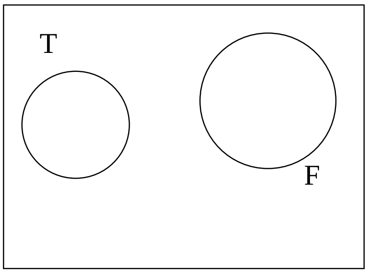
Section 2.6 The Number of Elements in a Finite Set

Recall that if A is a finite set then $n(A)$ is the number of elements in A.

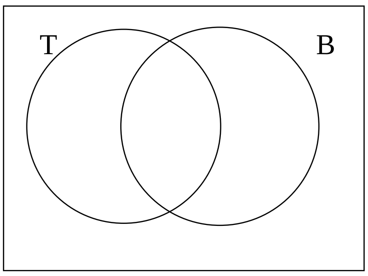
If two finite sets A and B are disjoint, then to count the number of elements in $A \cup B$ we can just count the number in A and the number in B and add: $n(A \cup B) = n(A) + n(B)$.

E.g., given a standard deck of cards (*i.e.*, a bridge, or poker, deck), let $T = \{\text{tens}\}$ and $F = \{\text{face cards}\}$. Then $T \cap F = \emptyset$, so $n(T \cup F) = n(T) + n(F) = 4 + 12 = 16$; that is, there are 16 cards that are either tens *or* face cards.



This procedure does not work when A and B have elements in common.

E.g., given a standard deck of cards, let $T = \{\text{tens}\}$ and $B = \{\text{black cards}\}$. How many cards are either a ten or a black card? There are four tens and twenty-six black cards; does this mean that there are $4 + 26 = 30$ cards that are one or the other? No: we've counted the two black tens twice. If we subtract two, then, we'll get the correct number, which is 28.



This is true in general. For any finite sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

E.g., In a standard deck, how many cards are either red cards or face cards? Let $R = \{\text{red cards}\}$ and $F = \{\text{face cards}\}$; then we are being asked for $n(R \cup F)$. There are 26 red cards, twelve face cards, and six cards that are both red and face cards, so $m(R \cup F) = n(R) + n(F) - n(R \cap F) = 26 + 12 - 6 = 32$.

Surveys II

E.g., A survey of 100 coffee drinkers found that 70 take sugar, 60 take cream, and 50 take both. How many take sugar or cream? (Note that this means *sugar or cream or both*.) [Formula, then Venn diagram]

E.g., A survey of 100 students gave the following information:

45	are taking	Math	(M)
40	are taking	Computer Science	(C)
41	are taking	Statistics	(S)
15	are taking	Math and Stat	
18	are taking	Math and CS	
17	are taking	Stat and CS	
7	are taking	all three	

How many are taking: (a) only Math?

(b) Math and CS but not Stat?

(c) exactly two of the three subjects?

(d) none of the three?

E.g., A survey of 103 drivers gave the following information. Two said that they had driven only Fords. Eighty-three said that they had driven Chevies, and seventy-three of those had also driven Toyotas. Eight people had driven both Fords and Chevies but not Toyotas. Seventy-five people had driven both Fords and Toyotas, and seventy of those had also driven Chevies. One driver had never driven any of the three. How many people had driven exactly two of the three brands?

Note. Usually when you're asked for the number of objects that are one thing *or* another, you're looking for the cardinality of a union of two sets.