

# Down on Sue's Farm

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**I. UNIT OVERVIEW & PURPOSE:**

On Sue's farm, she is presented with several real world scenarios that Sue needs to solve. She has been faced with different problems and needs solutions to each individual problem. This unit has five lessons that pose different real world situations that she needs to solve by making cost effective decisions.

Students are going to be guided through the proposed scenarios using the handouts provided to solve the problems the way Sue wants. Each of the five lessons incorporates concepts of surface area, volume, or both for three-dimensional objects such as sheds, hay bales, storage boxes, silos, and pools. Each lesson gets more rigorous from one to the other. Prior knowledge gained from each lesson will be needed to help students work comfortably and successfully through each subsequent lesson.

**II. UNIT AUTHOR:**

Shauna Knarr, Massaponax High School, Spotsylvania County  
Julia Schiesser, Massaponax High School, Spotsylvania County  
Monica Tomasik, Thomas Dale High School, Chesterfield County

**III. COURSE:**

Mathematical Modeling: Capstone Course

**IV. CONTENT STRAND:**

Algebra, Geometry and Data Analysis

**V. OBJECTIVES:**

- Find the surface area of a 3-dimensional shape.
- Use surface area to determine the amount of paint needed to cover the surface.
- Use knowledge of area to estimate the amount of materials needed to cover a surface.
- Use geometric skills in real world situations.
- Find the volume of a cylinder.
- Use data from a table to find an appropriate regression model.
- Make predictions using a regression model.
- Solve algebraic equations and cost analysis.
- Convert values using given conversion factors.
- Find volume of rectangular prisms.
- Determine most cost effective scenarios related to volume.
- Determine most available volume for different size rectangular prisms.

**VI. MATHEMATICS PERFORMANCE EXPECTATION(s):**

MPE 2: The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models.

MPE.6 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

MPE 12: The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction

**VII. CONTENT:**

This unit will also address problems surrounding the topic of cost analysis. This idea will be addressed throughout the entire unit.

**VIII. REFERENCE/RESOURCE MATERIALS:**

Classroom set of Geometry SOL formula sheets

Classroom set of graphing calculators (preferably TI-Nspire)

Classroom set of worksheets (each activity has a worksheet attached)

**IX. PRIMARY ASSESSMENT STRATEGIES:**

Assessments for each unit are included below. Assessments will be based on the student's work and written responses gathered from the worksheet. Additional assessment questions will be asked in some lessons. The students may also receive a participation grade based on the teacher's observations.

**X. EVALUATION CRITERIA:**

A key for each lesson's worksheet is included below. Appropriate weights for each response are also included.

**XI. INSTRUCTIONAL TIME:**

Lesson 1: 50 minutes

Lesson 2: 60 minutes

Lesson 3: 70 minutes

Lesson 4: 50 minutes

Lesson 5: 50-60 minutes



# Lesson 1: Painting Sue's Shed



## Strand

Algebra & Geometry

## Mathematical Objective(s)

Students will:

- Find the surface area of a 3-dimensional shape.
- Use surface area to determine the amount of paint needed to cover the surface.

This lesson will develop the student's ability to use geometric skills in real world situations they may encounter after high school.

## Mathematics Performance Expectation(s)

- MPE.6 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

## Related SOL

- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
- A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.

## NCTM Standards

- Analyze properties and determine attributes of two- and three-dimensional objects
- Make decisions about units and scales that are appropriate for problem situations involving measurement
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

## Materials/Resources

- A classroom set of calculators
- A classroom set of the attached worksheet: Painting Sue's Shed.

## Assumption of Prior Knowledge

- Students will need to know how to find that area of a rectangle and triangle.
- Students will need to know how to convert from inches to feet and vice versa.
- Students will express ideas about area, using such language as length, width, height, area, rectangle, and triangle.

- Students should be operating on the Abstraction level on the Van Hiele scale with respect to the concept of rectangles, triangles, and area.
- Students may have trouble interpreting and labeling the figure provided. They may also have a hard time finding the area of the paintable region of the shed.
- This lesson is the first of five lessons on surface area and volume. This lesson addresses surface area, followed with cost analysis.
- Students may face similar real life scenarios to the one in this lesson, especially if they became a homeowner, contractor, or worked in construction.

### **Introduction: Setting Up the Mathematical Task**

- In this lesson, you will investigate how to find the surface area of a shed. You will use the surface area to determine the amount of paint needed to paint the shed.
- The teacher will introduce the scenario by presenting the following problem to the students (project scene on whiteboard).

Sue lives on Circle Jay Farm. Several of Sue’s friends have been evacuated from the shore due to Hurricane Ezekiel, so she invited them to stay with her for a couple of weeks. Sue wants to impress them by putting a fresh coat of yellow paint on her shed. Sue wants to spend the least amount of money as possible on her project. If you are given the dimensions of the shed and the cost of the paint, can you figure out how much paint Sue needs to buy?

Ask the students to discuss the following:

- While looking at Sue’s shed, what will you need to use to solve her problem? (small group discussion for 5 minutes; large group discussion to follow for 5 minutes)
- The students might conclude that they need to find the area of rectangles and triangles, find the sum of all the areas while subtracting out areas of windows and doors since they are not painted. We will discuss the formulas for these and write them on the board.

### **Student Exploration 1:**

The students will be provided with a worksheet to solve the painting shed problem. The students will use their knowledge of surface area to complete the problem in small groups of 2-3 students. (30 minutes)

Small group work: the students will work through the provided worksheet on painting a shed (worksheet and answer key provided).

## Student/Teacher Actions

- Each student in the group is expected to participate. Every student will show their work and provide answers on a separate worksheet.
- The teacher will walk around the room and provide feedback/assistance as needed. The teacher will also correct any misconceptions the students may have. For example, some students may have forgotten to subtract the windows and doors.
- The teacher may have to assist the students as necessary. For example, the students may struggle in finding the area of rectangles and triangles found on the shed. Since the dimensions are provided in both inches and feet, the teacher may have to assist the students with conversions also.
- The teacher can represent this problem on a dry erase board, chalk board, or Smart Board to facilitate class discussion.
- When the students have finished working, they will share their findings with the whole class. Each group will explain how they solved the problem and present their final answer.

## Monitoring Student Responses

- While the students work through the worksheet they are expected to show their work. This will represent their mathematical thinking and understanding.
- The teacher will monitor the group work to guarantee that each student is working. Each student is expected to complete the entire worksheet.
- If a group of students is still experiencing difficulty after the teacher's initial assistance, then the teacher can do the following:
  - Clarify the directions of the problem
  - Assist the students in labeling their diagram
  - Assist the students in converting their measurements from inches to feet and vice versa
- At the conclusion of the lesson, the students are expected to informally summarize their findings to the class. It will take approximately 10 minutes for the students to discuss their ideas and reasoning with the whole class.
- To connect this lesson to real life situations the teacher could ask "When might you apply the ideas gathered from this lesson in the future?" The teacher could ask students who finish early to write up their response to this question.
- The teacher will provide feedback to the students and have evidence of their knowledge through the assessment.

## Assessment

- Students will be assessed on their completion of the attached worksheet. Each student is expected to turn in their own worksheet based on their group's findings. Students will

receive partial credit for incorrect responses if they show the work that goes along with the problem.

- The students will also receive a group participation grade. This grade will be based on the teacher’s observation of the group and feedback provided by the group based on the following rubric.

Group Participation Rubric

Member Name	<u>Contribution:</u> Provided useful ideas and relevant information.	<u>Working with Others:</u> Listened, shared, and worked well with peers.	<u>Focus:</u> Stayed focused on the task and what needed to be done.	POINTS EARNED
1.	3 2 1 0	3 2 1 0	3 2 1 0	
2.	3 2 1 0	3 2 1 0	3 2 1 0	
3.	3 2 1 0	3 2 1 0	3 2 1 0	

- The teacher will provide feedback to the students and have evidence of their knowledge throughout the assessment.

### Extensions and Connections (for all students)

Give the students the following assignment as an extension:

- Choose a room in your house that you want “paint.” Measure the room and find the dimensions of the paintable region. Determine the amount of paint you will need to purchase and the total cost.
- What if you wanted to apply two coats of paint?

### Strategies for Differentiation

- Students may need redirection throughout the lesson.
- Preferential seating may be needed for students with visual limitations. A braille copy of instructions and an elevated version of the diagram may be needed.

\*Painting Sue’s Shed Worksheet on the following page.



## Painting Sue's Shed Worksheet



Sue lives on Circle Jay Farm. Several of Sue's friends have been evacuated from the shore due to Hurricane Ezekiel, so she invited them to stay with her for a couple of weeks. Sue wants to impress them by putting a fresh coat of yellow paint on her shed. Sue wants to spend the least amount of money as possible on her project. If you are given the dimensions of the shed and the cost of the paint, can you figure out how much paint Sue needs to buy?

1. Use the following dimensions to label the shed:

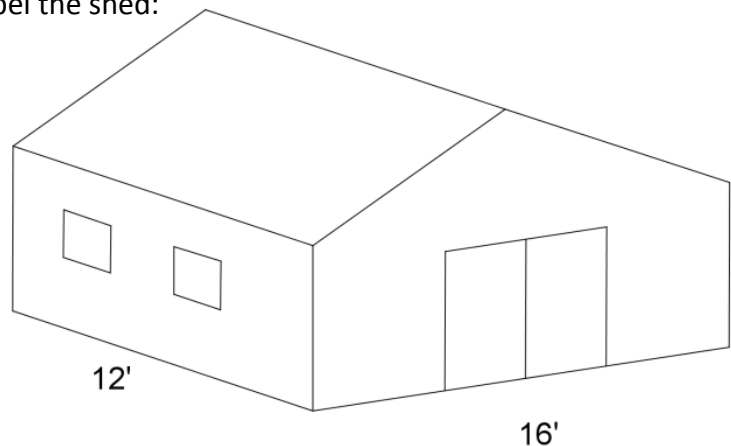
The shed is 16 ft. x 12 ft.

Door opening is 5 ft. 11in. x 5 ft. 4 in.

4 windows 2 ft. x 4ft.

Wall height 6 ft. 3 in.

Peak Height 10 ft. 8in.



2. What is the total surface area of this shed that Sue needs to paint? Give your answer in square feet (round your answers to two decimal points). Remember, we don't paint windows, doors, or the roof. Show all work!
  3. Sue has several options for sizes of paint cans that she can buy:
    - 1-Quart of paint covers about 75 sq. ft. and costs \$6
    - 1-Gallon of paint covers about 300 sq. ft. and costs \$20
    - 3-Gallons of paint covers about 900 sq. ft. and costs \$35
- a) How much coverage per dollar do you get for each option? Which is the "best value"?

- b) Since Sue knows how much area she needs to cover on her shed (one coat), which paint can(s) should she buy? Remember, you need to cover the whole area with as little left over paint as possible all while keeping the costs low.

\_\_\_\_ 1-Quart can(s) + \_\_\_\_ 1-Gallon can(s) + \_\_\_\_ 3-Gallon can(s) = Cost \_\_\_\_\_

- c) Was the “best value” paint also the best choice for Sue? Explain.
- d) Will Sue have enough paint left over to paint the doors to the shed also? If not, how much more will she need to buy? What will this bring her total cost to?



\*Answers to Painting Sue's Shed worksheet

2) 357.15 sq. ft.

3) a) 1 – Quart \$ .08 per sq. ft.    1 – Gallon \$ .067 per sq. ft.    3 – Gallon \$ .039 per sq. ft.

3 – Gallon is the best value

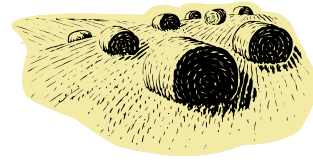
b) 1 1-Quart can(s) + 1 1-Gallon can(s) +        3-Gallon can(s) = Cost \$26

c) No, the best value paint was too much to cover Sue's shed and would end up costing more than buying the smaller cans.

d) No, she needs to cover 388.72 sq. ft. to include the door. She would need to purchase 1 more quart of paint to cover the door, bringing her total cost to \$32.

# Lesson 2: Baling Hay on Sue's Farm

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## Strand

Geometry & Algebra

## Mathematical Objective(s)

Students will:

- Find the volume of a cylinder.
- Find the area of three different quadrilaterals.
- Set up and solve algebraic equations.
- Use conversions to calculate weight and amount of hay consumed.

The goal of this lesson is for students to be able to find the area of quadrilateral given specific facts and the volume of a cylinder. Students will also need to solve basic algebraic equations and convert data given conversion factors.

## Mathematics Performance Expectation(s)

MPE 6: The student will use the formulas for volume of a three dimensional object to solve real-world problems. The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

## Related SOL

- A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.
- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

## NCTM Standards

- Carry out simple unit conversions, such as from centimeters to meters, within a system of measurement
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

- Apply and adapt a variety of appropriate strategies to solve problems
- Analyze properties and determine attributes of two- and three-dimensional objects
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

## Materials/Resources

- Classroom set of graphing calculators
- Classroom set of supplemental handouts

## Assumption of Prior Knowledge

- Students should be comfortable finding volume of cylinders, area of quadrilaterals and solving for specific variables inside those formulas.
- Students should be operating on Abstraction level on Van Hiele scale with respect to the volume of a cylinder and area of quadrilaterals.
- When finding the volume of the cylinder, students should begin to discuss how to convert the cubic feet into weight. They may also require some guidance from the teacher. While the students are discussing this topic, the teacher should be walking around the room answering any questions the students may have.
- The students may find difficulty finding the area of the three fields with the given information. They may also struggle with comparing and converting the volume of the hay bale with the weight and discovering how many bales of hay the three fields will create.
- This lesson builds on the area of quadrilaterals and the volume of three dimensional shapes.
- This lesson examines the production of hay bales and the consumption of hay by a horse. It requires students to predict the consumption rate and predict when the hay will run out.

## Introduction: Setting Up the Mathematical Task

- In this lesson, you will investigate how to find the area of a field and volume of a cylinder. You will use this information to determine how many hay bales can be produced from a certain area.
- While looking at Sue's field, what will you need to use to solve her problem? (small group discussion for 5 minutes; large group discussion to follow for 5 minutes)

- The students might conclude that they need to find the area of rectangles, triangles, trapezoids and parallelograms. Also they will need to find the volume of a cylinder. These images will be placed up on the board along with the formulas needed to find the area and volumes. If the students do not provide these, the teacher will remind them of the formula.

### **Student Exploration 1:**

The students will be provided with a worksheet to solve the Baling Hay on Sue’s Farm Problem. The students will use their knowledge of area and volume to complete the problem in small groups of 2-3 students. (40 minutes)

Small group work: the students will work through the provided worksheet on Baling Hay on Sue’s Farm (worksheet and answer key provided). The students will discuss ideas and methods for solving the problem while they work.

### **Student/Teacher Actions**

- Each student in the group is expected to participate. Every student will show their work and provide answers on a separate worksheet.
- Teachers may want to group students based on ability or behavior. Teachers may also add jobs to the students to insure each student participates constructively.
- The teacher will walk around the room and provide feedback/assistance as needed. The teacher will also correct any misconceptions the students may have.
- The teacher may have to assist the students as necessary. For example, the students may struggle in finding the area of rectangles, triangles, trapezoids, and parallelograms found in the field. Students could also require help finding the volume of the hay bales.
- The teacher can represent this problem on a dry erase board, chalk board, or Smart Board to facilitate class discussion.
- When the students have finished working, they will share their findings with the whole class. Each group will explain how they solved the problem and present their final answer. Their results will be presented on the SmartBoard or whiteboard.

### **Monitoring Student Responses**

- While the students work through the worksheet they are expected to show their work. This will represent their mathematical thinking and understanding.
- The teacher will monitor the group work to guarantee that each student is working.
- If a group of students is still experiencing difficulty after the teacher’s initial assistance, then the teacher can do the following:
  - Clarify the directions of the problem
  - Assist the students in labeling their diagram
  - Assist the students in converting their measurements from inches to feet and vice versa

- At the conclusion of the lesson, the students are expected to informally summarize their findings to the class. It will take approximately 10 minutes for the students to discuss their ideas and reasoning with the whole class.

## Assessment

- Students will be assessed on their completion of the attached worksheet. Each student is expected to turn in their own worksheet based on their group’s findings. Students will receive partial credit for incorrect responses if they show the work that goes along with the problem.
- The students will also receive a group participation grade. This grade will be based on the teacher’s observation of the group and feedback provided by the group based on the following rubric.

### Group Participation Rubric

Member Name	<u>Contribution:</u> Provided useful ideas and relevant information.	<u>Working with Others:</u> Listened, shared, and worked well with peers.	<u>Focus:</u> Stayed focused on the task and what needed to be done.	POINTS EARNED
1.	3 2 1 0	3 2 1 0	3 2 1 0	
2.	3 2 1 0	3 2 1 0	3 2 1 0	
3.	3 2 1 0	3 2 1 0	3 2 1 0	

## Extensions and Connections (for all students)

Give the students the following assignment as an extension:

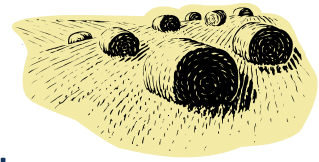
Research and answer the following questions:

- In this problem, area was measured in square feet. Usually property is measured in acres. Find out how many square feet are in an acre.
- How many acres of land make up Sue’s field?
- How many bales of hay does one acre produce

## Strategies for Differentiation

- Students may need redirection throughout the lesson.
- Preferential seating may be needed for students with visual limitations. A braille copy of instructions and an elevated version of the diagram may be needed.

\* Handout: Baling Hay on Sue’s Farm Worksheet on next page

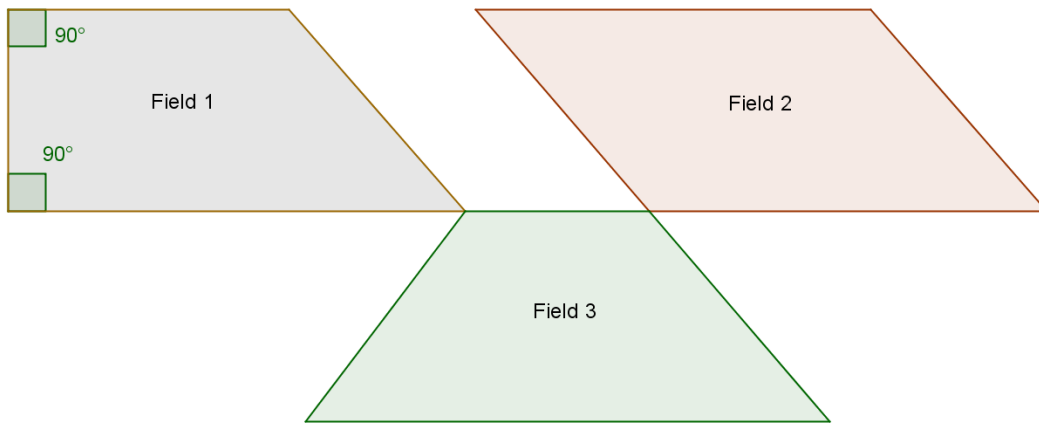


## Baling Hay on Sue's Farm Worksheet

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Sue's friend David needs hay for his horse farm. Sue has three fields of hay that she can sell to David. David only wants to buy the hay if it will be enough to last him 10 weeks. Sue's three hay fields are laid out as follows:

- The height of each field is 200 ft.
- The length across the top of field 1 is 200 ft.
- Each field has the same base of 370 ft.
- Hay field 2 is a parallelogram
- The distance across all three fields is 780 ft.



1) What is the area of each of Sue's hay fields? What is the total area of the three fields?

Field 1 \_\_\_\_\_ Field 2 \_\_\_\_\_ Field 3 \_\_\_\_\_ Total \_\_\_\_\_

2) If 10,000 square feet of field creates 3 cylindrical hay bales, how many bales will Sue's three fields create?

- 3) If each hay bale has a diameter of 6 ft. and a height of 5 ft., what is the volume of one hay bale? Round to one decimal place. Provide a sketch of the hay bale with your answer. ( $\pi = 3.14$ )
- 4) If each cubic foot of hay weighs 5 pounds (lbs.), how much does the hay bale weigh?
- 5) If David needs to feed each of his 20 horses 25 lbs. of hay per day, how much hay will David need to feed his horses for 10 weeks? Give your answer in both pounds and bales of hay.

Pounds \_\_\_\_\_

Bales \_\_\_\_\_

- 6) Will Sue's field produce enough hay to last David 10 weeks?

\*Answers to Baling hay on Sue's Farm Problem

1) Field 1 – 57,000 sq. ft.      Field 2 – 74,000 sq. ft.    Field 3 – 41,000 sq. ft.    Total – 172,000 sq. ft.

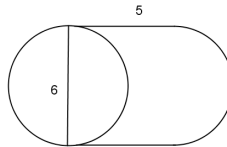
2) Approximately 51 hay bales

3) 141.4 cubic feet

4) 707 lbs.

5) 35,000 lbs and approximately 50 hay bales

6) Yes, Sue will produce 51 hay bales and David only needs 50 bales.





# Lesson 3 – Sue’s Storage Shed

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## Strand

Geometry

Algebra

## Mathematical Objective(s)

In this lesson students will be able to determine how much available space exists in a real world application by:

- Finding volume of rectangular prisms
- Determining how many smaller rectangular prisms can fit into one rectangular prism
- Finding costs of storage containers using volume of rectangular prisms
- Determining most cost effective scenarios related to volume
- Determining most available volume for different size storage containers

The specific skill taught in this lesson is maximizing volume and minimizing cost.

## Mathematics Performance Expectation

MPE 6: The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

## Related SOL

A.4 The student will solve multistep linear and quadratic equations in two variables, including:

- e) solving systems of two linear equations in two variables algebraically and graphically;
- f) solving real-world problems involving equations and systems of equations.

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

G.14 The student will use similar geometric objects in two- or three-dimensions to:

- a) compare ratios between side lengths, perimeters, areas, and volumes;
- b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;

## **NCTM Standards**

- Students should be able to analyze properties and determine attributes of two- and three-dimensional objects;
- Students should be able to explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;
- Students should be able to understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects;
- Students should be able to draw geometric objects with specified properties, such as side lengths or angle measures;
- Students should be able to use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;
- Students should be able to interpret representations of functions of two variables.
- Students should be able to write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases;
- Students should be able to carry out simple unit conversions, such as from centimeters to meters, within a system of measurement;
- Students should be able to understand and use formulas for the area, surface area, and volume of geometric figures;
- Students should be able to apply and adapt a variety of appropriate strategies to solve problems;
- Students should be able to apply and adapt a variety of appropriate strategies to solve problems;
- Students should be able to communicate mathematical thinking coherently and clearly to peers, teachers, and others.

## **Materials/Resources**

- Classroom set of Geometry SOL Formula Sheets
- Classroom set of calculator (no specified kind)
- Supplemental handout developed for lesson for every student
- Graph paper and rulers may be helpful when creating the drawings for the shed

## **Assumption of Prior Knowledge**

- Summarization of Prior Knowledge:
  - Students must be able to find the volume of a three-dimensional object.
  - Students must know how to convert from feet to inches or inches to feet (this is a choice they have to make).

- Students must know how to find cost given amount and price.
- Typical background of a student for this lesson would be a Van Heile Level 3 – Abstraction
- Students will begin to understand the importance of pricing analysis and deductive reasoning related to maximizing volume and minimizing cost.
- Students may have a hard time working with numbers that were converted to feet because they result in decimals. If students chose to convert to inches, numbers may be easier to manage.
- Relevant concepts that should have been explored prior to this lesson are volume of three-dimensional shapes, converting from unit to unit, and finding cost.
- Price analysis and volume analysis are demonstrated in this lesson.

### Introduction: Setting Up the Mathematical Task

- In this lesson, you will investigate how to find volume of rectangular prisms and use cost analysis to find the most cost effective storage boxes available for moving. Student will investigate how to maximize volume.
- Time Outline: Depends if school has 45-min periods or 90-min blocks.
  - Introduction – 10 minutes
  - Activity – 50 minutes
  - Questions at end – 10 minutes
- Introduce the task:
  - Begin by introducing the situation presented to the students.
  - This is the third lesson of a unit that builds on three-dimensional concepts and students will be challenged more as they continue through unit.
  - This situation presents itself when the students read the introduction on the handout.
  - Students should be able to understand the task at hand after reading the introduction situation.
- Groups of 2 or 3 students – These can be picked by teacher using numbering/counting off for random selection or teacher could chose the 2 to 3 students to work together.
- The handout guides the students through the activity prompting them with questions that they need to be answering related to the material required to understand the concepts involved.
- Teacher will walk around helping students with their thinking process. Teacher can ask questions to help if students are moving in the wrong direction, but with the format of the handout, most students should be able to figure out how to solve the different scenarios for moving and storage costs.

- Students will realize how important cost analysis is when making decisions on quantity of items even when certain prices seem cheaper to buy per unit, but in fact are not cost effective depending on a combination of factors.

### Student Exploration 1:

#### Small Group Work

Students will work through the handout investigating different volume scenarios of rectangular prisms centered on cost effective decisions.

#### Student/Teacher Actions:

- Students should be completing the handout by calculating the volumes of possible boxes needed for storage. They need to be determining how the different possibilities affect the roommate's decision on what they need to buy for their move so they can fit the most boxes in the shed for the least cost.
- Teachers should be making sure each student's work is correct and accurate while they are working. If students are struggling with concepts, the teacher should help guide them in the right direction.
- Handout Answers, converting everything to inches, attached at end of worksheet.

#### Monitoring Student Responses

- Expectations:
  - Students will complete the handout.
  - Students will communicate with their partner and help each other problem solve;
  - Teacher and students will continue to dialogue throughout the exercise to reduce the amount of confusion on the mathematical concepts needed to solve the problem;
  - Teacher will assist any students who have difficulties;
  - Teacher will have the students who have completed their assignments work with groups who are struggling.
- At the end of the exploration and assessment time, the teacher will collect the handouts that have been filled out by the students.
  - Once the groups are finished, the teacher should have the students write their solutions and explanations for their work. The teacher will then discuss the solutions to the entire class.

### Assessment

- Students will be assessed on their completion of the attached worksheet.

- Each student is expected to turn in their own worksheet based on their group's findings.
- Students will receive partial credit for incorrect responses if they show the work that goes along with the problem.
- Students will answer the assessment questions below. See worksheet.

### **Extensions and Connections (for all students)**

- Give the students an extension problem for homework or a follow-up problem. Teacher can determine what this could be based on how the lesson went.
  - If students struggled with finding volume, the teacher could give out a few different scenarios for finding volume.
  - If students struggled with cost analysis, the teacher could give out a few different scenarios for evaluating cost effectively in order to get best prices.

### **Strategies for Differentiation**

- Students who are visual learners should sketch the shed for each scenario presented and label the relevant information.
- Graph paper and rulers may be helpful when creating the drawings for the shed.
- Teacher could provide a drawing of the shed for students who are having a hard time with finding the volume of the shed. This can also be done for the boxes if they are struggling with figuring out how many boxes can fit into the shed.

# Sue's Storage Shed Worksheet

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Sue has graciously invited three of her grad school buddies to come and stay with her because they are in a bind and need somewhere to live for a month or so. These three guests have been roommates for a year but they have to move out of their apartment before the end of the month, which is two weeks away, because their lease is up.

Unfortunately for them, Hurricane Ezekiel is coming straight for their town in less than a week and is determined to be a category V hurricane. Since they are already moving to Sue's farm the week after next, they decide it would be better to get out while they can and make their move to the farm.

All three roommates' belongings are going to be boxed up and stored in Sue's shed while they live there. They need to find out possible pricing for storage boxes, how many boxes they need to buy in order to fit everything into Sue's shed, and then pick the best deal for their situation.

Sue's shed is 16' wide and 12' long with a maximum storage height of 6'. It has some space to spare above that 6' so their boxes should not be crammed into the shed. The three roommates choose two places to search for storage boxes: the local UPS store and the BJ's wholesale store in town.

## **PART I:**

<b><u>Options at UPS store:</u></b>	<b><u>PRICE per box</u></b>
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Medium Boxes: 18" x 18" x 16"	\$2.99 each
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Extra Large Boxes: 22" x 22" x 22"	\$4.99 each
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UPS will only allow you to buy either only medium boxes or only extra large boxes because there is a shortage on boxes because of the hurricane coming. You cannot buy a combination of boxes either. The three roommates just happen to be math teachers and know that they will figure out the best deal available to them. So they figure out everything needed to determine this! Remember to keep all units the same throughout the lesson and determine what you would prefer to use, inches or feet, throughout the lesson.

1) Find the total volume (cubic units) of Sue's shed.

**Answer the following questions with all work shown:**

<b>Medium Boxes</b>	<b>Extra Large Boxes</b>
2) How much volume is in one medium box?	3) How much volume is in one extra large box?
4) How many medium boxes will fit into the shed?	5) How many extra large boxes will fit into the shed?
6) How much volume do these boxes take up?	7) How much volume do these boxes take up?

8) Which is the better deal for these three roommates as far as availability of space? Basically, which boxes would take up more storage space so they could fit all of their belongings?	
9) How much does it cost to buy all medium boxes to fit inside the shed?	10) How much does it cost to buy all extra large boxes to fit inside the shed?
11) Which is the better deal for these three roommates as far as cost?	

**PART II:**

After figuring out the options at the UPS store, the three roommates decide to look one more place before making their purchase. The local wholesale store, BJ's, sells packages of eight boxes with six medium sized boxes and two extra large size boxes. The dimensions of each box are the same as the UPS store.



**Only Option at BJ's store:** **PRICE per package**

8 boxes per package: 6 Medium boxes & 2 XTL boxes \$24.00



<p>1) Determine how much each box costs based on the information below. You will need to set up a system equations for this problem.</p> <ul style="list-style-type: none"> <li>If the package says that together one medium and one extra large box costs \$7, find the cost of each box.</li> </ul>	
<p>2) What is the price per medium box?</p>	<p>3) What is the price per extra large box?</p>
<p>4) How much volume is in one package sold at BJ's?</p>	
<p>5) How many <u>packages</u>, not boxes, can fit into the shed?</p>	
<p>6) How much volume is in the total number of packages?</p>	
<p>7) How much does it cost to buy storage boxes from BJ's?</p>	

**Assessment Questions:**

- 1) Is it financially better to buy at BJ's wholesale store if only concerned about cost? Explain.
  
- 2) Is it financially better to buy at the UPS Store if only concerned about cost? Explain.
  
- 3) Is it better to buy at the UPS Store if only concerned about fitting all of their belongings into the shed? Explain.
  
- 4) Is it better to buy at BJ's wholesale store if only concerned about fitting all of their belongings into the shed? Explain.
  
- 5) Which scenario, Medium only, Extra Large only, or mixed package, should be eliminated from consideration? Why?
  
- 6) Which scenario, out of all three possible situations, would be the most cost effective AND volume effective for the three roommates? Explain.

**Handout answers:**

**PART I:**

- 1) 1,990,656 in<sup>3</sup>
- 2) 5184 in<sup>3</sup>
- 3) 10,648 in<sup>3</sup>
- 4) 320 boxes
- 5) 144 boxes
- 6) 1,658,880 in<sup>3</sup>
- 7) 1,533,312 in<sup>3</sup>
- 8) Medium boxes take up more space to fit more belongings but cost more.
- 9) \$956.80
- 10) \$718.56
- 11) Extra Large Boxes

**PART II:**

- 1) X = medium; Y = XTL  
 $6X + 2Y = 24$   
 $X + Y = 7$
- 2) X = \$2.50
- 3) Y = \$4.50
- 4) 52400 in<sup>3</sup> in one package
- 5) 37 packages
- 6) 1,938,800 in<sup>3</sup>
- 7) \$888.00

**Assessment Questions:**

- 1) No, the cost is in between the other two cost options; \$888 is between \$718.56 and \$956.80
- 2) Yes, the lowest price is from the UPS store at \$718.56
- 3) No, both volumes at UPS store are lower than BJ's volume
- 4) Yes, BJ's volume is the closest to actual available volume in the shed.
- 5) Medium boxes only situation should be removed as an option because it's the worst from both cost and volume.
- 6) With both cost and volume factored in, BJ's would be the best buy because it has the most volume and the cost is in between the other two costs

# Lesson 4: Sue's Cement Pond



## Strand

Geometry

## Mathematical Objective(s)

In this lesson students will be able to determine how much surface area a swimming pool has:

- Using area formulas of rectangles and trapezoids
- Determining most cost effective scenarios related to surface area

The specific skill taught in this lesson is minimizing product needed to buy.

## Mathematics Performance Expectation

MPE 6: The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

## Related SOL

G.9 The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

## NCTM Standards

- Students should be able to analyze properties and determine attributes of two- and three-dimensional objects;
- Students should be able to explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;
- Students should be able to use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;
- Students should be able to carry out simple unit conversions, such as from centimeters to meters, within a system of measurement;
- Students should be able to understand and use formulas for the area, surface area, and volume of geometric figures;
- Students should be able to communicate mathematical thinking coherently and clearly to peers, teachers, and others.

## Materials/Resources

- Geometry SOL Formula Sheet
- Classroom set of calculator (no specified kind)
- Supplemental handout developed for lesson

## Assumption of Prior Knowledge

- Summarization of Prior Knowledge:
  - Student must be able to find the surface area of a three-dimensional object.
  - How to convert from inches to feet
- Typical background of a student for this lesson would be a Van Heile Level 3 – Abstraction.
- Students will express ideas about surface area using language such as length, width, height, area, rectangle and trapezoid.
- Students may have a hard time working with the converted numbers depending on which way they were converted.
- Relevant concepts that should have been explored prior to this lesson:
  - Finding surface area
  - Converting from one unit to another
  - Relating amount of sealant to buy to the area needed to cover
- Price analysis is demonstrated in this lesson.
- This lesson builds on the ideas of surface area addressed in previous lessons.

## Introduction: Setting Up the Mathematical Task

- In this lesson, you will investigate how to find surface area of rectangular prisms and trapezoidal prisms and use cost analysis to find the most cost effective scenario.
- Time Outline:
  - Introduction – 10 minutes
  - Activity – 40minutes
  - Questions at end – 10 minutes
- Introduce the task:
  - Begin by introducing the situation presented to the students.
  - This is the second to last lesson of a unit that builds on three-dimensional concepts and students will be challenged more as they move from one lesson to another.
  - This situation presents itself when the students read the introduction on the handout.

- Students should be able to understand the task at hand after reading the introduction situation.
- Groups of 2 or 3 students. Students may be grouped at the teacher’s discretion.
- The handout guides the students through the activity prompting them with questions that they need to be answering related to the material required to understand the concepts involved.
- Teacher will walk around helping students in their approach to thinking. Teacher can ask questions to help if students are moving in the wrong direction, but with the format of the handout, most students should be able to figure out how to solve the different scenarios surface area of the pool.
- Students will need to ask the teacher about accuracy of their final answer in order to know if they’re on the right track. As the teacher walks around the room, they will offer suggestions to those students who may be doing the problem incorrectly.

## Student Exploration 1:

### Small Group Work

Students will work through the handout drawing 3-D pictures of individual sections of the pool to determine the surface area of the pool.

### Student/Teacher Actions:

- Students should be working through the handout figuring out surface areas of the pool’s sections.
- Teachers should be making sure the student’s work is correct and accurate while they move through the worksheet. If students are struggling with certain concepts, the teacher needs to help guide them in the right direction.
- Handout Answers: (Using feet for calculations)

## Monitoring Student Responses

- Expectations:
  - Students will complete the handout.
  - Students will communicate with their partner and help each other problem solve;
  - Teacher and students will continue to dialogue throughout the exercise to reduce the amount of confusion on the mathematical concepts needed to solve the problem;
  - Teacher will assist any students who have difficulties;

- Teacher will have the students who have completed their assignment work with groups that are struggling.
- Students who finish early could also move onto the extension problem.
- At the end of the exploration and assessment time, the teacher will collect the handouts that have been completed by the students.
  - Once the handouts are collected, the teacher should go over the solutions having the students guide him/her through the questions.

## Assessment

- Students will be assessed on their completion of the attached worksheet.
- Each student is expected to turn in their own worksheet based on their group's findings.
- Students will receive partial credit for incorrect responses if they show the work that goes along with the problem.

## Assessment Questions:

Students will use the sheet to answer questions about the swimming pool problem.

## Extensions and Connections (for all students)

- Give the students an extension problem for homework:
  - What is the volume of the swimming pool? Give your answers in cubic feet.
  - Look up the conversion from cubic feet to gallons. How many gallons of water do you need to fill the pool?

## Strategies for Differentiation

- Diagrams of the pool are provided for students who are more visual learners. Additional sketches of the pool for each scenario presented should also be used by students.
- Graph paper and rulers may be helpful when creating the drawings for the pool.

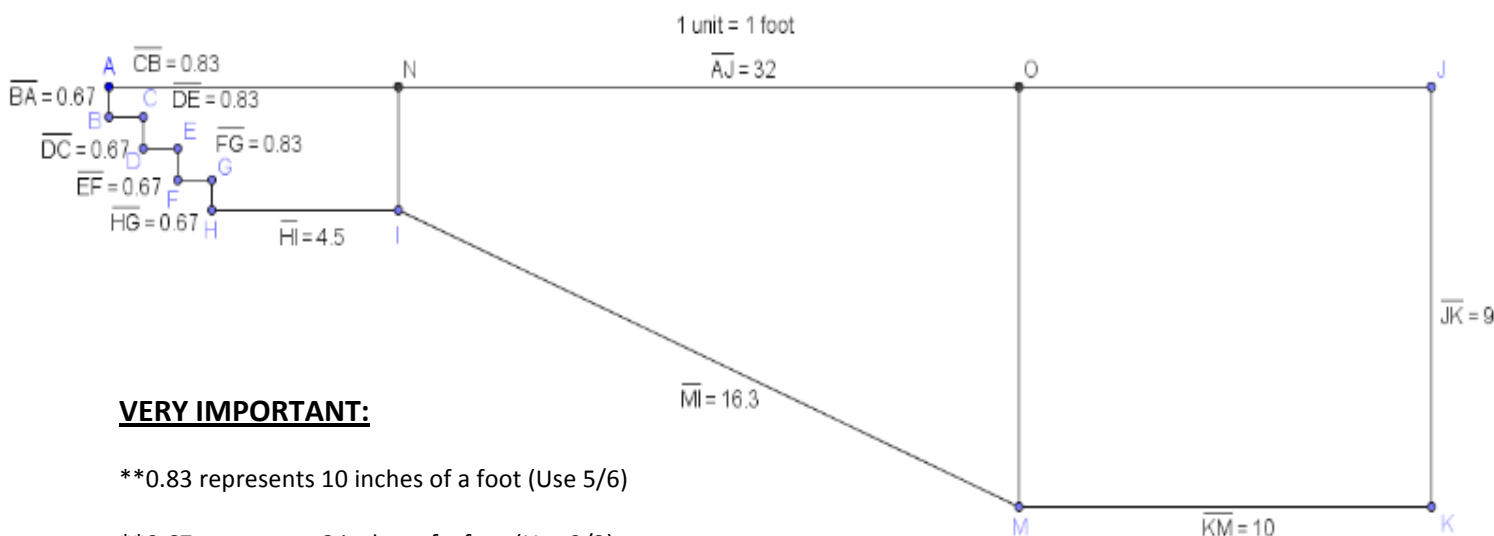
\*Worksheet: Sue's Cement Pond on following page

# Sue's Cement Pond Worksheet



Sue had a pool put in on her property, but to save some money she decided to paint the surface of the pool with a sealer herself instead of using the pool company. With the help of her new roommates, Sue was confident they would be able to complete the project within a few days. However, before they could start the job, there were several mathematical concepts to consider. Let's find some information in order to determine how much sealer is needed to buy!

## Side View of Sue's pool:



### **VERY IMPORTANT:**

- \*\*0.83 represents 10 inches of a foot (Use 5/6)
- \*\*0.67 represents 8 inches of a foot (Use 2/3)
- \*\* NO = 15
- \*\* The pool is 16 ft wide all the way across

### **Directions:**

Find the surface area of all 3-D shapes in the pool keeping in mind that pools are not closed shapes so there is no top to any of the 3-D shapes given. Determine how you will get the total surface area of the pool in order to seal the pool before adding water. Show all steps and decisions made to get to your final answer. Once you have your solution and all work shown, ask your teacher if your Surface Area answer is correct.

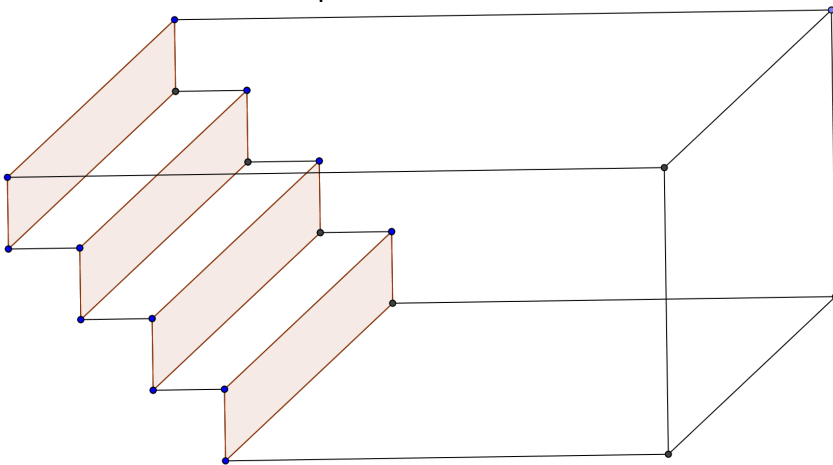


### Section By Section

Being that this problem is quite difficult, you have been provided with one of the 3-D structures of the pool. It's not drawn to scale. This picture has been given to you to help you think about every section of the pool that you need to consider in order to determine the surface area of this pool. You will have to draw your own 3-D sections for every section after the steps section. To start, add in all numbers needed from the above picture onto this picture to find the surface area of the stairs section.

### Surface Area of Stairs, Walls, and section to the right of the stairs: Hardest Part!

This picture has been provided for you because it may be difficult to draw the stairs in three dimension however you may need to draw pictures for the other sections yourself. Also show all work! Do not forget all sides of the pool that exist because some sides in this picture are invisible in this picture!



\*This is a possible picture you can use to help figure out the surface area of the stairs and walls as well as the region to the right of the stairs.\*

Remember to label and divide the picture into sections to figure out all surfaces area. Show all work!

1) Total Surface area from picture above  
(stairs and to the right of the stairs):

---

Now move onto the rest of the pool, doing section by section. Draw necessary pictures in 3-D to help figure out what to do.

**Ramp section into deep end:**

Draw picture and show work:

2) Total Surface Area of all faces:

\_\_\_\_\_

**Deep End Section:**

Draw picture and show work:

3) Total Surface Area of all faces:

\_\_\_\_\_

4) Add all three sections of area together. Total Surface Area of Pool: \_\_\_\_\_

5) Each gallon of sealant covers  $250 \text{ ft}^2$  to  $300 \text{ ft}^2$ . How many gallons of sealant must Sue purchase? \_\_\_\_\_

Lesson 4 answers to worksheet:

- 1)  $(268/3) \text{ ft}^2 + 96 \text{ ft}^2 =$  total surface area of both regions from first picture.
- 2)  $175 \text{ ft}^2$
- 3)  $340 \text{ ft}^2$
- 4)  $(2101/3) \text{ ft}^2$
- 5) 3 gallons of sealant

# Lesson 5: Sue's Dairy Cows

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## Strand

Geometry & Algebra



## Mathematical Objective(s)

Students will:

- Find the volume of a cylinder.
- Use data from a table to find an appropriate regression model.
- Make predictions using a regression model.
- Solve algebraic equations.
- Convert values using given conversion factors.

The goal of this lesson is for students to be able to find the volume of a cylinder and analyze given data to make predictions about the consumption rate of corn by dairy cows.. Students will also need to solve basic algebraic equations and convert data given conversion factors.

## Mathematics Performance Expectation(s)

MPE 2: The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models.

MPE 6: The student will use the formulas for volume of a three dimensional object to solve real-world problems.

MPE 12: The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction

## Related SOL

- A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.
- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

- AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.
- AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

## **NCTM Standards**

- Carry out simple unit conversions, such as from centimeters to meters, within a system of measurement
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders
- Apply and adapt a variety of appropriate strategies to solve problems
- Analyze properties and determine attributes of two- and three-dimensional objects
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

## **Materials/Resources**

- Classroom set of graphing calculators
- Classroom set of supplemental handouts

## **Assumption of Prior Knowledge**

- Students need to have a working knowledge of regression models, specifically linear regression
- Students should be comfortable finding volume and solving for specific variables inside the volume formula
- Students should be operating on Abstraction level (3) on Van Hiele scale with respect to the volume of a cylinder.
- When finding the volume of the cylinder, students should begin to discuss how to convert the cubic feet into gallons of corn. They will also need to understand the relationship between the volume and the height of the cylinder.
- The students may find difficulty when predicting when the cylinder will be empty as they will have to set the regression model to zero and solve for the constant. This will require

them to solve a linear equation. They may also struggle with comparing and converting the volume of the silo into gallons of corn.

- Prior to this lesson students should be familiar with volume of a cylinder.
- This lesson builds on the volume of three dimensional shapes and making predictions given data in a table.
- This lesson examines the consumption of corn on a dairy farm. It requires students to predict the consumption rate and predict how much corn is required to feed a specific amount of dairy cows.

## Introduction: Setting Up the Mathematical Task

- “In this lesson you will be finding volume of a cylinder and a regression model based off of given data. You will be making predictions based on your conclusions and solving a real world problem.”
- Introduction: 10 min  
Exploration: 30-40 min  
Assessment: 10 min
- Pass out the attached student handout and read the first paragraph to the students. Have the students plot the data on graph paper and ask them what kind of regression they would need to use. Briefly explain the task at hand and give the students a timeframe to complete the worksheet.
- Students will get in groups of 2 or 3 to complete the handout. You may have them chose their own groups or assign them to a group.
- While the students are working, the teacher will walk around the room answering questions and help any of the groups that are struggling.
- Students will be turning in their work to be checked by the teacher.

## Student Exploration 1:

**Small Group Work:** Students will complete the attached worksheet in groups of 2 or 3 on the provided handout.

### Student/Teacher Actions:

- Students should be working in groups to answer the questions on the handout.
- The teacher should be walking around the room answering questions and guiding the students through any difficulties they may be having with the lesson.
- The students may not realize that they will have to convert cubic feet into gallons and/or they may have difficulty with the conversion. They may also struggle with setting up the appropriate equations and solving for the missing information.
- The students can use the TI-Nspire to create a graph of the data and find the regression model.

## Monitoring Student Responses

- Expectations:
  - students will complete the activity per the handout;
  - students will communicate with those in their group and help each other problem solve
  - teacher and students will continue to dialogue throughout the exercise to reduce the amount of confusion on the mathematical concepts needed to solve the problem;
  - teacher will assist any students who have difficulties;
  - teacher will have the students who have completed their assignment work with groups that are struggling.
- At the end of the exploration and assessment time, the teacher will collect the handouts that have been filled out by the students.
  - Once the handouts are collected, the teacher should go over the solutions having the students guide him/her through the questions.

## Assessment

- Describe and attach the assessments for each lesson objective.
  - **Questions**
    - How many gallons of corn does Sue's silo hold?
    - What may happen to make your calculations false?
    - What if 5 cows die in July and 2 die in September? How much corn will Sue's cows consume at the end of a year?
  - **Evaluation**
    - 48,126.18 gallons
    - Some of the cows could stop producing milk (need less feed) or get sick/die
    - 33,886.2335 gallons

## Extensions and Connections (for all students)

- As a bell-ringer, give the students the following problem: If Sue's silo is a rectangular prism with a square base that has an area of 169 square feet and holds 40,000 gallons of corn, how high is the corn in the silo? (Remember that 1 cubic foot contains 7.48 gallons of corn)  
\*\*answer – 31.6426 feet high

- If Sue started her farm with 36 female cows, and each cow produces one calf a year. In 15 years, what would the graph of her yearly corn consumption make? What type of regression model would you use to describe the data?

**\*\*answer – exponential function**

### **Strategies for Differentiation**

- Students who are more visual learners should sketch the cylindrical silo and label the relevant information.

**\* Handout: Sue's Dairy Cows  
Located on the following page**



# Sue's Dairy Cows Worksheet



Sue has her corn silo filled in April. Her silo is 32 feet high as has a diameter of 16 feet. Once filled, the corn is 22 feet high inside the silo. Sue's dairy farm has six dozen lactating dairy cows and she is inexperienced at maintaining that many cows. She is not sure if she has purchased enough corn. Each month, Sue checks the height of the corn in the silo. Her data is in the table below:

April	May	June	July	August
22'	19.83'	17.64'	15.48'	13.28'

## Part I

- If April is year 0, find a regression model to fit this data. Is Sue going to run out of corn before her next scheduled delivery in a year?

Regression equation: \_\_\_\_\_

If Sue has enough corn for the year, skip to part II.

If Sue is going to run out of corn, what month will she run out? \_\_\_\_\_

## Part II

Sue is ready to order her next year's corn supply. She would like to order the correct amount of corn. Sue has a contract with her supplier for yearly shipments and ordering early will cost her more. If she orders too much, then the corn will rot before her cows can consume it and she loses money.

1. If one cubic foot contains 7.48 gallons of corn:
  - a. How much corn is consumed in a year at Sue's farm (in gallons)?
  
  
  
  
  
  
  
  
  
  
  - b. What would the height of the corn be in the silo if Sue ordered the amount of gallons in part a?



\*\*\*Answers

Part I:

Sue will run out of corn in March

Part II:

1.
  - a. 36,369.9 gallons
  - b. 24.183 feet
  - c. 3,030.82 gallons
  - d. 42.0947 gallons a month, 1.38 gallons a day
  - e. 42,426.7 gallons
  - f. 28.2103 feet