

Section 2.3: Proofs by Induction

Examples of inductive reasoning

1. 10, 20, 30, 40, 50, 60, 70

$P(1)$ $P(2)$ $P(3)$ $P(4)$ $P(5)$

$n=1$ $n=2$ $n=3$ $n=4$ $n=5$

2. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

$P(1)$ $P(2)$, $P(3)$

Steps to prove by induction

If $P(n)$ is an open sentence and:

Step 1: $P(1)$ MUST BE TRUE

Step 2: WHERE $P(k)$ IS TRUE, THEN $P(k+1)$ IS ALSO TRUE.

Then $P(n)$ is true for all values of n .

Examples of proofs using induction

1. Prove using induction

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \quad \text{GOAL: } \frac{(k+1)(k+2)}{2}$$

TEST HYPOTHESIS

$$n=3 \quad P(3) = 1+2+3 = 6 \quad \text{FORMULA: } \frac{3(3+1)}{2} = \frac{3(4)}{2} = 6$$

$$n=4 \quad P(4) = 1+2+3+4 = 10 \quad \text{FORMULA: } \frac{4(4+1)}{2} = \frac{4(5)}{2} = 10$$

$$n=5 \quad P(5) = 1+2+3+4+5 = 15 \quad \text{FORMULA: } \frac{5(5+1)}{2} = \frac{5(6)}{2} = 15$$

PROOF:

STEP 1: $n=1 \Rightarrow P(1) = 1$ FORMULA: $\frac{1(1+1)}{2} = \frac{1(2)}{2} = 1$ ✓

$$n=2 \Rightarrow P(2) = 3 \quad \text{FORMULA: } \frac{2(2+1)}{2} = \frac{2(3)}{2} = 3 \quad \checkmark$$

$$n=3 \Rightarrow P(3) = 6 \quad \text{FORMULA: } \frac{3(3+1)}{2} = \frac{3(4)}{2} = 6 \quad \checkmark$$

STEP 2: $n=k \Rightarrow P(k) = 1+2+3+4+\dots+k = \frac{k(k+1)}{2}$ ✓

$$n=k+1 \Rightarrow P(k+1) = 1+2+3+4+\dots+k + (k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

SINCE $P(1)$ WAS TRUE, $P(k)$ WAS TRUE
AND $P(k+1)$ WAS TRUE, THEN

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \quad \blacksquare$$