

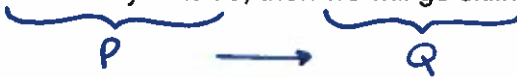
Section 2.2: Indirect proofs

An indirect proof is a proof created by using the contrapositive of a given conditional statement.

Recall, $P \rightarrow Q \equiv \sim Q \rightarrow \sim P$

Create the contrapositive for the following statements:

If it is sunny outside, then we will go skiing.



IF WE DO NOT GO SKIING, THEN IT IS NOT SUNNY OUTSIDE.

If n is even, the $2n$ is odd.



IF $2n$ IS EVEN, THEN n IS ODD.

Examples of indirect proofs.

1. Prove using an indirect proof the following statement.

$$\begin{array}{l} \text{If } 3n \text{ is odd, then } n \text{ is odd.} \quad P \rightarrow Q \\ \underbrace{\hspace{1.5cm}} \quad \underbrace{\hspace{1.5cm}} \\ P \rightarrow Q \\ \sim Q \rightarrow \sim P \end{array}$$

"IF n IS EVEN, THEN $3n$ IS EVEN."

<u>TEST HYPOTHESIS:</u>	$n = 4$	$3n = 3(4) = 12 \checkmark$
	$n = 10$	$3n = 3(10) = 30 \checkmark$
	$n = 20$	$3n = 3(20) = 60 \checkmark$

PROOF: LET n BE AN EVEN INTEGER. FROM THE DEFINITION OF EVEN, THERE EXISTS AN INTEGER k SUCH THAT $n = 2k$.

$$\begin{aligned} 3n &= 3(2k) \quad (\text{SUBSTITUTION}) \\ &= 6k \\ &= 2(3k) \end{aligned}$$

SINCE k IS AN INTEGER, $3k$ IS ALSO AN INTEGER. FROM THE DEFINITION OF EVEN, $2 \cdot$ INTEGER IS EVEN. THEREFORE $3n$ IS EVEN. SINCE THE CONTRAPOSITIVE WAS PROVEN TRUE, THEN THE ORIGINAL STATEMENT IS ALSO TRUE. ■

2. Prove using an indirect proof the following statement.

If $m \times n$ is even, then either m or n must be even.

$$\begin{array}{ccc} P & \longrightarrow & Q \\ \sim Q & \longrightarrow & \sim P \end{array}$$

"IF m AND n ARE ODD, THEN $m \times n$ IS ODD."

TEST HYPOTHESIS :

$m = 3$	$n = 5$	$m \cdot n = 3 \cdot 5 = 15 \checkmark$
$m = 11$	$n = 3$	$m \cdot n = 11 \cdot 3 = 33 \checkmark$
$m = 9$	$n = 7$	$m \cdot n = 9 \cdot 7 = 63 \checkmark$

PROOF: LET m BE ODD. FROM THE DEFINITION OF ODD, THERE EXISTS AN INTEGER L SUCH THAT $m = 2L + 1$. LET n ALSO BE ODD, FROM THE DEFINITION OF ODD, THERE EXISTS AN INTEGER K SUCH THAT $n = 2K + 1$.

$$\begin{aligned} m \cdot n &= (2L + 1)(2K + 1) \quad (\text{SUBSTITUTION}) \\ &= 4LK + 2K + 2L + 1 \\ &= 2(2LK + K + L) + 1 \end{aligned}$$

SINCE $K \neq L$ ARE INTEGERS, $2LK + K + L$ IS ALSO AN INTEGER. FROM THE DEFINITION OF ODD, $2 \cdot \text{INTEGER} + 1$ IS ODD. $m \cdot n = 2 \cdot \text{INTEGER} + 1$ WHICH IS ODD. SINCE THE CONTRADICTORY WAS TRUE, THEN THE ORIGINAL STATEMENT IS ALSO TRUE. ■