Section 2.2: Proof by cases

For some proofs, each and every case must be proven true for the statement to be true.

Example of a proof by cases.

1. For any integer n, n + 3n is always even.

TEST HYPOTHESIS: $n = 4$ $\quad n+3n = 4+3(4) = 4+12 = 16$ ✓

$n = 5$ $\quad n+3n = 5+3(5) = 5+15 = 20$ ✓

$n = 7$ $\quad n+3n = 7+3(7) = 7+21 = 28$ ✓

$n = 6$ $\quad n+3n = 6+3(6) = 6+18 = 24$ ✓

PROOF: CASE 1: LET n BE AN EVEN INTEGER. FROM THE DEFINITION OF EVEN, THERE EXISTS AN INTEGER k SUCH THAT $n = 2k$.

$$n+3n = 2k+3(2k) \text{ (SUBSTITUTION)}$$
$$= 2k+6k$$
$$= 8k$$
$$= 2(4k)$$

SINCE k IS AN INTEGER, THEN $4k$ IS AN INTEGER. FROM THE DEFINITION OF EVEN, 2·INTEGER IS EVEN. $n+3n = 2\cdot$INTEGER WHICH IS EVEN.

CASE 2: LET n BE AN ODD INTEGER. FROM THE DEFINITION OF ODD, THERE EXISTS AN INTEGER L SUCH THAT $n = 2L+1$.

$$n+3n = (2L+1)+3(2L+1) \text{ (SUBSTITUTION)}$$
$$= 2L+1+6L+3$$
$$= 8L+4$$
$$= 2(4L+2)$$

SINCE L IS AN INTEGER, THEN $4L+2$ IS ALSO AN INTEGER. FROM THE DEFINITION OF EVEN, 2·INTEGER IS EVEN. $n+3n = 2\cdot$INTEGER WHICH IS EVEN.

SINCE BOTH CASES WERE PROVEN TRUE, $n+3n$ IS ALWAYS EVEN FOR ANY INTEGER n. ∎