

Writing Equations from Patterns

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Grade Level: 6 – 10

Time Allotment: TWO – 50-minute class periods

Overview: This lesson is designed to introduce students to the idea of linear functions with emphasis on the algebraic representations as rules, data tables, and graphs. The mathematical notions of independent and dependent variables will be discussed as well as a review of domain and range. When introducing linear functions, it is important to emphasize that these types of functions have a constant rate of change. Whether the students are given a graph, table, or an equation, they should understand that if points lie in a line, there is a constant rate of change between the points. Students are asked to graph several sets of equations and determine a relationship between those graphs that are lines and those that are not. Students will discover patterns from the table and the plotted lines and use their new-found knowledge to write equations of lines given a table or graph. This experience will be useful when the more formal discussion of linear functions takes place. Emphasis is placed on the concepts of rate of change and the point where the line crosses the y -axis (versus the terms *slope* and *y-intercept*). This is to help students obtain a visual understanding and allow them to better make connections when the terms are introduced. Also, by having students explain what they see and how it relates to the linear equation promotes the communication of mathematical concepts.

Subject Matter: Algebra – Writing Linear Equations

Learning Objectives:

The student will be able to

- Determine if a given set of points lie in a line.
- Find a pattern on the domain and range in a table or a given set of points.
- Determine the rate of change for a line from either the table or graph.
- Determine how an equation is written from a table or graph.
- Use the TI-83 calculator to verify the equations (in table or graph form).

Pre-Requisite Knowledge:

- Identify domain and range and determine if a relation is a function.
- Make a table of values if given a function.
- Graph a set of ordered pairs.

Standards:

This lesson correlates with the following Virginia Standards of Learning:

- A.2 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Students will choose and appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.

- A.5 The student will create and use tabular, symbolic, graphical, verbal, and physical representations to analyze a give set of data for the existence of a pattern, determine the domain and range of relations, and identify the relations that are functions.
- A.6 The student will select, justify, and apply an appropriate technique to graph linear functions and linear inequalities in two variables. Techniques will include slope-intercept, x- and y-intercepts, graphing by transformation, and the use of a graphing calculator.
- A.7 The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined. The graphing calculator will be used to investigate the effect of changes in the slope on the graph of the line.
- A.8 The student will write an equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

Link to the NCTM Standards:

Understand patterns, relationships, and functions

- Represent, analyze, and generalize a variety of patterns with tables, graphs, works, and when possible, symbolic rules.
- Relate and compare different forms of representation for a relationship
- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

Represent and analyze mathematical situations and structures using algebraic symbols.

- Develop an initial conceptual understanding of different uses of variables.
- Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.
- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.

Use mathematical models to represent and understand quantitative relationships

- Model and solve contextualized problems using various representations, such as graphs, table, and equations

Analyze change in various contexts

- Use graphs to analyze the nature of changes in quantities in linear relationships.

Mathematics as communication.

- Students will reflect upon and clarify their thinking about mathematical ideas and relationships.
- The students will express and generalization discovered through investigation.

Mathematics as reasoning

- Students will make and test conjectures.

Mathematical connections

- The students will recognize equivalent representations of the same concept.

Media Components:

- Overhead projector (or projection unit with PowerPoint)
- TI-83 Graphing Calculator and view screen (or other graphing software)
- Helpful Web-Sites:
<http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=166>
“explore learning”—gizmos used to help with the following: graphing linear and non-linear equations exploring how “m” and “b” affect the graph
- <http://www.shodor.org/interactivate/activities/slopeslider/>
interactive website that helps students understand the effect of the slope on the graph of a line

Teacher Materials:

Introductory practice sheet
Transparencies

Prep for Teachers:

If using computer graphing software, make sure the projection unit is hooked up and ready before class begins. Otherwise, have the graphing calculator handy to minimize “down-time.”

If using transparencies, have a transparency ready for each exercise to minimize time spent plotting points and graphing lines.

Student Materials:

- Practice sheet for graphing lines by making a table of values.
- NOTES sheet for writing equations from patterns.

Introductory Activity: DAY ONE (not on video)

Step 1: Remind students what has been learned in previous lessons that will be pertinent to this lesson and/or have them begin to think about the words and ideas of this lesson:

- ✓ What is the domain of a relation? Range?
- ✓ How do we create a table of values?
- ✓ Without a ruler, how can we determine if a set of points lie in a line?
- ✓ How does the graph of a line differ from the graph of a curve by looking at the table of values?

Step 2: Have students create tables given a function rule and graph the set of values on a coordinate plane. Connect these points to identify *all* points that lie on the line.

Make a table and graph the solution set for each equation:

$$y = 2x - 4$$

$$y = \frac{1}{2}x + 1$$

$$y = x + 5$$

$$y = 4$$

Step 3: Verify your graphs using the graphing calculator.

Step 4: Determine which of the following tables represents the points on a linear graph?

x	y
-2	4
-1	6
0	8
1	10

linear

x	7	5	3
y	-1	-3	-4

not linear

x	y
0	-2
1	0
2	2
5	8

linear

Step 5: Each table below represents points on a linear graph. Complete each.

a	b
0	_____
1	8
2	12
3	16

4

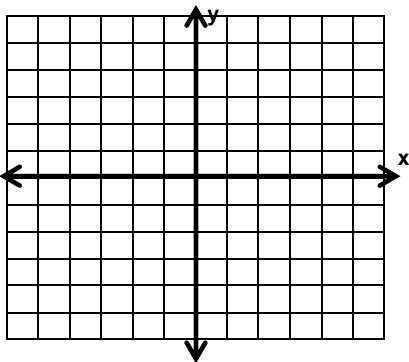
x	y
-1	0
1	3
3	_____
5	_____
7	12

6, 9

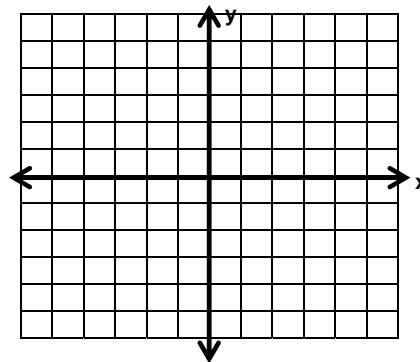
n	-6	-3	0	3	6
f(n)	_____	-10	-15	_____	_____

-5, ... -20, -25

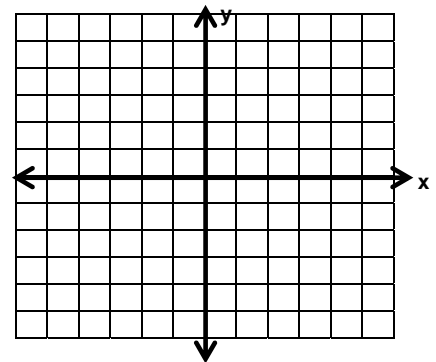
Step 6: Look at each graph below and determine if each set of points lie in a straight line.



NO



YES



YES

Without using a ruler, how can we tell that the points lie in a line?

We are looking for a constant rate of a change.

Step 7: Independent practice: Students are to create their own table of values and graph the following equations. Teacher will monitor students' progress and assist students who need help. Encourage students to check graphs using the graphing calculator.

$$y = 2x + 3$$

$$y = -x + 1$$

$$y = x^2 - 4$$

$$y = \frac{1}{3}x - 5$$

Which graph is different from all the others? Why? What is different about the equation? Is there a pattern in the table? Is there a constant rate of change between the points?

Step 8: Assign homework... "Equations, tables, and graphs" (attached)

DAY TWO: (VIDEO LESSON)

Step 1: Divide students into groups of 3 or 4. Have students compare the graphing assignment and discuss their answers to the questions on the bottom of the page.

Graph on the same coordinate plane:

1.) $y = 2x - 5$	1.) $y = 2x - 1$	1.) $y = 2x + 3$
2.) $y = -\frac{1}{2}x + 4$	2.) $y = -\frac{1}{2}x + 1$	2.) $y = -\frac{1}{2}x - 3$
3.) $y = x + 4$	3.) $y = \frac{1}{2}x + 4$	3.) $y = -3x + 4$

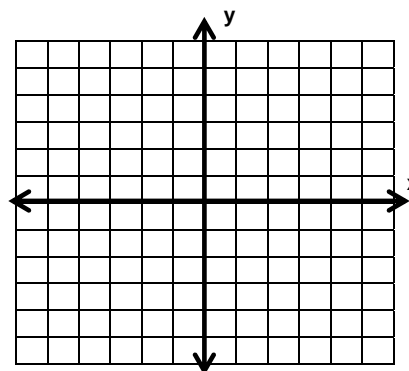
Step 2: Teacher will review answers to the questions, "Are all of the lines in the first graph similar? How are all of their equations similar? What makes the graphs/equations different? Can you figure out how the numbers in the equation affect the graph?" Group members will discuss their answers and explain the relationships they found between the graphs and the equations. One member from each group will report an answer and explain it to the class. The teacher will reiterate how working backwards and finding relationships and patterns is involved and give additional examples.

Step 3: Guide students to discover the relationship between the table and the graph. Concentrate on "rate of change" and "point where line crosses the y-axis."

Graph each set of points:

x	y
-4	-3
-2	0
0	3
2	6
4	9

+2 ↘ ↗ +3
+2 ↘ ↗ +3
+2 ↘ ↗ +3
+2 ↘ ↗ +3



How is the pattern in the table reflected in the graph?

How is the pattern (rate of change) evident in the equation? How do we find the constant that is either being added or subtracted?

Students are to explain how the pattern in the table relates to the graph. Lead students to discover rate of change is written as a ratio: change in y's divided by the change in x's. A mnemonic of "rise over run" may be used, but students need to understand that we are discussing a rate of change. Teacher will facilitate the writing of the linear equation (or function rule) by asking a series of questions and using visual cues to lead students to the conclusion that the pattern/slope is the "multiplier" and the y-intercept is the constant when the equation is written in the form $y = mx + b$ (although m and b have not been discussed).

Step 4: Give students some examples to practice on their own. (back of notes sheet)

x	y
0	5
1	8
2	11
3	14
4	17

rate of change: $\frac{\text{change in y-values}}{\text{change in x-values}} = \frac{3}{1} = 3$

So, we know that $y = 3x$ _____

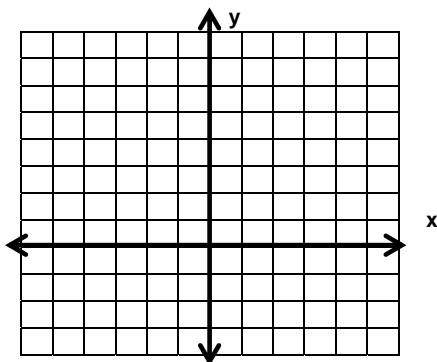
we need to find the number that is either added or subtracted to "3x" to give the corresponding range value in the table.

Choose a domain value..."2" substitute 2 in place of x in the equation $y = 3(2)$ _____
 3 times 2 is 6 → we need 11
 Therefore, we must add 5

Our equation is $y = 3x + 5$ Use another point to check the equation.

Some students will recognize that when $x = 0$, the line will cross the y-axis at 5 and that is the constant in the equation. Encourage the use of patterns to find the domain value of 0 if it is not given in the table.

Step 5: Find the equation from the graph.



Rate of change: $\frac{\text{change in y-values}}{\text{change in x-values}} = \frac{-2}{5}$

$y = -\frac{2}{5}x$ _____

Choose a point and substitute: (5, 4)

$y = -\frac{2}{5}(5)$ _____
 $= -2$ what do we add to get 4? **6**

Therefore, our equation is $y = -\frac{2}{5}x + 6$.

Culminating Activity:

[Operation Algebra: Introduction to Linear Relations](#): A Practical Example: Computing Time and Distance Using Linear Equations

This episode introduces the concept of linear relations, stressing that equations, graphs and tables, are merely three different representations for the same information. Using the real-world equation $T = n + 40$, where T = the outside temperature in degrees Fahrenheit and n is the number of times a snowy tree cricket chirps in 14 seconds, we create a table of values that satisfies the equation then use the table to create a graph. Students see how the constant increase pattern in a table manifests itself as a straight line on a graph. This leads to a discussion about linear relationships and how to recognize linear relationship as graphs, as tables and as equations. Students practice identifying linear relations as equations, tables, and graphs. Further connections amongst equations, tables and graphs are explored by looking closely at ordered pairs.

Extensions:

“NAME THAT FUNCTION” CONTEST – give students a series of graphs and tables and have them find the function rule (linear). Give fewer and fewer table values until students realize that they only need 2 points to find the equation of a line.

Discuss different rates of change: positive vs. negative, steep vs. shallow, what about horizontal and vertical lines? How would you describe their rate of change?

Linear Regressions: students are given a scatter plot where they will determine if there is a positive correlation, negative correlation, or no correlation. Then the students will find the line of best-fit, the equation of the line of best-fit, and make predictions based on the equation. Students will model real-world data and make predictions.