

Formulas from Trigonometry

1 Radians and Degrees

If θ is an angle expressed in radians, and α is the same angle measured in degrees, then

$$\frac{\theta}{\pi} = \frac{\alpha}{180} .$$

The length s of an arc on a circle of radius r opposite the angle θ , measured in radians, is given by the formula $s = r\theta$.

2 Definition of Trigonometric Functions

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \quad \cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} \quad \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

3 Values for Specific Angles

θ	0°	30°	45°	60°	90°
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan \theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	$\pm\infty$

4 Fundamental Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

5 Co-Functions

$$\sin \left(\frac{\pi}{2} - \theta \right) = \cos \theta \quad \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta$$

$$\tan \left(\frac{\pi}{2} - \theta \right) = \cot \theta \quad \cot \left(\frac{\pi}{2} - \theta \right) = \tan \theta$$

$$\sec \left(\frac{\pi}{2} - \theta \right) = \csc \theta \quad \csc \left(\frac{\pi}{2} - \theta \right) = \sec \theta$$

6 Symmetry, Periodicity

$$\begin{aligned} \sin(-\theta) &= -\sin \theta & \cos(-\theta) &= \cos \theta \\ \sin(\theta + 2\pi) &= \sin \theta & \sin(\theta + \pi) &= -\sin \theta \\ \cos(\theta + 2\pi) &= \cos \theta & \cos(\theta + \pi) &= -\cos \theta \\ \tan(\theta + \pi) &= \tan \theta & \tan(-\theta) &= -\tan \theta \end{aligned}$$

7 Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

8 Addition, Subtraction

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

9 Double Angles, Half Angles

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1\end{aligned}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin^2 \left(\frac{\theta}{2}\right) = \frac{1 - \cos \theta}{2} \quad \cos^2 \left(\frac{\theta}{2}\right) = \frac{1 + \cos \theta}{2}$$

$$\tan \left(\frac{\theta}{2}\right) = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$

10 Products

$$\sin \alpha \sin \beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos \alpha \cos \beta = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$\sin \alpha \cos \beta = \frac{1}{2}(\sin(\alpha - \beta) + \sin(\alpha + \beta))$$

11 Sinusoids

Sinusoids are functions of the form

$$y = a \sin(bx + c) + d$$

Amplitude	$ a $
Period	$2\pi/ b $
Phase shift	$-c/b$
Maximum value	$d + a $
Minimum value	$d - a $

12 Inverse Functions

For $-1 \leq x \leq 1$ we define:

$$y = \sin^{-1} x = \arcsin x \quad \text{if and only if} \\ -\pi/2 \leq y \leq \pi/2 \quad \text{and} \quad \sin y = x.$$

For $-1 \leq x \leq 1$ we define:

$$y = \cos^{-1} x = \arccos x \quad \text{if and only if} \\ 0 \leq y \leq \pi \quad \text{and} \quad \cos y = x.$$

For any real number x we define:

$$y = \tan^{-1} x = \arctan x \quad \text{if and only if} \\ -\pi/2 < y < \pi/2 \quad \text{and} \quad \tan y = x$$

$$\arcsin x + \arccos x = \frac{\pi}{2}$$

$$\arcsin(-x) = -\arcsin x$$

$$\arccos(-x) = \pi - \arccos x$$

$$\arctan(-x) = -\arctan x$$

13 Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

14 Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

15 Miscellaneous Formulas

Area A of a triangle: $A = \frac{1}{2}ab \sin \gamma$

Heron's Formula:

With $s = \frac{1}{2}(a + b + c)$ (= semiperimeter) the area A of a triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$