Reliability Homework Answers

The data bellow represent the pool of responses obtained by giving the same test to a group of participants with a 3 month delay between presentations.

Time 1

Subject #	Question 1	Question 2	Question 3	Question 4	Question 5	Subject Mean (X_1)
1	2	2	1	1	3	1.8
2	5	5	4	3	1	3.6
3	6	3	1	5	2	3.4

Time 2

Subject #	Question 1	Question 2	Question 3	Question 4	Question 5	Subject Mean (X_2)
1	2	2	1	1	3	1.8
2	1	1	1	3	1	1.4
3	3	7	4	5	2	4.2

1. Calculate the test-retest reliability between X_1 and X_2.

- What % of the variance is attributable to True Score and what % is attributable to error?

Answer with Computational Formula

	Subject	X_1	X_2	X_1^2	X_2^2	X_1X_2		
	s1	1.8	1.8	3.24	3.24	3.24	$COV_{xx'}$	0.853333
	s2	3.6	1.4	12.96	1.96	5.04	$TotalVar_{xx'}$	2.988095
	s3	3.4	4.2	11.56	17.64	14.28	$r_{xx'}$	0.285578
Σ		8.8	7.4	27.76	22.84	22.56		
n		3						
		ΣX_1	ΣX_2	ΣX_1^2	ΣX_2^2	ΣX_1X_2		

$$r_{xx'} = \frac{\sum X_1X_2 - \frac{(\sum x_1)(\sum x_2)}{n}}{\sqrt{\sum X_1^2 - \frac{(\sum x_1)^2}{n}}\sqrt{\sum X_2^2 - \frac{(\sum x_2)^2}{n}}} = \frac{22.56 - \frac{(8.8)(7.4)}{3}}{\sqrt{27.76 - \frac{(8.8)^2}{3}}\sqrt{22.84 - \frac{(7.4)^2}{3}}} = \frac{22.56 - \frac{65.12}{3}}{\sqrt{27.76 - \frac{77.44}{3}}\sqrt{22.84 - \frac{54.76}{3}}} =$$

$$\frac{22.56 - 21.7067}{\sqrt{27.76 - 25.8133}\sqrt{22.84 - 18.2533}} = \frac{.8533}{\sqrt{1.9467}\sqrt{4.5867}} = \frac{.8533}{(1.3952)(2.1417)} = \frac{.8533}{2.9881} = .2856$$

Answer with Definitional Formula

Subject	X_1	X_2	$X_1 - \bar{x}_1$	$X_2 - \bar{x}_2$	$(X_1 - \bar{x}_1)^2$	$(X_2 - \bar{x}_2)^2$	$(X_1 - \bar{x}_1)(X_2 - \bar{x}_2)$
s1	1.8	1.8	-1.13333	-0.66667	1.284444	0.444444	0.755556
s2	3.6	1.4	0.666667	-1.06667	0.444444	1.137778	-0.71111
s3	3.4	4.2	0.466667	1.733333	0.217778	3.004444	0.808889
Σ	8.8	7.4	-6.7E-16	0	1.946667	4.586667	0.853333
n	3						
$\bar{x}_i$	2.933333	2.466667					

Note: Your numbers may differ from mine due to rounding error. You should carry your calculations out to 4 decimals, I did this in excel and it took the calculations much further.

$COV_{xx'}$	0.853333
$TotalVar_{xx'}$	2.988095
$r_{xx'}$	0.285578

$$r_{xx'} = \frac{\sum (X_1 - \bar{X}_1)(X_2 - \bar{X}_2)}{\sqrt{\sum (X_1 - \bar{X}_1)^2}\sqrt{\sum (X_2 - \bar{X}_2)^2}} = \frac{.85333}{\sqrt{1.9467}\sqrt{4.5867}} = \frac{.8533}{(1.3952)(2.1417)} = \frac{.8533}{2.9881} = .2856$$

- % Variance Attributable to True Score = 28.56%
- % Variance Attributable to Error = 71.44%

2. Calculate the Split-Half reliability between Odd and Even items at time 1. Ignore the fact that there are more odd items than there are even items.

	X_o	X_e	X_o^2	X_e^2	X_oX_e
	2	1.5	4	2.25	3
	3.3333	4	11.1109	16	13.3332
	3	4	9	16	12
Σ	8.3333	9.5	24.1109	34.25	28.3332
n	3				
	ΣX_o	ΣX_e	ΣX_o^2	ΣXe^2	ΣX_oX_e

COV_{oe}	1.944444
$TotalVar_{oe}$	2.003084
r_{oe}	0.970725

$$r_{oe} = \frac{\sum X_oX_e - \frac{(\sum x_o(\sum x_e))}{n}}{\sqrt{\sum X_o^2 - \frac{(\sum x_o)^2}{n}}\sqrt{\sum X_e^2 - \frac{(\sum x_e)^2}{n}}} = \frac{28.3332 - \frac{(8.3333)(9.5)}{3}}{\sqrt{24.1109 - \frac{(8.3333)^2}{3}}\sqrt{34.25 - \frac{(9.5)^2}{3}}} = \frac{28.3332 - \frac{79.1664}{3}}{\sqrt{24.1109 - \frac{69.4439}{3}}\sqrt{34.25 - \frac{90.25}{3}}} =$$

$$\frac{28.3332 - 26.3888}{\sqrt{24.1109 - 23.1480}\sqrt{34.25 - 30.0833}} = \frac{1.9444}{\sqrt{.9629}\sqrt{4.1667}} = \frac{1.9444}{(.9813)(2.0412)} = \frac{1.9444}{2.0030} = .9707$$

3. Use the Spearman-Brown Prophesy formula to estimate the reliability for the whole test.

$$r_{xx'} = \frac{2(r_{oe})}{1 + (r_{oe})} = \frac{2(.9707)}{1 + .9707} = \frac{1.9414}{1.9707} = .9851$$

4. Calculate the Split-Half reliability between the End Items (1 and 5) and the Middle Items (2, 3, and 4) for Time 1.

	X_e	X_m	X_e^2	X_m^2	X_eX_m		
	2.5	1.3333	6.25	1.7777	3.3333	COV_{em}	0.944444
	3	4	9	16	12	$TotalVar_{em}$	2.057807
	4	3	16	9	12	r_{em}	0.458957
Σ	9.5	8.3333	31.25	26.7777	27.3333		
n	3						
	ΣX_e	ΣX_m	ΣX_e^2	ΣX_m^2	ΣX_eX_m		

Note: rounding error may cause your answer to be slightly different than mine

Why is the split-half reliability for the End vs Middle Items different from the split-half reliability for the Odd and Even items?

- The Unreliable items are is distributed such that the Odd/Even split distributed unreliability equally across the two halves. The Middle/End split grouped unreliability in an unequal manner.

The data bellow represent the pool of responses obtained by giving an measure of South Park Knowledge to a group of participants. Each item is scored as correct or incorrect and it is assumed that each item is of equal difficulty. Items are as follows:

1. Who Invented Cheezy Poofs? (A: Edward H. Cheezy)
2. What is Chef's real name? (A: Lavar Burton)
3. Why shouldn't you do drugs? (A: Because if you do drugs, you're a hippy. And, Hippies suck)
4. Will a Pig and Elephant's DNA splice? (A: Pig and Elephant DNA just won't splice)
5. Who Shot Mr. Burns (A: Maggie Simpson)

	Quest 1	Quest 2	Quest 3	Quest 4	Quest 5	Subject Total (X)
Kyle	0	0	0	0	0	0
Stan	0	0	1	1	1	3
Kenny	1	1	1	1	1	5
Eric	1	1	1	0	0	3
Butters (aka: Prof. Chaos)	1	0	1	1	1	4

5. Estimate the Reliability of this Exam using KR21.

X	X^2
0	0
3	9
5	25
3	9
4	16
15	59
ΣX	ΣX^2

$\bar{x} = 15/5 = 3$
K = 5

$$\sigma^2 = \frac{\sum X^2 - \frac{(\sum x)^2}{n}}{n} = \frac{59 - \frac{(15)^2}{5}}{5} = \frac{59 - \frac{225}{5}}{5} = \frac{59-45}{5} = \frac{14}{5} = 2.8$$

$$KR21 = \left(\frac{K}{K-1}\right)\left(1 - \frac{\bar{X}(K-\bar{X})}{K(\sigma^2)}\right) = \left(\frac{5}{5-1}\right)\left(1 - \frac{3(5-3)}{5(2.8)}\right) = (1.25)\left(1 - \frac{3(2)}{14}\right) =$$

$$= (1.25)\left(1 - \frac{6)}{14}\right) = (1.25)(1-.4286) = (1.25)(.5714) = .7143$$

6. Estimate the Reliability of this Exam using KR20.

Subject	Q1	Q2	Q3	Q4	Q5
s1	0	0	0	0	0
s2	0	0	1	1	1
s3	1	1	1	1	1
s4	1	1	1	0	0
s5	1	0	1	1	1
p	0.6	0.4	0.8	0.6	0.6
q	0.4	0.6	0.2	0.4	0.4
pq	0.24	0.24	0.16	0.24	0.24

$\sum pq = .24 + .24 + .16 + .24 + .24 = 1.04$

$$KR20 = \left(\frac{K}{K-1}\right)\left(1 - \frac{\sum pq}{\sigma^2}\right) = \left(\frac{5}{5-1}\right)\left(1 - \frac{1.12}{2.8}\right) =$$

$$= (1.25)(1-.4) = (1.25)(.6) = .75$$

7. Using the Standard Deviation calculated in Problem 5, and the Reliability Coefficient calculated in Problem 5, calculate the Standard Error of the Measure for the 68% Confidence Level, 95% Confident Level, and the 99% Confidence Level.

$$\sigma = \sqrt{\sigma^2} = \sqrt{.28} = 1.6733$$

68% Confidence Level

$$SEM = \sigma\sqrt{1 - r_{xx'}} = 1.6733\sqrt{1 - .7143} = 1.6733\sqrt{.2857} = .7483(.5345) = .8944$$

95% Confidence Level

$$SEM_{\alpha/2} = Z_{\alpha/2}(SEM) = 1.96(.8944) = 1.7530$$

99% Confidence Level

$$SEM_{\alpha/2} = Z_{\alpha/2}(SEM) = 2.58(.8944) = 2.3076$$

8. Two raters have coded 51 participant's behavior using a 4 point rating system. Use Cohen's Kappa to estimate the inter-rater reliability. (The data appear below)

	Rater 1 1	Rater 1 2	Rater 1 3	Rater 1 4
Rater 2 1	n = 8	n = 2	n = 1	n = 2
Rater 2 2	n = 3	n = 6	n = 0	n = 1
Rater 2 3	n = 2	n = 1	n = 8	n = 4
Rater 2 4	n = 2	n = 1	n = 1	n = 9

You get to forget about this one.