Homework 6 (Measures of Variability) Answers

1a. range = 14 - 1 = 13

1b.

	X	X^2	(X-X)	$(X-X)_2$
	1	1	-5.5500	30.8025
	2	4	-4.5500	20.7025
	2	4	-4.5500	20.7025
	3	9	-3.5500	12.6025
	3	9	-3.5500	12.6025
	3	9	-3.5500	12.6025
	4	16	-2.5500	6.5025
	4	16	-2.5500	6.5025
	6	36	-0.5500	0.3025
	6	36	-0.5500	0.3025
	7	49	0.4500	0.2025
	7	49	0.4500	0.2025
	7	49	0.4500	0.2025
	9	81	2.4500	6.0025
	9	81	2.4500	6.0025
	10	100	3.4500	11.9025
	11	121	4.4500	19.8025
	11	121	4.4500	19.8025
	12	144	5.4500	29.7025
	14	196	7.4500	55.5025
Σ	131	1131	0.0000	272.9500
Mean (X)	6.5500			

1b. Sample Variance and Standard Deviation

Using the Mean Deviation Formula

$$s^2 = \frac{\left(X - \overline{X}\right)^2}{n-1} = \frac{272.95}{19} = 14.3658$$

$$s = \sqrt{s^2} = \sqrt{14.3658} = 3.7902$$

Using the Computational Formula

$$s^2 = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{N}}{n-1} = \frac{1131 - \frac{(131)^2}{20}}{20-1} = \frac{1131 - \frac{17{,}161}{20}}{19} = \frac{1131 - 858.05}{19}$$

$$\frac{272.95}{19} = 14.3658$$

$$s = \sqrt{s^2} = \sqrt{14.3658} = 3.7902$$

1c. Population Variance and Standard Deviation
Using the Mean Deviation Formula

$$\sigma^2 = \frac{\left(X - \overline{X}\right)^2}{N} = \frac{272.95}{20} = 13.6475$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{13.6475} = 3.6943$$

Using the Computational Formula

$$\sigma^2 = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{N}}{N} = \frac{1131 - \frac{(131)^2}{20}}{20} = \frac{1131 - \frac{17{,}161}{20}}{20} = \frac{1131 - 858.05}{20}$$

$$\frac{272.95}{20} = 13.6475$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{13.6475} = 3.6943$$

1c. The sample standard deviation is larger than the population standard deviation because the numerator of for the sample standard deviation (n–1) is smaller than the numerator or the pop sd (N).

2a. range = 29 - 3 = 26

2b.

	X	X^2	(X-X)	$(X-X)_2$
	3	9	-11.2800	127.2384
	5	25	-9.2800	86.1184
	6	36	-8.2800	68.5584
	7	49	-7.2800	52.9984
	8	64	-6.2800	39.4384
	8	64	-6.2800	39.4384
	9	81	-5.2800	27.8784
	10	100	-4.2800	18.3184
	10	100	-4.2800	18.3184
	11	121	-3.2800	10.7584
	11	121	-3.2800	10.7584
	12	144	-2.2800	5.1984
	14	196	-0.2800	0.0784
	14	196	-0.2800	0.0784
	15	225	0.7200	0.5184
	16	256	1.7200	2.9584
	17	289	2.7200	7.3984
	18	324	3.7200	13.8384
	18	324	3.7200	13.8384
	21	441	6.7200	45.1584
	22	484	7.7200	59.5984
	23	529	8.7200	76.0384
	23	529	8.7200	76.0384
	27	729	12.7200	161.7984
	29	841	14.7200	216.6784
Σ	357	6277	0.0000	1179.0400
Mean (X)	14.2800			

2b. Population Variance and Standard Deviation
Using the Mean Deviation Formula

$$\sigma^2 = \frac{\left(X - \bar{X}\right)^2}{N} = \frac{1179.04}{25} = 47.1616$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{47.1616} = 6.8674$$

Using the Computational Formula

$$\sigma^2 = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{N}}{N} = \frac{6277 - \frac{(357)^2}{25}}{25} = \frac{6277 - \frac{127{,}449}{25}}{25} = \frac{6277 - 5{,}097.96}{25}$$

$$\frac{1{,}179.04}{25} = 47.1616$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{47.1616} = 6.8674$$

2c. Sample Variance and Standard Deviation
Using the Mean Deviation Formula

$$s^2 = \frac{\left(X - \bar{X}\right)^2}{n - 1} = \frac{1179.04}{24} = 49.1267$$

$$s = \sqrt{s^2} = \sqrt{49.1267} = 7.0090$$

Using the Computational Formula

$$s^2 = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{n}}{n - 1} = \frac{6277 - \frac{(357)^2}{25}}{24} = \frac{6277 - \frac{127{,}449}{25}}{24} = \frac{6277 - 5{,}097.96}{24}$$

$$\frac{1{,}179.04}{24} = 49.1267$$

$$s = \sqrt{s^2} = \sqrt{49.1267} = 7.0090$$

2d. You can ignore this question.

3a. range = 12 - 1 = 11

	X	X^2	(X-X)	$(X-X)_2$
	1	1	-5.0667	25.6714
	2	4	-4.0667	16.5380
	2	4	-4.0667	16.5380
	3	9	-3.0667	9.4046
	3	9	-3.0667	9.4046
	4	16	-2.0667	4.2712
	4	16	-2.0667	4.2712
	6	36	-0.0667	0.0044
	6	36	-0.0667	0.0044
	7	49	0.9333	0.8710
	9	81	2.9333	8.6042
	10	100	3.9333	15.4708
	11	121	4.9333	24.3374
	11	121	4.9333	24.3374
	12	144	5.9333	35.2040
Σ	91	747	-0.0005	194.9333
Mean (X)	6.0667			

3b. Sample Variance and Standard Deviation
Using the Mean Deviation Formula

$$s^2 = \frac{\left(X - \overline{X}\right)^2}{n-1} = \frac{194.933}{14} = 13.9238$$

$$s = \sqrt{s^2} = \sqrt{13.9328} = 3.7315$$

Using the Computational Formula

$$s^2 = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{n}}{n-1} = \frac{747 - \frac{(91)^2}{15}}{15-1} = \frac{747 - \frac{8{,}281}{15}}{14} = \frac{747 - 552.0667}{14}$$

$$\frac{194.9333}{14} = 13.9238$$

$$s = \sqrt{s^2} = \sqrt{13.9238} = 3.7315$$

3c. Population Variance and Standard Deviation
Using the Mean Deviation Formula

$$\sigma^2 = \frac{\left(X - \overline{X}\right)^2}{N} = \frac{194.9333}{25} = 12.9956$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{12.9956} = 3.6049$$

Using the Computational Formula

$$s^2 = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{n}}{n-1} = \frac{747 - \frac{(91)^2}{15}}{15-1} = \frac{747 - \frac{8{,}281}{15}}{14} = \frac{747 - 552.0667}{14}$$

$$\frac{194.9333}{14} = 13.9238$$

$$s = \sqrt{s^2} = \sqrt{13.9238} = 3.7315$$

4a.

	X	X^2	(X-X)	$(X-X)_2$
	10	100	-10.0000	100.0000
	11	121	-9.0000	81.0000
	13	169	-7.0000	49.0000
	14	196	-6.0000	36.0000
	15	225	-5.0000	25.0000
	17	289	-3.0000	9.0000
	18	324	-2.0000	4.0000
	18	324	-2.0000	4.0000
	19	361	-1.0000	1.0000
	19	361	-1.0000	1.0000
	20	400	0.0000	0.0000
	20	400	0.0000	0.0000
	21	441	1.0000	1.0000
	21	441	1.0000	1.0000
	22	484	2.0000	4.0000
	22	484	2.0000	4.0000
	23	529	3.0000	9.0000
	25	625	5.0000	25.0000
	26	676	6.0000	36.0000
	27	729	7.0000	49.0000
	29	841	9.0000	81.0000
	30	900	10.0000	100.0000
Σ	440	9420	0.0000	620.0000
Mean (X)	20.0000			

4a. Sample Variance and Standard Deviation
Using the Mean Deviation Formula

$$s^2 = \frac{\left(X - \overline{X}\right)^2}{n-1} = \frac{620}{21} = 29.5238$$

$$s = \sqrt{s^2} = \sqrt{29.5238} = 5.4336$$

Using the Computational Formula

$$s^2 = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{n}}{n-1} = \frac{9{,}420 - \frac{(440)^2}{22}}{22-1} = \frac{9{,}420 - \frac{193{,}600}{22}}{21} = \frac{9{,}420 - 8{,}800}{21}$$

$$\frac{620}{22} = 29.5238$$

$$s = \sqrt{s^2} = \sqrt{29.5238} = 5.4336$$

4a

	X	X^2	(X-X)	$(X-X)_2$
	1	1	-19.0000	361.0000
	1	1	-19.0000	361.0000
	5	25	-15.0000	225.0000
	6	36	-14.0000	196.0000
	9	81	-11.0000	121.0000
	11	121	-9.0000	81.0000
	11	121	-9.0000	81.0000
	13	169	-7.0000	49.0000
	16	256	-4.0000	16.0000
	19	361	-1.0000	1.0000
	20	400	0.0000	0.0000
	20	400	0.0000	0.0000
	21	441	1.0000	1.0000
	24	576	4.0000	16.0000
	27	729	7.0000	49.0000
	29	841	9.0000	81.0000
	29	841	9.0000	81.0000
	31	961	11.0000	121.0000
	34	1156	14.0000	196.0000
	35	1225	15.0000	225.0000
	39	1521	19.0000	361.0000
	39	1521	19.0000	361.0000
Σ	440	11784	0.0000	2984.0000
Mean (X)	20.0000			

4b. Sample Variance and Standard Deviation

Using the Mean Deviation Formula

$$s^2 = \frac{\left(X - \overline{X}\right)^2}{n-1} = \frac{2984}{21} = 142.0952$$

$$s = \sqrt{s^2} = \sqrt{142.0952} = 11.9204$$

Using the Computational Formula

$$s^2 = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{n}}{n-1} = \frac{11{,}784 - \frac{(440)^2}{22}}{22-1} = \frac{11{,}784 - \frac{193{,}600}{22}}{21} = \frac{11{,}784 - 8{,}800}{21}$$

$$\frac{2{,}984}{21} = 142.0952$$

$$s = \sqrt{s^2} = \sqrt{142.0952} = 11.9204$$

4c. The mean number of penguins seen by stats students and english students is the same. However, this is greater variability in the number of penguins seen by english students, compared to the number of penguins seen by stats students.