Reliability Homework Answers

The data below represent the pool of responses obtained by giving the same test to a group of participants with a 3 month delay between presentations.

<table>
<thead>
<tr>
<th>Time 1</th>
<th>Subject #</th>
<th>Question 1</th>
<th>Question 2</th>
<th>Question 3</th>
<th>Question 4</th>
<th>Question 5</th>
<th>Subject Mean (X₁)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>3.6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>3.4</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Time 2</th>
<th>Subject #</th>
<th>Question 1</th>
<th>Question 2</th>
<th>Question 3</th>
<th>Question 4</th>
<th>Question 5</th>
<th>Subject Mean (X₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>4.2</td>
<td></td>
</tr>
</tbody>
</table>

1. Calculate the test-retest reliability between X₁ and X₂.
   - What % of the variance is attributable to True Score and what % is attributable to error?
   Answer with Computational Formula

\[ r_{xx'} = \frac{\sum X_1 X_2 - \frac{(\sum X_1)(\sum X_2)}{n}}{\sqrt{\sum X_1^2 - \frac{(\sum X_1)^2}{n}} \sqrt{\sum X_2^2 - \frac{(\sum X_2)^2}{n}}} \]

\[ \text{TotalVar}_{xx'} = 2.988095 \]

\[ r_{xx'} = 0.285578 \]

\[ \text{COV}_{xx'} = 0.853333 \]

\[ \text{Var}_{X_1} = 2.77 \]

\[ \text{Var}_{X_2} = 17.74 \]

\[ \text{Var}_{X_1} < \text{Var}_{X_2} \]

\[ \text{E} = 8.8 \]

\[ \text{E}X_1 = 7.4 \]

\[ \text{E}X_2 = 22.84 \]

\[ \text{E}X_1X_2 = 22.56 \]

\[ \Sigma X_1X_2 = 22.56 \]

\[ \Sigma X_1 = 8.8 \]

\[ \Sigma X_2 = 7.4 \]

\[ \Sigma X_1^2 = 27.76 \]

\[ \Sigma X_2^2 = 22.84 \]

\[ \Sigma X_1X_2 = 22.56 \]

\[ \Sigma X_1 = 8.8 \]

\[ \Sigma X_2 = 7.4 \]

\[ \Sigma X_1^2 = 27.76 \]

\[ \Sigma X_2^2 = 22.84 \]

\[ \Sigma X_1X_2 = 22.56 \]

\[ \Sigma X_1 = 8.8 \]

\[ \Sigma X_2 = 7.4 \]

\[ \Sigma X_1^2 = 27.76 \]

\[ \Sigma X_2^2 = 22.84 \]

\[ \Sigma X_1X_2 = 22.56 \]

\[ \Sigma X_1 = 8.8 \]

\[ \Sigma X_2 = 7.4 \]
### Answer with Definitional Formula

<table>
<thead>
<tr>
<th>Subject</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_1 - \bar{X}_1$</th>
<th>$X_2 - \bar{X}_2$</th>
<th>$(X_1 - \bar{X}_1)^2$</th>
<th>$(X_2 - \bar{X}_2)^2$</th>
<th>$(X_1 - \bar{X}_1)(X_2 - \bar{X}_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>s1</td>
<td>1.8</td>
<td>1.8</td>
<td>-1.13333</td>
<td>-0.66667</td>
<td>1.284444</td>
<td>0.444444</td>
<td>0.755556</td>
</tr>
<tr>
<td>s2</td>
<td>3.6</td>
<td>1.4</td>
<td>0.66667</td>
<td>-1.06667</td>
<td>0.444444</td>
<td>1.137778</td>
<td>-0.71111</td>
</tr>
<tr>
<td>s3</td>
<td>3.4</td>
<td>4.2</td>
<td>0.46667</td>
<td>1.733333</td>
<td>0.217778</td>
<td>3.004444</td>
<td>0.808889</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>8.8</td>
<td>7.4</td>
<td>-6.7E-16</td>
<td>0</td>
<td>1.946667</td>
<td>4.586667</td>
<td>0.853333</td>
</tr>
</tbody>
</table>

Note: Your numbers may differ from mine due to rounding error. You should carry your calculations out to 4 decimals, I did this in excel and it took the calculations much further.

$COV_{xx'} = 0.853333$

$TotalVar_{xx'} = 2.988095$

$r_{xx'} = \sqrt{\frac{\sum(X_1 - \bar{X}_1)(X_2 - \bar{X}_2)}{\sqrt{\sum (X_1 - \bar{X}_1)^2 \sum (X_2 - \bar{X}_2)^2}}} = \sqrt{\frac{0.853333}{\sqrt{1.9467 \times 4.5867}}} = \sqrt{\frac{0.8533}{(1.3952)(2.1417)}} = \frac{0.8533}{2.9881} = .2856$

- % Variance Attributable to True Score = 28.56%
- % Variance Attributable to Error = 71.44%

2. Calculate the Split-Half reliability between Odd and Even items at time 1. Ignore the fact that there are more odd items than there are even items.

<table>
<thead>
<tr>
<th>$X_o$</th>
<th>$X_e$</th>
<th>$X_o^2$</th>
<th>$X_e^2$</th>
<th>$X_oX_e$</th>
<th>$COV_{oe}$</th>
<th>$TotalVar_{oe}$</th>
<th>$r_{oe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.5</td>
<td>4</td>
<td>2.25</td>
<td>3</td>
<td>1.944444</td>
<td>2.003084</td>
<td>0.970725</td>
</tr>
<tr>
<td>3.3333</td>
<td>4</td>
<td>11.1109</td>
<td>16</td>
<td>13.3332</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>8.3333</td>
<td>9.5</td>
<td>24.1109</td>
<td>34.25</td>
<td>28.3332</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$n = 3$

$\Sigma X_o = \Sigma X_e = \Sigma X_o^2 = \Sigma X_e^2 = \Sigma X_oX_e$

$r_{oe} = \frac{\sum X_oX_e - \frac{1}{n}\left(\sum X_o\sum X_e\right)}{\sqrt{\sum X_o^2 - \frac{1}{n}\left(\sum X_o^2\right)}\sqrt{\sum X_e^2 - \frac{1}{n}\left(\sum X_e^2\right)}} = \sqrt{24.1109 - \frac{(8.3333)(9.5)}{3}} = \sqrt{24.1109 - \frac{(8.3333)^2}{3}} = \sqrt{79.1664 - \frac{90.25}{3}} = 8.3333 - \frac{9.5}{3} = \frac{28.3332}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = 8.3333 - \frac{9.5}{3} = \frac{28.3332}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - \frac{90.25}{3} = \frac{79.1664}{3} = \frac{90.25}{3} = \frac{28.3332}{3} = 24.1109 - 23.1480 = \sqrt{2.3033} = \frac{9.813}{2.0412} = .9707$

$\sum X_oX_e = 28.3332 - 23.1480 = \sqrt{2.3033} = \frac{9.813}{2.0412} = .9707$

$r_{oe} = \frac{2(r_{oe})}{1 + (r_{oe})} = \frac{2(0.9707)}{1 + 0.9707} = \frac{1.9414}{1.9707} = .9851$

3. Use the Spearman-Brown Prophesy formula to estimate the reliability for the whole test.
4. Calculate the Split-Half reliability between the End Items (1 and 5) and the Middle Items (2, 3, and 4) for Time 1.

\[
\begin{array}{cccccc}
X_e & X_m & X_e^2 & X_m^2 & X_eX_m & COV_{em} \\
2.5 & 1.3333 & 6.25 & 1.7777 & 3.3333 & 0.944444 \\
3 & 4 & 9 & 16 & 12 & TotalVar_{em} 2.057807 \\
4 & 3 & 16 & 9 & 12 & r_{em} 0.458957 \\
\end{array}
\]

\[
\begin{array}{cccccc}
\Sigma & \Sigma X_e & \Sigma X_m & \Sigma X_e^2 & \Sigma X_m^2 & \Sigma X_eX_m \\
9.5 & 8.3333 & 31.25 & 26.7777 & 27.3333 & n 3 \\
\end{array}
\]

Note: rounding error may cause your answer to be slightly different than mine

Why is the split-half reliability for the End vs Middle Items different from the split-half reliability for the Odd and Even items?

- The Unreliable items are distributed such that the Odd/Even split distributed unreliability equally across the two halves. The Middle/End split grouped unreliability in an unequal manner.

The data below represent the pool of responses obtained by giving an measure of South Park Knowledge to a group of participants. Each item is scored as correct or incorrect and it is assumed that each item is of equal difficulty. Items are as follows:

2. What is Chef’s real name? (A: Lavar Burton)
3. Why shouldn’t you do drugs? (A: Because if you do drugs, you’re a hippy. And, Hippies suck)
4. Will a Pig and Elephant’s DNA splice? (A: Pig and Elephant DNA just won’t splice)
5. Who Shot Mr. Burns (A: Maggie Simpson)

<table>
<thead>
<tr>
<th></th>
<th>Quest 1</th>
<th>Quest 2</th>
<th>Quest 3</th>
<th>Quest 4</th>
<th>Quest 5</th>
<th>Subject Total (X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kyle</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Stan</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Kenny</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Eric</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Butters (aka: Prof. Chaos)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c|c}
\chi & \chi^2 \\
\hline
0 & 0 \\
3 & 9 \\
5 & 25 \\
3 & 9 \\
4 & 16 \\
\hline
15 & 59 \\
\hline
\Sigma\chi & \Sigma\chi^2
\end{array}
\]

\[
\overline{\chi} = \frac{15}{5} = 3
\]

K = 5

\[
\sigma^2 = \frac{\sum X^2 - \left(\frac{\sum X}{n}\right)^2}{n} = \frac{59 - \left(\frac{15}{5}\right)^2}{5} = \frac{59 - \frac{225}{5}}{5} = \frac{59 - 45}{5} = \frac{14}{5} = 2.8
\]

\[
KR21 = \left(\frac{K}{K-1}\right)\left(1 - \frac{\overline{X}(K - \overline{X})}{K\sigma^2}\right) = \left(\frac{5}{5-1}\right)\left(1 - \frac{3(5 - 3)}{5(2.8)}\right) = (1.25)\left(1 - \frac{3(2)}{14}\right) = \\
(1.25)\left(1 - \frac{6}{14}\right) = (1.25)(1 - 0.4286) = (1.25)(0.5714) = 0.7143
\]


\[
\begin{array}{c|c|c|c|c|c}
\text{Subject} & Q1 & Q2 & Q3 & Q4 & Q5 \\
\hline
s1 & 0 & 0 & 0 & 0 & 0 \\
\hline
s2 & 0 & 0 & 1 & 1 & 1 \\
\hline
s3 & 1 & 1 & 1 & 1 & 1 \\
\hline
s4 & 1 & 1 & 1 & 0 & 0 \\
\hline
s5 & 1 & 0 & 1 & 1 & 1 \\
\hline
\text{p} & 0.6 & 0.4 & 0.8 & 0.6 & 0.6 \\
\text{q} & 0.4 & 0.6 & 0.2 & 0.4 & 0.4 \\
\text{pq} & 0.24 & 0.24 & 0.16 & 0.24 & 0.24 \\
\hline
\text{p} + \text{q} & 1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

\[
\sum pq = 0.24 + 0.24 + 0.16 + 0.24 + 0.24 = 1.04
\]

\[
KR20 = \left(\frac{K}{K-1}\right)\left(1 - \frac{\sum pq}{\sigma^2}\right) = \left(\frac{5}{5-1}\right)\left(1 - \frac{1.12}{2.8}\right) = \\
(1.25)(1 - 0.4) = (1.25)(0.6) = 0.75
\]

7. Using the Standard Deviation calculated in Problem 5, and the Reliability Coefficient calculated in Problem 5, calculate the Standard Error of the Measure for the 68% Confidence Level, 95% Confident Level, and the 99% Confidence Level.

\[
\sigma = \sqrt{\sigma^2} = \sqrt{2.8} = 1.6733
\]
68% Confidence Level

\[ SEM = \sigma \sqrt{1 - r_{xx}} = 1.6733 \sqrt{1 - .7143} = 1.6733 \sqrt{.2857} = .7483(.5345) = .8944 \]

95% Confidence Level

\[ SEM_{\alpha/2} = Z_{\alpha/2}(SEM) = 1.96(.8944) = 1.7530 \]

99% Confidence Level

\[ SEM_{\alpha/2} = Z_{\alpha/2}(SEM) = 2.58(.8944) = 2.3076 \]

8. Two raters have coded 51 participant’s behavior using a 4 point rating system. Use Cohen’s Kappa to estimate the inter-rater reliability. (The data appear below)

<table>
<thead>
<tr>
<th></th>
<th>Rater 1 1</th>
<th>Rater 1 2</th>
<th>Rater 1 3</th>
<th>Rater 1 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rater 2 1</td>
<td>n = 8</td>
<td>n = 2</td>
<td>n = 1</td>
<td>n = 2</td>
</tr>
<tr>
<td>Rater 2 2</td>
<td>n = 3</td>
<td>n = 6</td>
<td>n = 0</td>
<td>n = 1</td>
</tr>
<tr>
<td>Rater 2 3</td>
<td>n = 2</td>
<td>n = 1</td>
<td>n = 8</td>
<td>n = 4</td>
</tr>
<tr>
<td>Rater 2 4</td>
<td>n = 2</td>
<td>n = 1</td>
<td>n = 1</td>
<td>n = 9</td>
</tr>
</tbody>
</table>

You get to forget about this one.