

### Section 1.3: Truth Tables (Conjunctions and Disjunctions)

What is a truth table?

A TRUTH TABLE IS AN ANALYTICAL TOOL TO DETERMINE THE TRUTH VALUE OF ALL COMBINATIONS OF A COMPOUND STATEMENT.

#### The Conjunction ("and" statement) Truth Table

Given: P = Today is ~~Monday~~ <sup>WEDNESDAY.</sup> Q = It is cloudy outside.

Case 1: Today is ~~Monday~~ <sup>WEDNESDAY</sup> and it is cloudy outside. (T) (T) (T)

Case 2: Today is ~~Monday~~ <sup>WEDNESDAY</sup> and it is snowing. (T) (F) (F)

Case 3: Today is ~~Sunday~~ <sup>Sunday</sup> and it is cloudy outside. (F) (T) (F)

Case 4: Today is ~~Sunday~~ <sup>Sunday</sup> and it is raining. (F) (F) (F)

General Rule for the Conjunction Truth Table:

FOR A CONJUNCTION, ALL PORTIONS OF THE STATEMENT MUST BE TRUE FOR THE STATEMENT TO BE TRUE.

### The Disjunction (or statement) Truth Table

Given: P = You are a student at Radford University. Q = It is January.

Case 1: You are a student at Radford University or it is January.

Case 2: You are a student at Radford University or it is July.

Case 3: You are a student at Virginia Tech or it is January.

Case 4: You are a student at Virginia Tech or it is July.

(T)  
(T)  
(F)  
(T)  
(T)  
(F)

General Rule for the Disjunction Truth Table:

IF ONE PART OF A DISJUNCTION IS TRUE, THEN THE ENTIRE DISJUNCTION IS TRUE.

Create a Truth Table for each statement

1.  $P \vee Q$

<u>P</u>	<u>Q</u>
T	T
T	F
F	T
F	F

<u>P ∨ Q</u>
T
T
T
F

2.  $P \wedge Q$

<u>P</u>	<u>Q</u>
T	T
T	F
F	T
F	F

<u>P ∧ Q</u>
T
F
F
F

3.  $\sim P \vee \sim Q$

<u>P</u>	<u>Q</u>	<u><math>\sim P</math></u>	<u><math>\sim Q</math></u>	<u><math>\sim P \vee \sim Q</math></u>
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

4.  $\sim Q \wedge P$

<u>P</u>	<u>Q</u>	<u><math>\sim Q</math></u>	<u><math>\sim Q \wedge P</math></u>
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

5.  $\sim(P \wedge \sim Q)$

<u>P</u>	<u>Q</u>	<u><math>\sim Q</math></u>	<u><math>(P \wedge \sim Q)</math></u>	<u><math>\sim(P \wedge \sim Q)</math></u>
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

6.  $(P \vee \sim Q) \wedge (\sim P \wedge Q)$

$P$	$Q$	$\sim P$	$\sim Q$	$(P \vee \sim Q)$	$(\sim P \wedge Q)$	$(P \vee \sim Q) \wedge (\sim P \wedge Q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	F	F

↑  
CONTRADICTION

Vocabulary of Truth Table results:

Tautology : ALL COMBINATIONS ARE TRUE.

Contradiction : ALL COMBINATIONS ARE FALSE.

Logically Equivalent : ARE STATEMENTS HAVE THE SAME TRUTH TABLE VALUES. SYMBOL: " $\equiv$ "

7. Is  $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$ ?

$P$	$Q$	$\sim P$	$\sim Q$
T	T	F	F
T	F	F	T
F	T	T	F
F	F	T	T

$(P \vee Q)$
T
T
T
F

$\sim(P \vee Q)$
F
F
F
T

$(\sim P \wedge \sim Q)$
F
F
F
T

YES, LOGICALLY EQUIVALENT!

8. Is  $(P \vee Q) \wedge R$  a tautology, contradiction, or neither?

$P$	$Q$	$R$
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

$(P \vee Q)$
T
T
T
T
T
T
T
F

$(P \vee Q) \wedge R$
T
F
T
F
T
F
F
F

NEITHER