2. Prove using induction

$$3 + 6 + 9 + \ldots\ldots\ldots 3n = \frac{3n(n+1)}{2}$$ GOAL: $\frac{3(k+1)(k+2)}{2}$

TEST HYPOTHESIS: $n = 3$ $P(3) = 3+6+9 = 18$ FORMULA: $\frac{3(3)(3+1)}{2} = \frac{36}{2} = 18$ ✓

$n = 4$ $P(4) = 3+6+9+12 = 30$ FORMULA: $\frac{3(4)(4+1)}{2} = \frac{60}{2} = 30$ ✓

$n = 5$ $P(5) = 3+6+9+12+15 = 45$ FORMULA: $\frac{3(5)(5+1)}{2} = \frac{90}{2} = 45$ ✓

PROOF: $n = 1$ $P(1) = 3$ FORMULA: $\frac{3(1)(1+1)}{2} = \frac{6}{2} = 3$ ✓

$n = 2$ $P(2) = 9$ FORMULA: $\frac{3(2)(2+1)}{2} = \frac{18}{2} = 9$ ✓

$n = k$ $P(k) = 3+6+9+\ldots. k = \frac{3k(k+1)}{2}$ ✓

$n = k+1$ $P(k+1) = 3+6+9+\ldots.. k + k+1 = \frac{3k(k+1)}{2} + k+1$

$$= \frac{3k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{(k+1)(3k+2)}{2}$$

SINCE $P(1)$ WAS TRUE, $P(k)$ WAS TRUE

AND $P(k+1)$ WAS TRUE, THEN

$$3+6+9+\ldots.+3n = \frac{3n(n+1)}{2}. \blacksquare$$

3. Prove using induction

$(k+1)(k+2)$

$$2 + 4 + 6 + \ldots\ldots + 2n = n(n+1)$$

TEST HYPOTHESIS: $n = 3$ $P(3)$: $2+4+6 = 12$ FORMULA: $3(3+1) = 3(4) = 12$ ✓

$n = 4$ $P(4)$: $2+4+6+8 = 20$ FORMULA: $4(4+1) = 4(5) = 20$ ✓

$n = 5$ $P(5)$: $2+4+6+8+10 = 30$ FORMULA: $5(5+1) = 5(6) = 30$ ✓

PROOF: STEP 1: $n = 1$ $P(1) = 2$ FORMULA: $1(1+1) = 1(2) = 2$ ✓ +1

$n = 2$ $P(2) = 6$ FORMULA: $2(2+1) = 2(3) = 6$ ✓

STEP 2: $n = k$ $2 + 4 + 6 + \ldots\ldots + 2k = k(k+1)$ +1

$n = k+1$ $2 + 4 + 6 + \ldots\ldots + 2(k+1) = k(\cancel{k+1}) + 2(\cancel{k+1})$

+2 $= (k+1)(k+2)$

SINCE $P(1)$ WAS TRUE, $P(k)$ WAS TRUE, $P(k+1)$ WAS TRUE, THEN

$2+4+6+\ldots 2n = n(n+1)$. ∎ +1