Math 142
Test #2 Practice Proofs
Spring 2024

1. If m is divisible by 4 and n is any even integer, then $m \cdot n$ is divisible by 8.

Test Hypothesis: $m = 8 \quad n = 2 \qquad m \cdot n = 8 \cdot 2 = 16 = 8(2)$ ✓

$m = 4 \quad n = 6 \qquad m \cdot n = 4 \cdot 6 = 24 = 8(3)$ ✓

$m = 12 \quad n = 2 \qquad m \cdot n = 12 \cdot 2 = 24 = 8(3)$ ✓

PROOF: LET m BE DIVISIBLE BY 4. FROM THE DEFINITION OF DIVISIBILITY, THERE EXISTS AN INTEGER K SUCH THAT $m = 4K$. +1

LET n BE AN EVEN INTEGER. FROM THE DEFINITION OF EVEN, THERE EXISTS AN INTEGER L SUCH THAT $n = 2L$. +1

+1 $m \cdot n = 4K \cdot 2L = 8KL$ (SUBSTITUTION)

$m \cdot n = 8KL$

+1 SINCE L & K ARE INTEGERS, K·L IS ALSO AN INTEGER. FROM THE DEFINITION OF DIVISIBILITY, 8 · INTEGER IS DIVISIBLE BY 8.

$m \cdot n = 8 \cdot$ INTEGER AND IS DIVISIBLE BY 8. ■ +1

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$\sim Q \longrightarrow \sim P$

$P \longrightarrow Q$

2. If n^2 is even, then n is even. (**HINT**: **Use an indirect proof**).

CONTRAPOSITIVE: "IF n IS ODD, THEN n^2 IS ODD."

Test Hypothesis: $n = 5$ $\quad n^2 = 5^2 = 25$ ✓

$n = 11$ $\quad n^2 = 11^2 = 121$ ✓

$n = 9$ $\quad n^2 = 9^2 = 81$ ✓

PROOF: LET n BE AN ODD INTEGER. FROM THE DEFINITION OF ODD, THERE EXISTS AN INTEGER K SUCH THAT $n = 2K+1$.

$$\begin{aligned} n^2 &= (2k+1)^2 \\ &= (2k+1)(2k+1) \\ &= 4k^2 + 2k + 2k + 1 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \end{aligned}$$

SINCE K IS AN INTEGER, $2k^2 + 2k$ IS ALSO AN INTEGER. FROM THE DEFINITION OF ODD, $2 \cdot \text{INTEGER} + 1$ IS ODD. $n^2 = 2 \cdot \text{INTEGER} + 1$ WHICH MEANS n^2 IS ODD. SINCE THE CONTRAPOSITIVE WAS TRUE, THEN THE ORIGINAL STATEMENT IS ALSO TRUE. IF n^2 IS EVEN, THEN n IS EVEN. ■