Section 1.4: Predicates and Quantifiers

The concept of a Predicate

A predicate, denoted as P(x), is a statement that incorporates a variable x such that when x is replaced the a value, the predicate is either true or false.

Examples:

Let P(x) be the predicate for “x is even”.

Let x = 2, 17, 22.

P (2):

P (17):

P (22):

Let P(x) be the predicate for “x > 15”.

Let x = 4, 11, 15.

P (4):

P (11):

P (15):

Let D = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Which elements satisfy each predicate and predicate negation?

Predicate	Values of D	Negated Predicate	Values of D
$x \geq 8$			
$(x > 5 \text{ } and \text{ } x \text{ } is \text{ } even)$			
$x \geq x^2$			

Quantifiers (Definitions and Symbols)

A universal quantifier are words such as "all" and "every" that are added to a statement to make a statement true for every instance.

Notation: ∀ = "all", "every"

Example:

Statement: Men are lazy.

Universal Quantifier:

An existential quantifier are words such as "there exists" and "some" that are added to statements to make a statement true for at least one instance.

Notation: ∃ = "Some", "there exists"

Example:

Statement: Sharks attack humans.

Existentially Quantified:

The symbol ∈ indicates membership in a set.

Example:

n ∈ D: "n is an element in set D."

Example: $D = \{5, 10, 15, 20, 25\}$.

$\forall n \in D, n < 20$. True or False?

$\forall n \in D, (n < 5) \vee (n$ is a multiple of $5)$. True or false?

Example: $D = \{-2, -1, 0, 1, 2\}$.

$\exists n \in D, n \geq -2$. True or False?

$\forall n \in D, n > -2$. True or False?

$\forall n \in D, (n > -3) \wedge (n < 3)$. True or False?

$\exists n \in D, (n > 10) \vee (n \leq -2)$. True or False?