

**Math 142**  
**Test #2**  
**Spring**  
**2023**

Name: \_\_\_\_\_

Multiple Choice: Choose the answer that best fits as the solution to the question.

1. If you were to be asked to write to proof using an *indirect proof* (as the method) for the statement below, what would be trying to prove?

**Given Statement: "If  $2^{n-1}$  is a prime number, then  $n < 2$ ."**

- A) If  $2^{n-1}$  is not a prime number, then  $n \geq 2$ .
- B) If  $n \geq 2$ , then  $2^{n-1}$  is a not prime number.
- C) If  $n > 2$ , then  $2^{n-1}$  is a not prime number.
- D) If  $n < 2$ , then  $2^{n-1}$  is a not prime number.
- E) None of these

2. Suppose  $n$  is an integer and is not divisible by 9. How many remainders are possible for integer,  $n$ , when divided by 9?

- A) 7
- B) 8
- C) 9
- D) 6
- E) None of these

3. Compute:  $(45 \bmod 10) - (18 \bmod 7) =$

- A) 2
- B) 5
- C) 4
- D) 1
- E) None of these

**For questions 4-8, us the following information:**

$U = \{-1,0,1,2,3,4,5,6,7,8,9,10\}$  ,  $A = \{3,4,5,6,8\}$  ,  $B = \{2,5,7,10\}$   $C = \{5,6,7,8\}$   $D = \{7,8,9,10\}$  ,

4.  $(A \cap B \cap D)$

- A) {8}
- B)  $U$
- C) { }
- D) {-1, 0, 1, 2, 3, 4, 5, 6, 7, 9, 10}
- E) None of these

5.  $D - A =$

- A) {7, 8, 9, 10}
- B) {7, 9, 10}
- C) {2, 5}
- D) {8, 9}
- E) None of these

6.  $(B \cup C)' \cap D$

- A) {2, 5, 6, 7, 8}
- B) {9}
- C) {1, 3, 4, 9, 10}
- D) {2, 4, 6, 8}
- E) None of these

7.  $(A \cup B \cup C \cup D)'$

- A) {1}      B) {-1, 0, 1}      C) {2, 3, 4, 5, 6, 7, 8, 9, 10}      D) {1, 2}      E) None of these

8.  $n(A \cup B \cup C)' =$

- A) {-1, 0, 1, 9}
- B) 7
- C) 4
- D) {2, 3, 4, 5, 6, 7, 10}
- E) None of these

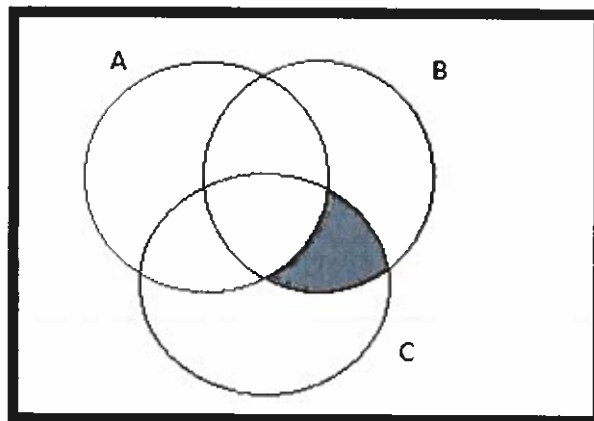
9. How many subsets exist for the empty set?

- A) 0 subsets
- B) 1 subset
- C) 2 subsets
- D) An infinite number of subsets
- E) None of these

10. Given  $Z$  is the set of integers.  $U = \{Z\}$ ;  $B = \{x : x \in Z \text{ and } x < -1\}$ , find  $B'$ .

- A)  $\{-1, 0, 1, 2, 3, 4, \dots\}$
- B)  $\{0, 1, 2, 3, 4, \dots, 99\}$
- C)  $\{0, 1, 2, 3, 4, 5, \dots\}$
- D)  $Z'$
- E) None of these

11. Given Set A is the set of people who listen to music online, Set B is people who listen to music on CD's, and Set C is people who listen to music via the radio. What would the shaded region represent?



- A) People who listen to music online and on CD's, but not on the radio.
- B) People who listen to music on the radio and CD's, but not online.
- C) People who listen to music online or on CD's, but not the radio.
- D) People who listen to music online, CD's, and the radio.
- E) None of these.

**Answer questions 12-14 given the following information:**

A survey of 200 students at Radford University was taken to determine how students followed the latest news. The following information was obtained:

- 100 students followed the news via online.
- 70 students followed the news via a newspaper.
- 35 students followed the news via television.
- 20 students followed the news via both television and the newspaper.
- 15 students followed the news via both television and online.
- 18 students followed the news via both online and the newspaper.
- 10 students followed the news via all three news mediums.

12. How many students used two of the mediums to follow the news?

- A) 23
- B) 87
- C) 42
- D) 129
- E) None of these

13. How many students did not use television to follow the news?

- A) 165
- B) 38
- C) 127
- D) 165
- E) None of these

14. How many students used only online to follow the news?

- A) 119
- B) 42
- C) 77
- D) 10
- E) None of these

15. Which of the following statements are true given that  $Z$  is the set of integers,  $N$  is the set of natural numbers,  $Q$  is the set of rational numbers, and  $R$  is the set of real numbers.

- i.  $R \subseteq Z^+$
- ii.  $Z \subseteq Q$
- iii.  $Z^- \subseteq R$
- iv.  $R \subseteq Q$

- A) All of the statements are true.
- B) None of the statements are true.
- C) Only i, ii, iii are true.
- D) Only ii and iii are true.
- E) Only ii, iii, and iv are true.

**Math 142  
Test #2  
Spring 2021  
(Proof Portion)**

*Prove any three of the following four statements. Your choice!*

*Circle the 3 proofs you would like me to grade.*

*Should you wish, \*star\* the 4<sup>th</sup> proof of your choice for me to grade as extra credit.*

Use the given method where indicated. (Each proof is worth 5 points).

1. Prove:

If  $m$  is even, then  $m^3 + 2m$  is divisible by 4.

**Test Hypothesis:**

**Proof:**

2. Prove using *an indirect proof*:

If  $n^3$  is even, then  $n$  is even.

[HINT: Remember that  $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ ]

**Test Hypothesis:**

**Proof:**

3. Prove for any integer,  $x^2 + x$  is always even. [**HINT:** Use two cases for x].

**Test Hypothesis:**

**Proof:**



4. Using induction, prove that:

$$1 + 3 + 6 + 10 + \dots \dots \dots \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$$

**Test Hypothesis:**

**Proof using Induction:**