Section 5.2: Permutations

The number of r permutations from a set of n elements is given by: $\frac{n!}{(n-r)!} = {}_nP_r$

Where there are n ways to permute r items.

******ORDER IS IMPORTANT**********

Examples:

1. How many ways are possible to set a 4-digit bike lock (digits 0-9).

✓ Counting Principle (with repetition allowed):

$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$ WAYS

Counting Principle (with no repetition allowed):

$10 \cdot 9 \cdot 8 \cdot 7 = 5040$ WAYS

Permutation Formula:

$${}_nP_r = \frac{n!}{(n-r)!} = {}_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}}{\cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}$$

$$= 10\cdot 9\cdot 8\cdot 7$$

$$= 5040 \text{ WAYS}$$

2. How many ways are possible to pick three students from the class to speak to a group of high school seniors at RU?

Counting Principle: $\underset{7}{\_\_} \cdot \underset{6}{\_\_} \cdot \underset{5}{\_\_} = 210$ WAYS

Permutation Formula: $_nP_r = \frac{n!}{(n-r)!} = {_7P_3} = \frac{7!}{(7-3)!} = \frac{7!}{4!}$

$$= \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$

$= 210$ WAYS

Combinations

A combination is a distinct group of objects without regard for arrangement. The number of r combinations from n elements is:

$$ {}_{n}C_{r} = \frac{n!}{r!\,(n-r)!} $$

******ORDER IS NOT IMPORTANT**********

Examples:

1. How many doubles combinations can a tennis coach choose from 5 players?

$P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_5$

NO FORMULA:

$P_1 - P_2$	$P_2 - P_3$	$P_3 - P_4$	$P_4 - P_5$
$P_1 - P_3$	$P_2 - P_4$	$P_3 - P_5$	
$P_1 - P_4$	$P_2 - P_5$		
$P_1 - P_5$			

10 DOUBLES TEAMS ✓

FORMULA:

$$ {}_{n}C_{r} = \frac{n!}{r!\,(n-r)!} = {}_{5}C_{2} = \frac{5!}{2!\,(5-2)!} = \frac{5!}{2!\,(3!)} $$

$$ = \frac{5\cdot 4\cdot \cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{(2\cdot 1)(\cancel{3}\cdot\cancel{2}\cdot\cancel{1})} $$

$$ = \frac{20}{2} $$

$$ = 10 \checkmark $$

2. How many ways can a coach choose 3 captains from a roster of 12 players?

$$_nC_r = \frac{n!}{r!(n-r)!} = {}_{12}C_3 = \frac{12!}{3!(12-3)!} = \frac{12!}{3!\cdot\cancel{9!}}$$

$$= \frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1} = \frac{1320}{6}$$

$$= 220 \text{ WAYS}$$

3. The play the Virginia Cash 5 Lottery (to win $100,000) a player must choose 5 numbers from a list of 34. How many possible ways can a player choose their numbers?

$$_nC_r = \frac{n!}{r!(n-r)!} = {}_{34}C_5 = \frac{34!}{5!(34-5)!} = \frac{34!}{5!\,(\cancel{29})!}$$

$$= \frac{34\cdot 33\cdot 32\cdot 31\cdot 30}{5\cdot 4\cdot 3\cdot 2\cdot 1}$$

$$= \frac{33390720}{120}$$

$$= \underbrace{278,256}_{\text{POSSIBLE WAYS!}}$$

EACH COSTS $1