation.

$7521.03 \quad = 7.52103 \times 10^3$

$0.003521 \quad = 3.521 \times 10^{-3}$

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2. Put the following numbers in order from least to greatest.

$3.14 \times 10^{-4} \qquad 0.00213 \qquad 23.7 \times 10^{-5} \qquad 0.00001 \qquad 0.003 \times 10^4$

Change all the numbers to decimal form for easy comparison. The order is $0.00001, 0.000237, 0.000314, 0.00213, 30$.

_____3. Last year a certain coat cost \$360, but the price has gone up 15% since then. What is the price now?

(a) \$306.00

(b) \$375.00

(c) \$414.00

(d) \$666.00

15% of 360 is 54, so the new price is $360 + 54 = 414$.

_____4. An item in a store was marked down 25%. If its sale price was \$54.54, what was its original price, to the nearest cent?

(a) \$79.54

(b) \$72.72

(c) \$95.45

(d) \$68.18

Let $x =$ the original price. Then we need 75% of x to be \$54.54; as an equation, $0.75x = 54.54$. Thus $x = 72.72$.

_____5. Of the equations listed below, which has a graph that is a parabola opening up?

(a) $y = 2 - 3x$

(b) $y = 2 - 3x^2$

(c) $y = 3x^2 - 2$

(d) $y = 3x - 2$

$y = 3x^2 - 2$ has the form $y = ax^2 + bx + c$ with $a > 0$, so its graph is a parabola opening up.

_____6. John has been keeping careful track of the amount of lemonade he sells at his lemonade stand, and has determined that if he sees n customers in a day, then he sells

$$L = 0.28n$$

gallons of lemonade that day. What is the meaning of slope in John's equation?

(a) For each extra customer, John sells 0.28 more gallons of lemonade.

(b) For every 0.28 customers, John sells an extra gallon of lemonade.

(c) The ratio of lemonade to customers is 0.28:1.

(d) John is not selling any lemonade.

The slope—in this case 0.28—is the answer to the question, "How does the output change when the input increases by 1?" Here the input is the number of customers and the output is the number of gallons of lemonade sold, so the answer is (a).

_____7. The vertex of the parabola $y = 8x - 2x^2$ is at

(a) $(-2, -8)$

(b) $(2, -8)$

(c) $(-2, 4)$

(d) $(2, 8)$

Rewrite as $y = -2x^2 + 8x$. The x-coordinate of the vertex is at $\frac{-b}{2a} = \frac{-8}{2(-2)} = 2$, and the y-coordinate is at $-2(2)^2 + 8(2) = 8$.

_____8. Suppose that a kicked football follows a path given by $y = -0.1x^2 + 1.6x$, where x is the distance in feet from where the ball was kicked and y is the height of the ball above the ground, also in feet. What is the maximum height the ball reaches?

(a) 8 feet

(b) 6.4 feet

(c) 1.7 feet

(d) 64 feet

The maximum height is the y-coordinate of the vertex. The x-coordinate is $\frac{-1.6}{2(-0.1)} = 8$, so the y-coordinate is $-0.1(8)^2 + 1.6(8) = 6.4$.

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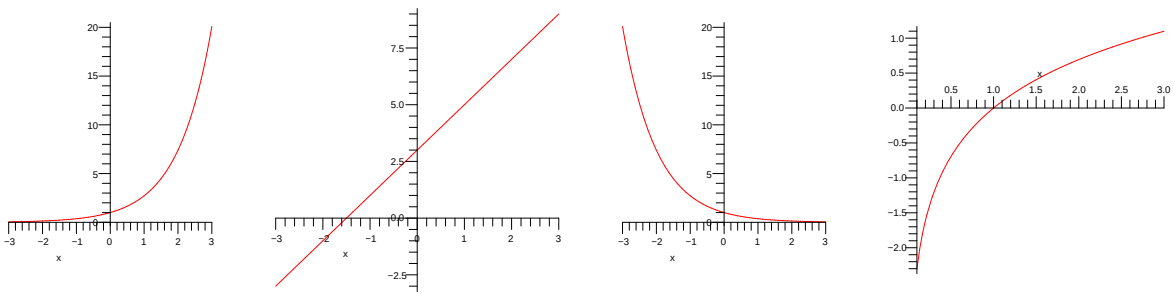
9. Which of the curves below is decreasing exponentially?



The third graph looks like e^{-x}. It is also the only one that is decreasing at all.

_____10. Which of the following is equivalent to $13^5 = 371293$?

(a) $\log_{13} 371293 = 5$

(b) $\log_5 13 = 371293$

(c) $\log_5 371293 = 13$

(d) $\log_{13} 5 = 371293$

(a)

_____11. The approximate percentage of a boy's adult height that he has reached at age x years is given by $P = 61.8 + 36\log(x-4)$ for $5 \le x \le 15$. What percentage of his adult height has a boy reached at age 8? Round your answer to three decimal places.

(a) 94.311

(b) 83.474

(c) 111,707

(d) 97.800

The percentage is $P = 61.8 + 36\log(8-4) \approx 83.474$.

_____12. Suppose you deposit \$2000 into an account paying 3.5% simple annual interest. If you make no other deposits or withdrawals, what is the value of the account after five years?

(a) \$2381.89

(b) \$2350.00

(c) \$2392.52

(d) \$2375.37

Use the simple interest formula: $A = P(1+rt) = 2000(1+0.035 \cdot 5) = 2350$.

_____13. Given the Excel spreadsheet pictured below, which formula would you type into cell D2 to find the amount A in an account paying 3.7% simple annual interest if \$3500 is deposited and left alone for five years?

	A	B	C	D
1	P	r	t	A
2	3500	3.7	5	
3				

(a) A2*POWER(1+B2,C2)

(b) A2*B2*C2

(c) A2*(1+(B2/100)*C2)

(d) A2*POWER(1+B2/12,12*C2)

You want the formula that implements the simple interest formula $A = P(1+rt)$, which is A2*(1+(B2/100)*C2).

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14. Jane has made the following table to compute the finance charge for her March credit card bill, which is due April 10.

Date	**Payments/ purchases**	**Balance each day**	**Number of days until the balance changes**	**Unpaid balance times the number of days**
Jul 10–12		541	3	
Jul 13–18	+325	866	6	
Jul 19–25	$+127-250$	743	7	
Jul 26–Aug 9	-250	493	15	
Total			31	

If the annual interest rate on the card is 24%, what will the finance charge be?

Her ADB is $\frac{19415}{31} \approx 626.29$. Her monthly rate is $\frac{0.24}{12} = 0.02$, so the finance charge is $(0.02)(626.29) = 12.53$.

_____15. Suppose you buy a \$2500 certificate of deposit paying 3.7% annual interest compounded annually. To the nearest cent, how much is it worth at the end of four years?

(a) \$2898.12

(b) \$2870.00

(c) \$8806.88

(d) \$2891.05

Use the compound interest formula: $A = P\left(1+\frac{r}{n}\right)^{nt} = 2500\left(1+\frac{0.037}{1}\right)^{1\cdot 4} \approx 2891.05$.

_____16. Suppose you deposit \$2000 in an account paying 4.1% annual interest, compounded monthly. If you make no other deposits or withdrawals, what is the value of the account after four years, to the nearest cent?

(a) \$2421.53

(b) \$2355.77

(c) \$2348.73

(d) \$2328.00

Use the compound interest formula: $A = P\left(1+\frac{r}{n}\right)^{nt} = 2000\left(1+\frac{0.041}{12}\right)^{12\cdot 4} \approx 2355.77$.

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17. If the nominal rate on a CD is 4%, what is the effective rate? Assume monthly compounding, and round your answer to the nearest hundredth of a percent.

The effective rate is $\left(1+\frac{0.04}{12}\right)^{12}-1 \approx 0.0407$, or 4.07%.

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18. What is the present value of a payment of $10,000 due ten years from today, assuming an annual discount rate of 5.3%?

$\frac{10000}{1.053^{10}} \approx 5966.45$

_____19. John computed the current price of a ten-year, $10,000 bond using a discount rate of 4.6%, but he has just learned that he should have used a discount rate of 4.7%. Will the new current price be higher or lower than the old one?

(a) higher

(b) lower

(c) neither—the price will not change

The current price of a bond is its present value. The present value of the $10,000 at 4.6% is the amount you must invest at 4.6% to end up with $10,000 in ten years. The present value of the $10,000 at 4.7% is the amount you must invest at 4.7% to end up with $10,000 in ten years. At a higher rate you need to invest less money, so the present value will decrease.

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20. For five years Jed has been making monthly payments of $877.60 on his 20-year, $125,000 mortgage at 5.75% annual interest, and his principal is down to $105,683.56. How much of his next monthly payment will go to pay down his principal?

Jed's monthly interest rate is $\frac{0.0575}{12}$, so his interest payment on $105,683.56 is $\left(\frac{0.0575}{12}\right)105\,683.56 \approx 506.40$. The amount of his payment that will go to the principal is therefore $877.60 - 506.40 = 371.20$.

_____21. Suppose you buy a new car. You put down $5000 in cash and finance the remaining $21,600 with a four-year loan at 4.5% annual interest compounded monthly. To the nearest cent, what will your monthly payments be?

(a) $606.57

(b) $536.63

(c) $492.56

(d) $511.27

Use the loan payment formula: $P = \frac{A\left(\frac{r}{n}\right)}{1-\left(1+\frac{r}{n}\right)^{-nt}} = \frac{21\,600\left(\frac{0.045}{12}\right)}{1-\left(1+\frac{0.045}{12}\right)^{-12\cdot 4}} \approx 492.56.$

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22. Mark each data set described below 'numerical' or 'categorical'.

- bowling scores — numerical
- RU ID numbers — categorical (you can't average them)
- hat sizes — numerical (or categorical if you think of Small, Medium, Large)
- lengths of text messages — numerical

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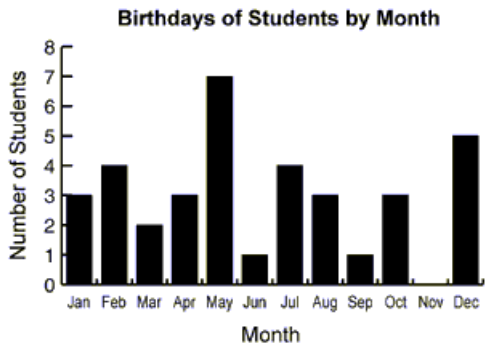
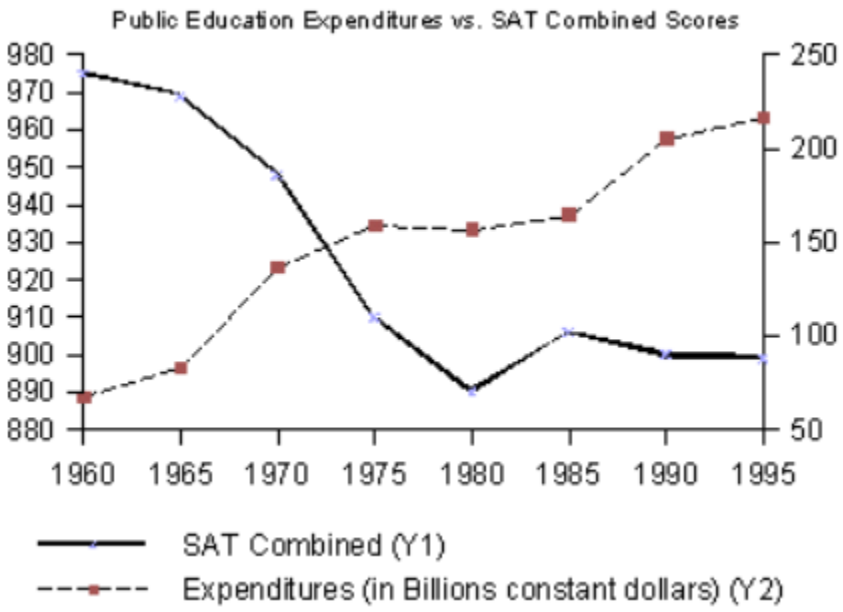
23. The bar graph below shows data about the birthdays of students in Mrs. Jones' class. How many students have birthdays before April 1?



$3+4+2=9.$

_____24. Approximately what was the percentage increase in public expenditure on education between 1960 and 1995, according to the double line graph shown?



(a) 214

(b) 150

(c) 85

(d) 140

% change = $\frac{\text{final amount} - \text{initial amount}}{\text{initial amount}} \times 100 = \frac{220-70}{70} \times 100 \approx 214$

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25. The heights of ten different trees of the same species were measured, with the following results (in feet):

13, 13.5, 17, 16.3, 15.4, 14.1, 12.7, 16.7, 14.1, 13

What is the mean height?

Adding all the values and dividing by 10 gives 14.58 ft.

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26. The heights of ten different trees of the same species were measured, with the following results (in feet):

13, 13.5, 17, 16.3, 15.4, 14.1, 12.7, 16.7, 14.1, 13

What is the median height?

Sort the values in ascending order: 12.7, 13, 13, 13.5, 14.1, 14.1, 15.4, 16.3, 16.7, 17. There are 10 data points. $\frac{10+1}{2} = 5.5$, so we average the values in the 5^{th} and 6^{th} positions. Thus the median is $\frac{14.1+14.1}{2} = 14.1$.

_____27. Ed has surveyed ten smokers to find out how many cigarettes they smoke each day. His findings:

10, 14, 40, 25, 30, 25, 30, 23, 17, 35

Which measure of central tendency should be reported for this data?

(a) mode

(b) mean

(c) range

(d) median

The mean. This is numerical data and there are no outliers.

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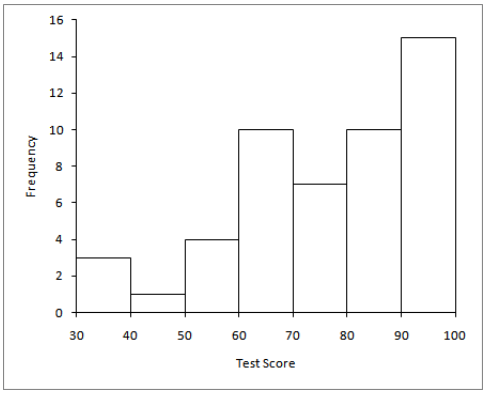
28. For a particular sample of 50 scores on a Math 114 test, the following results were obtained. (Each test score was an integer between 0 and 100.)

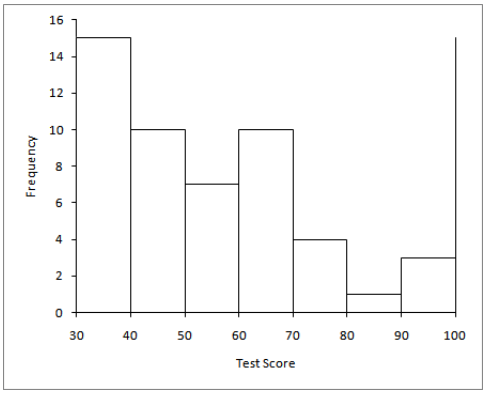
Mean: 77.18	Variance: 317.2	Mode: 97	Median: 81.5
Standard deviation: 17.8	Range: 67	Min: 33	Max: 100

(a) Which score was obtained by more students than any other?

The mode: 97

(b) Which of the following histograms best matches this data?





The one on the left. There are several ways to see this; perhaps the easiest is that the modal class of the histogram on the right does not contain 97.

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29. Consider the following data sets:

A:	13	17	21	42	35	46	16
B:	25	29	33	54	47	58	28

Note that **B** can be obtained from **A** by adding 12 to each point of **A**. If the standard deviation of set **A** is s, what is the standard deviation of set **B**? Why?

It is the same. The spread, or variation, in **B** is identical to that in **A**.

_____30. Jane has counted the number of words on her blogs and found that the mean number of words is 123 while the standard deviation is 8. Which of the following is a reasonable statement by the usual standards of statistics?

(a) On average, the number of words in one of Jane's blogs differs from 123 by 8 words.

(b) The median of Jane's data set is larger than the mean.

(c) The histogram for Jane's data set is symmetrical.

(d) On average, the number of words in one of Jane's blogs differs from 123 by 64 words.

(a). This is our usual interpretation of standard deviation.

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31. For each of the following, state whether it is a measure of central tendency or of spread.

- mean — central tendency
- mode— central tendency
- variance — spread
- range— spread
- standard deviation— spread

_____32. The Excel spreadsheet shown below has been set up to find, to the nearest cent, the value of a five-year, $2000 CD that pays 2.7% annual interest compounded monthly. Which formula should be typed into cell D2 to compute this amount?

	A	B	C	D
1	P	r	t	A
2	2000	0.027	5	

(a) ROUND(A2*POWER(1+B2,C2))

(b) A2*POWER(1+B2/12,12*C2)

(c) ROUND(A2*POWER(1+B2/12,12*C2),2)

(d) A2*POWER(ROUND(1+B2/12,12*C2,2))

You want the formula that implements $A = P\left(1+\frac{r}{12}\right)^{12t}$, rounded to the nearest cent, which is ROUND(A2*POWER(1+B2/12,12*C2),2).

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33. Find the median and range of the data in the stem-and-leaf diagram below.

Key: 8	1 = 810

Stem	Leaf
7	0 0 1 2 4
8	1 3 5 5 6 8
9	2 3 4 7 7
10	1 4 4 6
11	0 0 1

There are 23 data points. The smallest is 700 and the largest is 1110, so the range is $1110 - 700 = 410$. The median is the point in the $\frac{23+1}{2} = 12^{\text{th}}$ position, which is 920.

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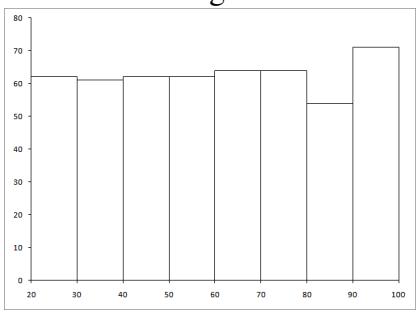
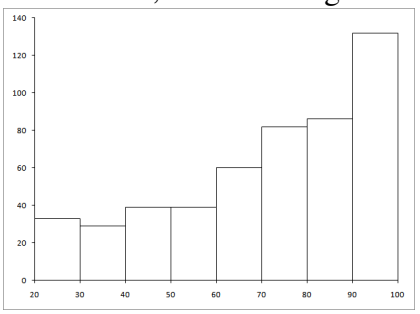
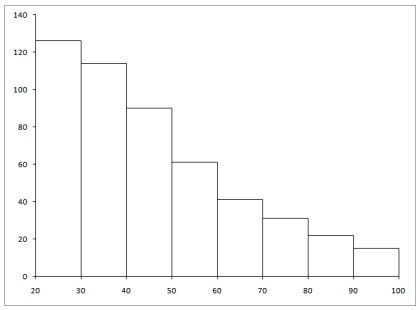
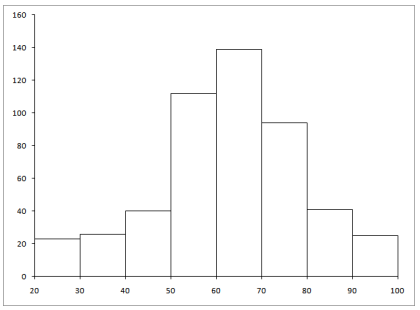
34. Mark each histogram as uniform, symmetric, bimodal, skewed left, or skewed right.









Top left: uniform. Top right: skewed left. Bottom left: skewed right. Bottom right: symmetric.

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35. Find the mean, median, and mode of the data represented in the following frequency table.

Interval	Frequency
0–12	10
12–24	8
24–36	7
36–48	6
48–60	4

For the mean, compute the midpoint of each interval, multiply each by the number of data points in the interval, sum, and divide by the number of data points:

$$\frac{6\cdot 10+18\cdot 8+30\cdot 7+42\cdot 6+54\cdot 4}{35}=25.2$$

As there are 35 data points, the median will be the one in the $\frac{35+1}{2}=18^{\text{th}}$ position. We count the number in each interval, starting at the one with the smallest data values, until we reach 18, and use the midpoint of the interval containing the 18^{th} point as the median. Thus the median here is 18.
The modal class is the one with the most data points, in this case the very first interval. We take the midpoint of that interval for the mode, so the mode is 6.

_____36. Xavier, Yvonne, and Zenobia are running in an election in which the voting method is plurality. If Xavier gets 165 votes and Yvonne gets 414 votes, how many votes does Zenobia need to win?

(a) 580

(b) Zenobia cannot win this election

(c) 166

(d) 415

415. She needs only one vote more than anyone else.

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37. A club with ten members used a Borda count to elect its president this year. Rankings were as follows.

Member	1	2	3	4	5	6	7	8	9	10
Ranking	ACB	BCA	CBA	BCA	CBA	ACB	BCA	CBA	BAC	ACB

How many points did candidate A receive?
$3+1+1+1+1+3+1+1+2+3=17$

_____38. Which of the following describes the Condorcet Criterion of social choice theory?

(a) If a candidate receives a majority of the votes, that candidate should win.

(b) If candidate A wins a first election, then some candidates are removed and a second election is held, then candidate A should win the second election.

(c) A candidate who wins a first election and gains additional support (without losing any of his original support) should win a second election.

(d) If a candidate is favored when compared one-on-one with every other candidate, then that candidate should win.

(d)

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39. An election with three candidates gave the following results.

Ranking	ABC	ACB	BAC	BCA	CAB	CBA
Voters	13	21	32	19	15	10

Who is the Condorcet candidate?

There are 110 voters altogether. A is preferred to B by $13+21+15=49$ voters, so B is preferred to A by 61 voters; thus, B could be the Condorcet candidate, but A cannot be. B is preferred to C by $13+32+19=64$ voters. B beats both A and C one-to-one, so B is the Condorcet candidate.