Math 142
Test #2
Spring
2024

Name: _____

Multiple Choice: Choose the answer that best fits as the solution to the question.

1. If you were to be asked to write to proof using an **indirect proof** (as the method) for the statement below, what would be trying to prove?

Given Statement: "If 2^{n-1} is a prime number, then n < 2."

A) If 2^{n-1} is not a prime number, then n $\geq$ 2.
B) If n $\geq$ 2, then 2^{n-1} is a not prime number.
C) If n > 2, then 2^{n-1} is a not prime number.
D) If n < 2, then 2^{n-1} is a not prime number.
E) None of these

2. Suppose n is an integer and is **_not divisible_** by 9. How many remainders are possible for integer, n, when divided by 9?

A) 7
B) 8
C) 9
D) 6
E) None of these

3. Compute: (45 mod 10) - (18 mod 7) =

A) 2
B) 5
C) 4
D) 1
E) None of these

$U = \{-1,0,1,2,3,4,5,6,7,8,9,10\}$, $A = \{3,4,5,6,8\}$, $B = \{2,5,7,10\}$ $C = \{5,6,7,8\}$ $D = \{7,8,9,10\}$,

4. $(A \cap B \cap D)$

    A) $\{8\}$
    B) U
    C) $\{\ \}$
    D) $\{-1, 0, 1, 2, 3, 4, 5, 6, 7, 9, 10\}$
    E) None of these

5. $D - A =$

    A) $\{7, 8, 9, 10\}$
    B) $\{7, 9, 10\}$
    C) $\{2, 5\}$
    D) $\{8, 9\}$
    E) None of these

6. $(B \cup C)' \cap D$

    A) $\{2, 5, 6, 7, 8\}$
    B) $\{9\}$
    C) $\{1, 3, 4, 9, 10\}$
    D) $\{2, 4, 6, 8\}$
    E) None of these

7. $(A \cup B \cup C \cup D)'$

    A) $\{1\}$ B) $\{-1, 0, 1\}$ C) $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ D) $\{1, 2\}$ E) None of these

8. $n(A \cup B \cup C)' =$

    A) $\{-1, 0-, 1, 9\}$
    B) 7
    C) 4
   D) $\{2, 3, 4, 5, 6, 7, 10\}$
    E) None of these

9. How many subsets exist for the empty set?

    A) 0 subsets
    B) 1 subset
    C) 2 subsets
    D) An infinite number of subsets
    E) None of these

10. Given Z is the set of integers. $U = \{Z\}$; $B = \{x : x \in Z \text{ and } x < -1\}$, find B'.

    A) $\{-1, 0, 1, 2, 3, 4\ldots\ldots\}$
    B) $\{0, 1, 2, 3, 4 \ldots\ldots 99\}$
    C) $\{0, 1, 2, 3, 4, 5\ldots\ldots\ldots\}$
    D) Z'
    E) None of these

11. Given Set A is the set of people who listen to music online, Set B is people who listen to music on CD's, and Set C is people who listen to music via the radio. What would the shaded region represent?
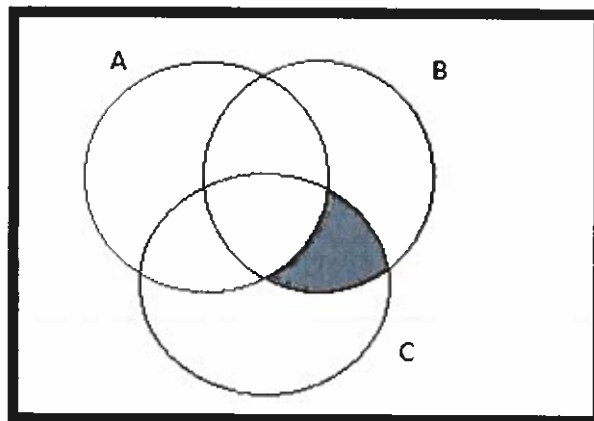
    A) People who listen to music online and on CD's, but not on the radio.
    B) People who listen to music on the radio and CD's, but not online.
    C) People who listen to music online or on CD's, but not the radio.
    D) People who listen to music online, CD's, and the radio.
    E) None of these.

A survey of 200 students at Radford University was taking to determine how students followed the latest news. The following information was obtained:

100 students followed the news via online.
70 students followed the news via a newspaper.
35 students followed the news via television.
20 students followed the news via both television and the newspaper.
15 students followed the news via both television and online.
18 students followed the news via both online and the newspaper.
10 students followed the news via all three news mediums.

12. How many students used two of the mediums to follow the news?

A) 23
B) 87
C) 42
D) 129
E) None of these

13. How many students did not use television to follow the news?

A) 165
B) 38
C) 127
D) 165
E) None of these

14. How many students used only online to follow the news?

A) 119
B) 42
C) 77
D) 10
E) None of these

15. Which of the following statements are true given that Z is the set of integers, N is the set of natural numbers, Q is the set of rational numbers, and R is the set of real numbers.

i. $R \subseteq Z^+$

ii. $Z \subseteq Q$

iii. $Z^- \subseteq R$

iv. $R \subseteq Q$

A) All of the statements are true.
B) None of the statements are true.
C) Only i, ii, iii are true.
D) Only ii and iii are true.
E) Only ii, iii, and iv are true.

Prove any <u>three of the following four</u> statements. Your choice!

Circle the 3 proofs you would like me to grade.

<u>*Should you wish, *star* the 4th proof of your choice for me to grade as extra credit.*</u>

Use the given method where indicated. (Each proof is worth 5 points).

1. Prove:

   If m is even, then $m^3 + 2m$ is divisible by 4.

<u>Test Hypothesis:</u>

<u>Proof:</u>

2. Prove using *an indirect proof:*

If n^3 is even, then n is even.

[HINT: Remember that $(x + y)^3 = x^3 + 3x^2 y + 3xy^2 + y^3$]

Test Hypothesis:

Proof:

3. Prove for any integer, $x^2 + x$ is always even.   [**HINT:** Use two cases for x].

Test Hypothesis:

Proof:

4. Using induction, prove that:

$$1 + 3 + 6 + 10 + \cdots \ldots \ldots \ldots \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$$

Test Hypothesis:

Proof using Induction: