Using Chi Square

I. Knowing which statistic to use to test the relationship between each variable depends on the type of data you have (and sometimes the type of question you want to answer). Chi Square can be used when you have two Discrete Variables

II. A. Single Discrete Variable

Goodness of Fit $X^2$ = Allows us to test whether the group frequencies differ from chance patterns (base rate frequencies: the frequency instances naturally occur in the environment). ($df = k - 1$)

$$X^2 = \sum \left( \frac{(O_i - E_i)^2}{E_i} \right)$$

where $O_i$ = observed frequency for each separate group

$E_i$ = expected group frequencies (based on chance)

Calculating Expected Frequencies

For Equal Chances Assumed: $E_i = n/k$ $k$ = number of groups

For Unequal Chances Assumed: $E_i = n(p_i)$ $p_i$ = the probability of occurrence for outcome i.

$p = \% / 100$. 20% = .20

$\Sigma O = n, \Sigma E = n, \Sigma O = \Sigma E$

Statistical Hypotheses:

$H_0 : O_i = E_i$

$H_a : O_i \neq E_i$

Example Research Question: Does number of people who say they like cheezy poofs (Yes = 1) vs. those who do not like cheezy poofs (No = 0), differ significantly from the number expected by chance alone?

The Decision to Reject $H_0$:

If $X^2$ obtained is $\geq X^2$ critical @ $p \leq .05$ for $df = k - 1$;

Then reject $H_0$ (Fail to reject $H_a$)

If $X^2$ obtained is $< X^2$ critical @ $p \leq .05$ for $df = k - 1$;

Then Fail to Reject $H_0$ (Reject $H_a$)

B. Discrete X Discrete

**Pearson’s $X^2$** = Allows us to test whether the cross tabulation pattern of two nominal variables differs from the patterns expected by chance. If one variable is ordinal then $t$ or $F$ are normally used.

($df = (R-1)(C-1)$) where $R$ = # of rows & $C$ = # of columns.

$$X^2 = \sum \left( \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \right)$$

where $i = \text{the different groups for Variable 1}$

$j = \text{the different groups for Variable 2}$

$$E_{ij} = \frac{R_iC_j}{N}$$

where $R_i$ = Row total of row $i$

$C_j$ = Column total of column $j$
Statistical Hypotheses: \( H_0 : O_f = E_f \)
\( H_a : O_f \neq E_f \)

**Example Research Question:** Does the number of people who think they are Eric Cartman (yes = 1, no = 0), relative to whether or not they eat cheezy poofs (yes = 1, no = 0), significantly differ from the frequencies that are expected by chance alone.

Note: a significant two way chi square will not tell you which cells are different. A series of post-hoc or planned goodness of fit chi square analyses would need to be performed to identify exact differences. However, this is rarely done and is just “eye-balled” instead.

C. **Limitations on \( \chi^2 \):**
1) Responses must be independent and mutually exclusive and exhaustive. Each case from the sample should fit into one and only one cell of the cross tab matrix. (This also means that repeated measure designs can not be tested using chi square).
2) Low expected Frequencies limit the validity of \( \chi^2 \).
   - If \( df = 1 \) (e.g., 2x2 matrix), then no expected frequency can be less than 5.
   - Also, If \( df = 2 \), all expected frequencies should exceed 2.
   - If \( df=3 \) or greater, then all expected frequencies except one should be 5 or greater and the one cell needs to have an expected frequency of 1 or greater.

D. **Assessing the Strength of the relationship between 2 Nominal Variables.**

1) **Phi Coefficient** (if 2X2 matrix) correlation coefficient that estimates the strength of the relationship between two dichotomous nominal variables. Note: Phi can not estimate the direction (e.g., positive linear vs. negative linear) of the relationship between 2 nominal variables because the numerical values are arbitrary (direction is meaningless).
   - This correlation coefficient can be calculated exactly like Pearson’s \( r \) (correlation coefficient) or can be estimated using the \( \chi^2 \) statistic. Thus any \( \chi^2 \) can be converted to Phi or Phi can be converted to \( \chi^2 \). (significance should be determined using \( \chi^2 \) tables)

\[
\phi = \sqrt{\frac{\chi^2}{N}} \quad \text{and} \quad \chi^2 = \phi^2 N
\]

**Statistical Hypotheses:**
\( H_0 : \phi = 0 \)
\( H_a : \phi \neq 0 \)

**Example Research Question:** What is the strength of the relationship between whether one thinks they are Eric Cartman or not (Yes = 1, No = 0) and whether one eats cheezy poofs (Yes = 1, No = 0).

2) **Cramer’s Phi or \( V \)** (If 2X3 matrix or larger) correlation coefficient that estimates the strength of the relationship between two discrete nominal variables (correcting for the influence of the number of groups). (Significance should be determined using \( \chi^2 \) tables). Note: Phi can not estimate the direction (positive vs. negative) of the relationship between 2 nominal variables because the numerical values are arbitrary (direction is meaningless)

\[
\phi_{cramer} = \sqrt{\frac{\chi^2}{N(k-1)}} \quad \text{where} \ k \ \text{is the smaller of} \ R \ \text{or} \ C \ (or \ when \ r = c, k = r = c)
\]
Statistical Hypotheses: 
Ho : Phi = 0  
Ha : Phi \neq 0

Example Research Question: What is the strength of the relationship between whether one thinks they are Eric Cartman or not (Yes = 1, No = 0) and whether one prefers cheezy poofs (1), HooHoo Dillies (2), or coa-coa yum-yum’s (3).