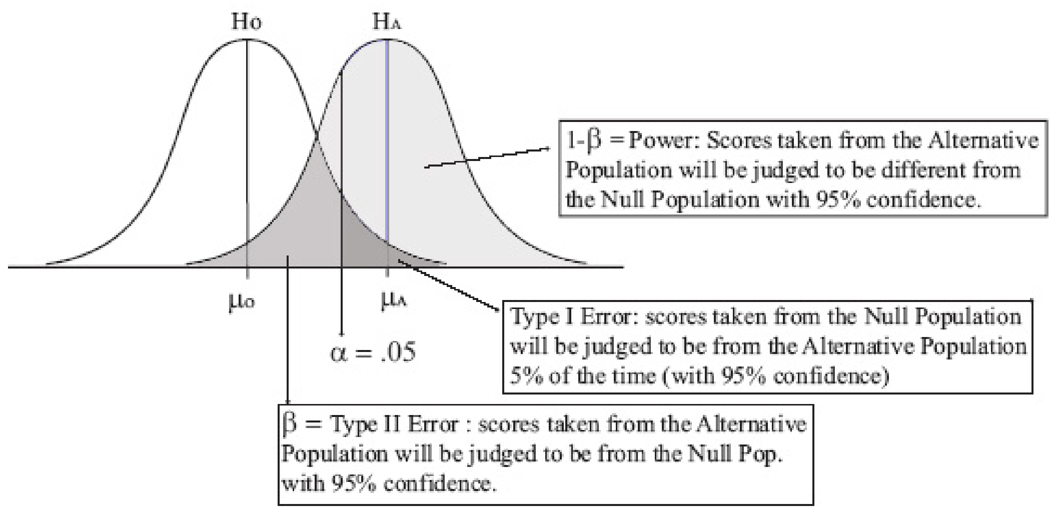
Power = Effect Size, Sample Size, & Alpha
Handout

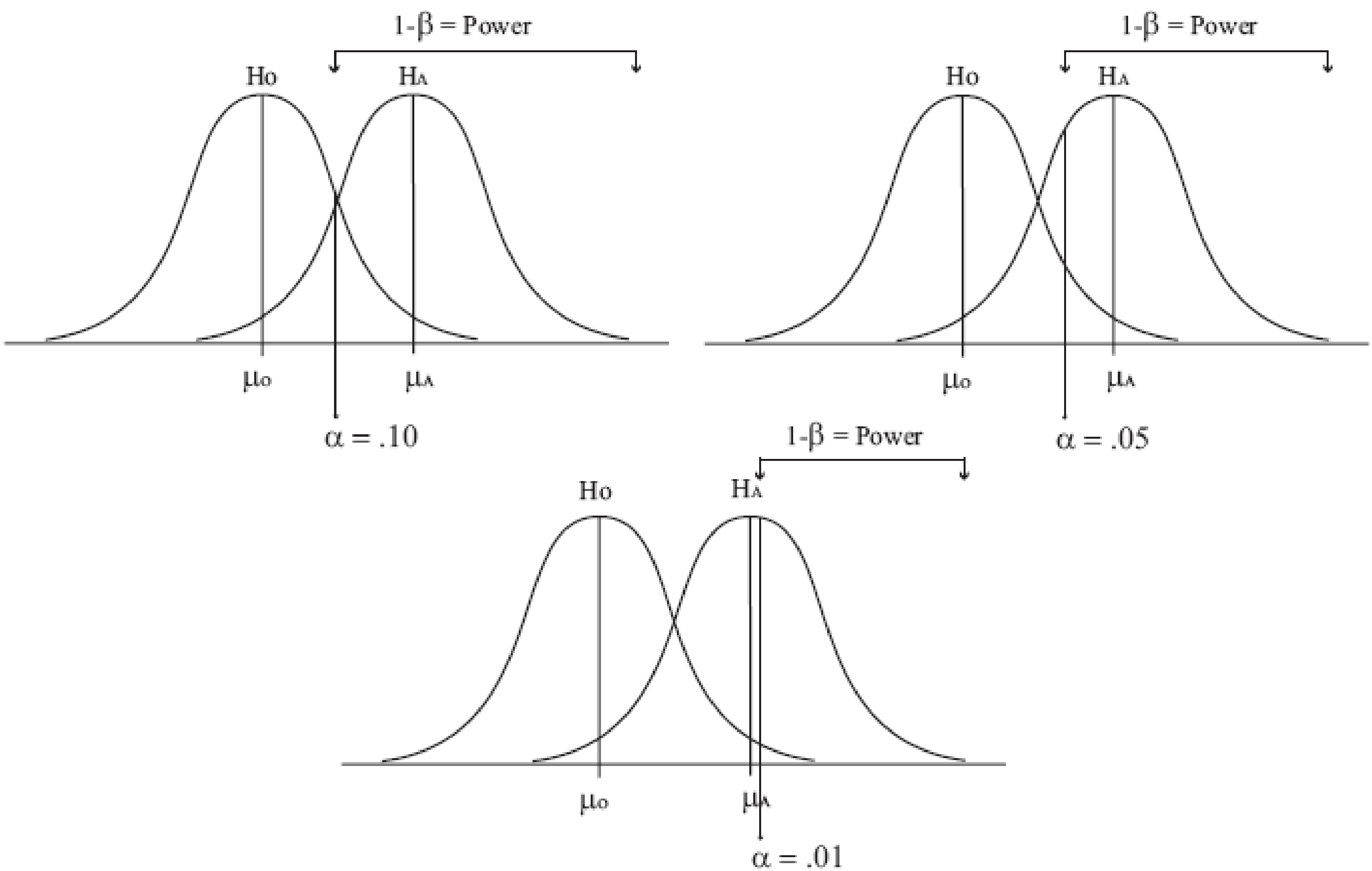
I. $Power = 1 - \beta$

II. Power is the probability that you will identify a significant relationship between 2 variables when in fact a significant relationship exists.

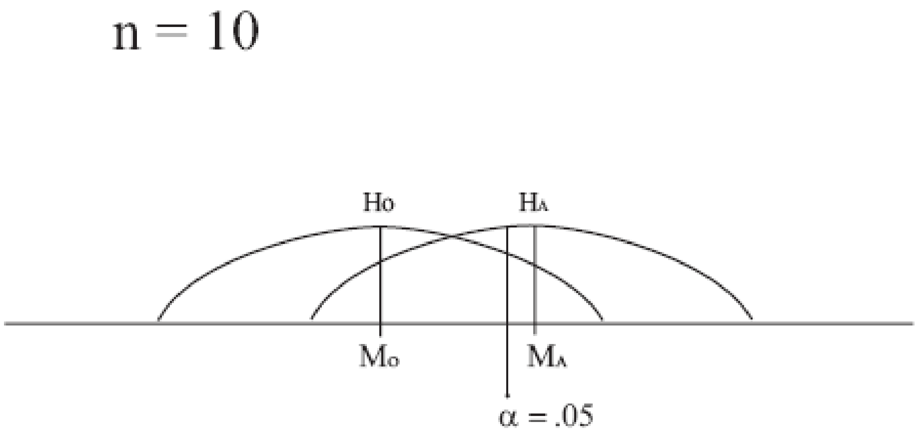


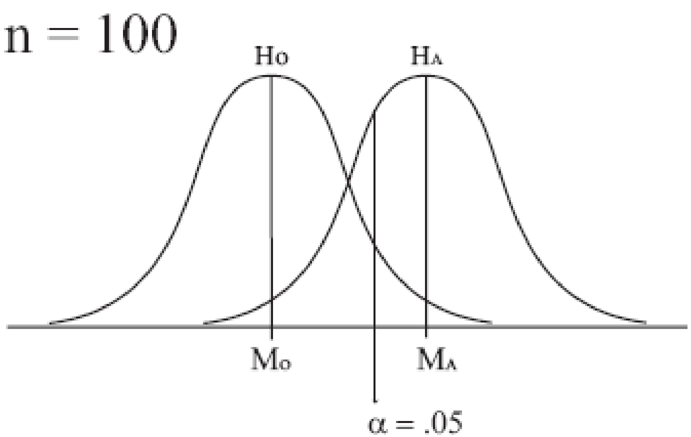
III. Power Determined by:

1. Alpha Level (Probability of Type II Error)



2. Sample Size





3. Effect Size (d, r^2, R^2, η^2, ω^2 (omega), ϕ^2)

- d is the standardized distance between two means (Howell, 2002).
- r^2, R^2, η^2 & ϕ^2 are all essentially squared Correlation Coefficients.
- ω^2 (omega) is an adjusted version of a squared Pearson Correlation Coefficient.
- All of the these can be converted to d'

IV. Effect Size: Amount of Change in DV attributable to changes in the IV

1. d conceptualizes effect size as the standardized distance between two means

$$d = \left[\frac{\mu_1 - \mu_2}{\sigma} \right]$$

Single Sample and repeated measures t :

$$d = \frac{t}{\sqrt{df}}$$

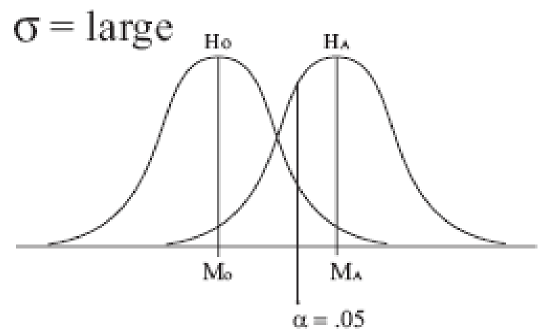
Independent sample *t* :

$$d = \frac{2t}{\sqrt{df}}$$ When sample sizes are equal (or must be assumed to be)

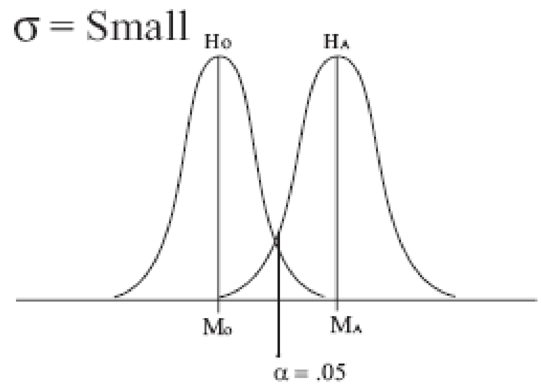
$$d = \frac{t(n_1 + n_2)}{\sqrt{df}\sqrt{n_1 n_2}}$$ For unequal sample sizes

2. As Variance Decreases - Effect Sizes Increase and Power Increases

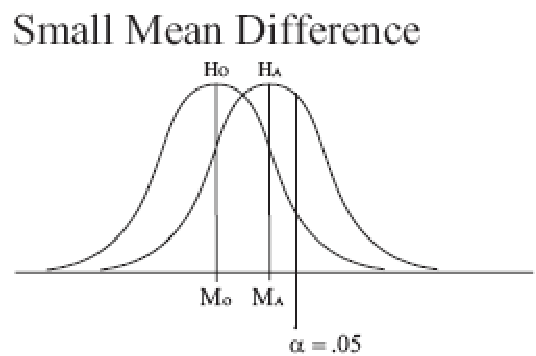
- More variance spreads out the distribution and results in more overlap between the distributions.

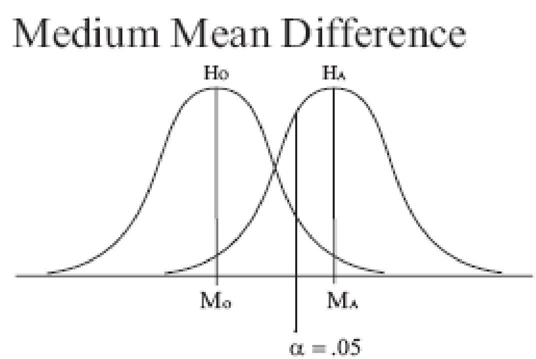


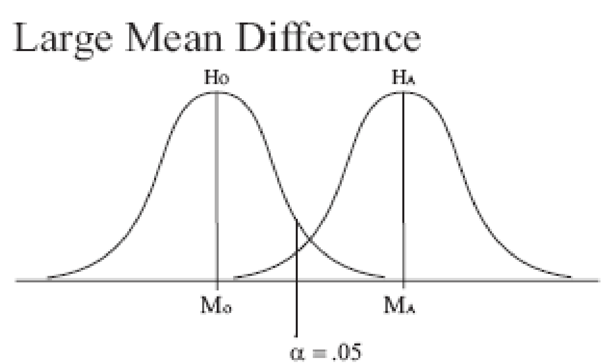
- Less variance makes the distribution more peaked and reduces the overlap between distributions and therefore results in more power.



3. As Mean Differences Increase - Effect Size Increases and Power Increases







4. Cohen (1988, 1992) suggests the following framework to interpret effect sizes.

Effect Size	*d*	% overlap of Dists.
Small	.20	85%
Medium	.50	67%
Large	.80	53%

V. Calculating Power

- Power can be estimated using power table like the one Howell (2002) provides
- This table is based on δ (lower case delta), which is a Non-Centrality Parameter that expresses how wrong the Null Hypothesis is (see Howell, 2002, p. 231 for a full discussion).

1. Power for *t*-Tests

a. Single Sample t and Repeated Measures/Matched Sample t

$\delta = d\sqrt{N}$ where N = number of sample observations

Independent Sample t with equal group sizes assumed

$\delta = d\sqrt{\frac{N}{4}}$ where $N = n_1 + n_2$

For Unequal Group sizes use the following -

$\delta = d\sqrt{\frac{n_h}{2}} = where = n_h = \frac{2n_1n_2}{n_1 + n_2}$ which is the Harmonic mean.

VI. Using Power to Calculate the Sample Size Need to Achieve a Desired Level of Power

N desired for Single Sample/Repeated Measures *t*: $N_{desired} = \left(\frac{\delta}{d}\right)^2$

When finding total N desired for Independent Sample t : $N_{desired} = 4\left(\frac{\delta}{d}\right)^2$

$n_i = (N/2)$

Following Cohen's (1988) framework, we would expect:
Given Power of .80 @ α=.05 = δ = 2.8

Effect Size	d	N for 1 sample t	N for 2 sample t
Small	.20	196	784
Medium	.50	32	128
Large	.80	13	52

VII. More Effect Sizes

1. Effect Size for Correlations:

r^2 = Coefficient of multiple determination

$$r^2 = \frac{SS_{betweengroups}}{SS_{total}} = r^2 = \left(\frac{\sum (X_x - \bar{X}_x)(X_y - \bar{X}_y)}{\sqrt{\sum (X_x - \bar{X}_x)^2}\sqrt{\sum (X_y - \bar{X}_y)^2}} \right)^2$$

Obtaining r^2 from d : $r^2 = \frac{d^2}{d^2 + 4}$

Obtaining d from r : $d = \frac{2r}{\sqrt{1 - r^2}}$

Noncentrality Parameter for r^2, convert to d' first, then: $\delta = d\sqrt{\frac{N}{4}}$

N desired : $N_{desired} = 4\left(\frac{\delta}{d}\right)^2$

Converting R^2 and r^2 to ω^2 (corrects for tendency of R^2 and r^2 to overestimate the population parameter):

$$\omega^2 = 1 - \frac{N - 1}{N - a}\left(1 - r^2\right)$$

2. Cohen's 1992 standards for effect sizes with correlation

Effect Size	r	r^2
Small	.10	.01
Medium	.30	.09
Large	.50	.25

3. Effect Size for X^2 (Chi-Square)

2x2 Matrix : $\phi^2 = \frac{X^2}{N}$

Noncentrality Parameter for 2x2 X^2: $\delta = \left(\frac{2\phi}{\sqrt{1 - \phi^2}}\right)\sqrt{\frac{N}{4}}$

Larger than 2.x2 Matrix : Cramer's $\phi^2 = \sqrt{\frac{X^2}{N(K - 1)}}$

where K = the smaller of the # of rows or # of columns

Noncentrality Parameter for X^2 larger than 2 x 2 = ?

4. Effect Size for Anova = R^2 (eta squared)

$$d = \frac{2\sqrt{F}}{\sqrt{df_{within}}}$$ Because $F = t^2$ (if you have two groups with equal group sizes)

$$d = \frac{\sqrt{F}(n_1 + n_2)}{\sqrt{df_{within}}\sqrt{n_1 n_2}}$$ For two groups with unequal group sizes.

For more than 2 groups

$$R^2 = \frac{SS_{betweengroups}}{SS_{total}}$$ Taken from Anova Tables.

Or

$$R^2 = \frac{F(df_{between})}{(F(df_{between})) + (df_{within})}$$

Noncentrality Parameter for R^2 for ANOVA with 2 equal sized Groups: $\delta = \left(\frac{2R}{\sqrt{1 - R^2}}\right)\sqrt{\frac{N}{4}}$

Desired Sample Size for ANOVA for ANOVA with 2 Groups

$$N_{desired} = 4\left(\frac{\delta}{d}\right)^2$$ Where k is the number of groups.

n_i = sample size for each group = $N_{desired}$ / k

Ancillary Formulas

Z test

$$z = \frac{\bar{X} - \mu}{\sigma}$$

Single Sample t

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$ where $df = n - 1$

Independent Sample *t*-test

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$ Where $df = n_1 + n_2 - 2$

Repeated Measures *t*-test (match sample, correlated sample, difference *t*)

$$t = \frac{\left(\sum (X_1 - X_2)\right)/n}{\sqrt{\left(\frac{\sum\left((X_1 - X_2) - \sum (X_1 - X_2)/n\right)^2}{n-1}\right)\left(\frac{1}{n}\right)}}$$

Where *df* = *n* - 1

Pearsons Product Moment Correlation

$$r = \frac{\Sigma XY^2 - \frac{(\Sigma X)(\Sigma Y)}{n}}{\sqrt{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}\sqrt{\Sigma Y^2 - \frac{(\Sigma Y)^2}{n}}}$$

or

$$r = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sqrt{\sum (X - \bar{X})^2}\sqrt{\sum (Y - \bar{Y})^2}}$$

Where *df* = *n* - 2

One Way ANOVA Summary Table

Source	*SS*	*df*	*MS*	*F*	*p*
Between	?	k - 1	SS_{btw}/df_{btw}	MS_{btw}/MS_{within}	?
Within (error)	?	n - k - 1	SS_{within}/df_{within}		
TOTAL	$SS_{btw} + SS_{within}$	n - 1			