Section 1.5: Defining the Converse, Inverse, and Contrapositive of a Conditional

Given the Implication: $P \to Q$

The Converse: $Q \to P$

The Inverse: $\sim P \to \sim Q$

The Contrapositive: $\sim Q \to \sim P$

NEGATION: $\sim(P \to Q) \equiv P \wedge \sim Q$

Examples: Create each given the conditionals:

$\underbrace{\text{If it snows}}_{P}$, then $\underbrace{\text{we will go skiing}}_{Q}$.

$P \longrightarrow Q$

The converse: $Q \to P$

IF WE GO SKIING, THEN IT SNOWS.

The inverse: $\sim P \to \sim Q$

IF IT DOES NOT SNOW, THEN WE WILL NOT GO SKIING.

The Contrapositive: $\sim Q \to \sim P$

IF WE WILL NOT GO SKIING, THEN IT DID NOT SNOW.

NEGATION: $P \wedge \sim Q$

~~WE WILL GO SKIING AND~~

IT SNOWS AND WE WILL NOT GO SKIING.

If it does not stop raining, then we cannot go outside.

$$\sim P \longrightarrow \sim Q$$

The converse:

IF WE CANNOT GO OUTSIDE, THEN IT DOES NOT STOP RAINING.

The inverse:

IF IT DOES STOP RAINING, THEN WE CAN GO OUTSIDE.

The Contrapositive:

IF WE CAN GO OUTSIDE, THEN IT DOES STOP RAINING.

NEGATION: $P \wedge \sim Q$

IT DOES NOT STOP RAINING AND WE CAN GO OUTSIDE.

Truth Tables and comparison for each.

P	Q	$P \to Q$	$Q \to P$	$\sim P$	$\sim Q$	$\sim P \to \sim Q$	$\sim Q \to \sim P$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

General rule for logically equivalent statements of the conditional.

THE IMPLICATION AND THE CONTRAPOSITIVE ARE LOGICALLY EQUIVALE

THE CONVERSE AND THE INVERSE ARE LOGICALLY EQUIVALENT.

Logical Equivalence Relationships of the Conditional

Given: If you do not eat your veggies, then you cannot watch TV.

The Converse: IF I CANNOT WATCH TV, THEN I DID NOT EAT MY VEGGIES.

The Inverse:

IF I DO EAT MY VEGGIES, THEN I CAN WATCH TV.

The Contrapositive.

IF I CAN WATCH TV, THEN I DID EAT MY VEGGIES.

Which of the above is logically equivalent to the original conditional?

Truth tables for the given implication, the conditional, the inverse, and the contrapositive.

The given implication:

The converse:

The inverse:

The contrapositive:

Create logically equivalent statements for each.

1. If today is not sunny, then we cannot go to the beach.

Logically Equivalent: (CONTRAPOSITIVE)

IF WE CAN GO TO THE BEACH, THEN TODAY IS SUNNY.

2. Today is Wednesday and you have Math 142. $\sim P \vee \sim Q$

$\sim (P \wedge Q)$

Logically Equivalent:

TODAY IS NOT WEDNESDAY OR YOU DO NOT HAVE MATH 142.

3. If $x \geq 7$, then $y < 10$.

Logically equivalent statement: (CONTRAPOSITIVE)

IF $y \geq 10$, THEN $x < 7$.

4. If $m = 17$, the $n > 12$.

Logically equivalent statement: (CONTRAPOSITIVE)

IF $n \leq 12$, THEN $m \neq 17$.