Chapter 5: Combinatorics

The Counting Principle, Permutations and Combinations

Section 5.1: The Counting Principle

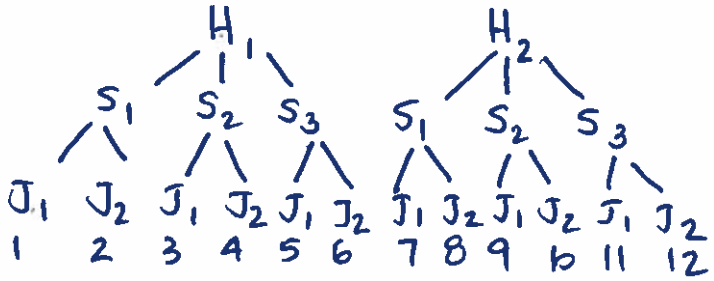
If a process can be completed in k steps with.....

- n_1 ways of doing step 1
- n_2 ways of doing step 2
- n_3 ways of doing step 3

There are $n_1 \cdot n_2 \cdot n_3 \cdot \ldots\ldots n_n$ ways of completing process k.

Examples:

1. How many ways can you wear 2 hats, 3 scarfs, and 2 jackets, 1 item at a time?



(2 HATS) x (3 SCARFS) x 2 JACKETS

= 12 WAYS

2. How many possible outfits are possible for the clothes in your closet? How many years will have be able to wear a different outfit each day?

25 SHIRTS

30 PANTS

50 SHOES

100 ACCESSORIES

$25 \times 30 \times 50 \times 100$

$= \frac{3,750,000}{365} = 10,274$ YEARS

3. How many ways can you create a Subway sandwich?

SIZE: 2
BREAD: 7
MEAT: 9
CHEESE: 7
TOASTED: 3
VEGGIES: 13
CONDIMENTS: 19

$2 \cdot 7 \cdot 9 \cdot 7 \cdot 3 \cdot 13 \cdot 19 = 653{,}562$ SANDWICHES

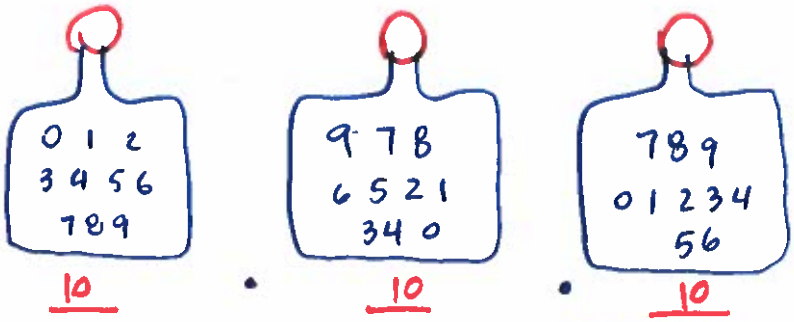
$\frac{653{,}562}{365}$

$= 1790$ YEARS

4. How many 8-character passwords exist to protect and email account where repetition is not allowed?

$46 \cdot 45 \cdot 44 \cdot 43 \cdot 42 \cdot 41 \cdot 40 \cdot 39 = 1.05 \times 10^{13}$

5. Guaranteed Virginia Lottery Pick-3 game to win $500. A player must pick three numbers and match in exact order from the lottery drawing held twice a day in Virginia.



10 · 10 · 10 = 1000 OUTCOMES (COSTS $1)

6. How many Virginia license plates are possible (with repetition allowed) if the first three characters much be letters and the next four characters must be numbers (X cannot be used as the first letter and 0 cannot be used as the first number).

$25 \cdot 26 \cdot 26 \cdot 9 \cdot 10 \cdot 10 \cdot 10 = 152{,}100{,}000$