

## Additional Definitions for Proofs

Definition of Divisibility: An integer  $n$  is divisible by a non-zero integer  $k$  if there is an integer  $q$  (the quotient) such that  $n = k \cdot q$  where  $k$  is a factor of  $n$ .

Examples of Divisibility:  $n = 10$

$$10 = 2 \cdot 5$$

← FACTOR  
← QUOTIENT

"10 is divisible by 2."

$n = 18$

$$18 = 9 \cdot 2$$

← FACTOR  
← QUOTIENT

"18 is divisible by 9."

$n = 40$

$$40 = 4 \cdot 10$$

← FACTOR  
← QUOTIENT

"40 is divisible by 4."

The Division Theorem: For all integers  $a$  and  $b$  (where  $b > 0$ ), THERE EXISTS AN INTEGER  $Q$  (THE QUOTIENT) WHEN  $a$  IS DIVIDED BY  $b$ . THERE CAN EXIST AN INTEGER  $r$  CALLED THE REMAINDER.

$$a = b \cdot Q + r \quad \text{WHERE } 0 \leq r < b$$

Examples of the Division Theorem:

$a = 25$	$25 = 5 \cdot 5 + 0 \checkmark$
$a = 11$	$11 = 5 \cdot 2 + 1 \checkmark$
$a = 32$	$32 = 5 \cdot 6 + 2 \checkmark$
$a = 13$	$13 = 5 \cdot 2 + 3 \checkmark$
$a = 24$	$24 = 5 \cdot 4 + 4 \checkmark$

The Division Theorem written in Mod notation Examples

① $19 \text{ MOD } 3 = 1$	$19 = 3 \cdot 6 + 1$
② $27 \text{ MOD } 5 = 2$	$27 = 5 \cdot 5 + 2$
③ $37 \text{ MOD } 6 = 1$	$37 = 6 \cdot 6 + 1$
④ $31 \text{ MOD } 3 = 1$	$31 = 3 \cdot 10 + 1$
⑤ $26 \text{ MOD } 7 = 5$	$26 = 7 \cdot 3 + 5$
⑥ $8 \text{ MOD } 11 = 8$	$8 = 11 \cdot 0 + 8$