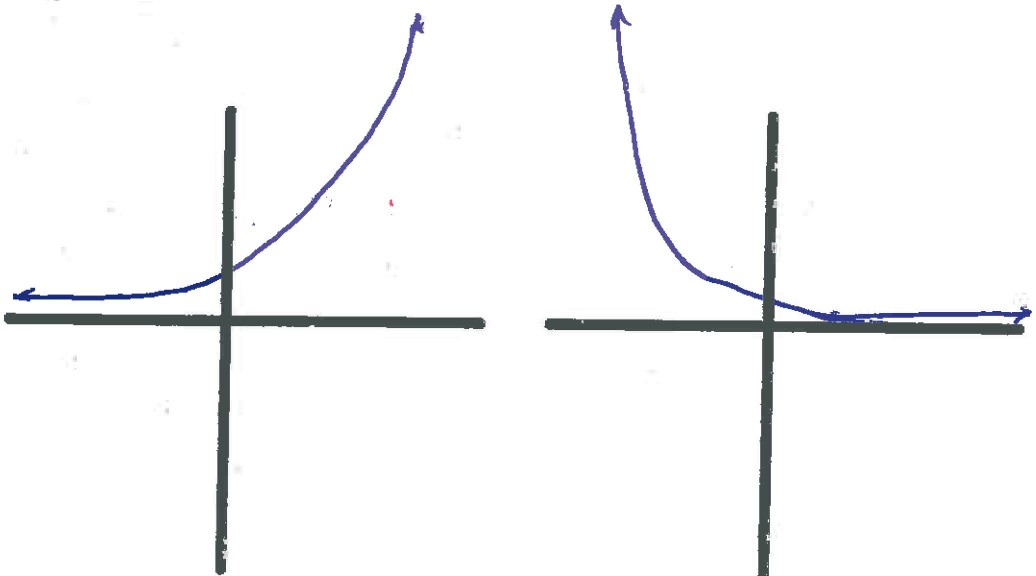
"EXPONENT"

Section 1.6: Exponential Models

An exponential model is written in standard form: $y = b^{x}$ (EXPONENT)

Part I: The graphs of exponential models are as follows:



EXPONENTIAL GROWTH $(b > 1)$

EXPONENTIAL DECAY $(0 < b < 1)$

Part II: A special constant in exponential models: "NATURAL LOGARITHM"

"e" = 2.718

Part III: Evaluation of Exponents on Calculators

Use a calculator to evaluate each.

2^5 : 2 [^] 5 = 32 | 2 [y^x] 5 = 32

$(\frac{1}{2})^4$: (1 ÷ 2) [^] 4 = 0.0625 | (1 ÷ 2) [x^y] 4 = 0.0625

e^4 (2.718) [^] 4 = 54.6 | [e^x] 4 = 54.6

$(e)^{\frac{1}{3}}$ (2.718 [^] (1 ÷ 3) = 1.4 | [e^x] (1 ÷ 3) = 1.4

$2e^{\frac{-1}{2}}$ (2.718) [^] (−1 ÷ 2) = 0.607 × 2 = 1.4

[e^x] (−1 ÷ 2) = 0.607 × 2 = 1.4

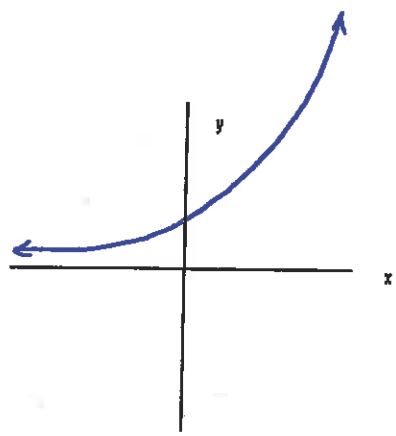
Part IV: Graphing of exponential models.

Sketch the graph of the exponential models below.

1. $y = 3^x$

b = 3

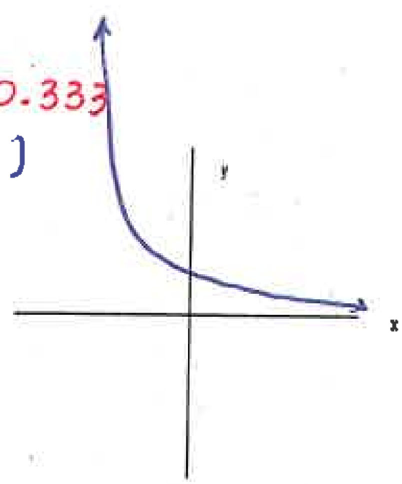
(b > 1)



2. $y = (\frac{1}{3})^x$

$b = \frac{1}{3} = 0.333$

(0 < b < 1)



Part V: Exponential Model Examples (Applications)

1. You have a summer job with 2 payment options.

   Option #1: 6 week job, 5 days a week for $3000 for the summer job.

   Option #2: DAY 1: 2¢
   DAY 2: 4¢
   DAY 3: 8¢
   DAY 4: 16¢
   DAY 5: 32¢
   ⋮
   DAY 30:

   $PAY = 2^x = 2^{30}$

   PAY = 1,073,741,824¢

   PAY = $10,737,418.24

2. Coffee cools in degrees Fahrenheit following the model: $T = 70 + 90e^{-0.045t}$ where t is the number of minutes the coffee has been sitting at room temperature and T is the temperature of the coffee.

   What is the temperature of the coffee after 2 minutes sitting at room temperature?

   $T = 70 + 90e^{-0.045t}$

   $T = 70 + 90(2.718)^{-0.045(2)}$

   $T = 70 + 90(2.718)^{-0.09}$

   $T = 70 + 90(0.914)$

   $T = 70 + 82$

   $T = 152°F$

   What is the temperature of the coffee after 20 minutes sitting at room temperature?

   $T = 70 + 90e^{-0.045t}$

   $T = 70 + 90(2.718)^{-0.045(20)}$

   $T = 70 + 90(2.718)^{-0.9}$

   $T = 70 + 90(0.41)$

   $T = 70 + 37°$

   $T = 107°F$

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3. In 1970, the population of the United States was 205,000,000. What was the predicted population of the United States in 1991 using the model $P = P_0e^{kt}$ given the k value for the United States of 0.89%? HINT: Don't forget to convert 0.89% into a decimal before entering it into the model.

$P = P_0 e^{kt}$

P = FUTURE POPULATION

P_0 = INITIAL POPULATION

e = 2.718

k = GROWTH RATE (0.89% = 0.0089)

t = TIME IN YEARS

$P = 205{,}000{,}000\,(2.718)^{0.0089(21)}$

$P = 205{,}000{,}000\,(2.718)^{0.1869}$

$P = 205{,}000{,}000\,(1.21)$

P = 248,050,000 PEOPLE (PREDICTED)

EXTRA 43,000,000 PEOPLE

ACTUAL POPULATION IN 1991 = 253,000,000