Section 1.4: Predicates and Quantifiers

The concept of a Predicate

A predicate, denoted as P(x), is a statement that incorporates a variable x such that when x is replaced the a value, the predicate is either true or false.

Examples:

Let P(x) be the predicate for "x is even".

Let x = 2, 17, 22.

P (2): TRUE

P (17): FALSE

P (22): TRUE

Let P(x) be the predicate for "x > 15".

Let x = 4, 11, 15.

P (4): FALSE

P (11): FALSE

P (15): FALSE

Let D = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Which elements satisfy each predicate and predicate negation?

Predicate	Values of D	Negated Predicate	Values of D
$x \geq 8$	8, 9, 10	$x < 8$	1, 2, 3, 4, 5, 6, 7
$(x > 5 \text{ and } x \text{ is even})$	6, 8, 10	$(x \leq 5$ OR x IS ODD)	1, 2, 3, 4, 5, 7, 9
$x \geq x^2$	1	$x < x^2$	2, 3, 4, 5, 6, 7, 8, 9, 10

Quantifiers (Definitions and Symbols)

A universal quantifier are words such as "all" and "every" that are added to a statement to make a statement true for every instance.

Notation: $\forall$ = "all", "every"

Example:

Statement: Men are lazy. IF IT IS A MAN, THEN IT IS LAZY.

Universal Quantifier:

ALL MEN ARE LAZY.

An existential quantifier are words such as "there exists" and "some" that are added to statements to make a statement true for at least one instance.

Notation: $\exists$ = "Some", "there exists"

Example:

Statement: Sharks attack humans.

Existentially Quantified: SOME SHARKS ATTACK HUMANS.

The symbol $\in$ indicates membership in a set.

Example:

$n \in D$: "n is an element in set D."

Example: D = {5, 10, 15, 20, 25}.

$\forall n \in D, n < 20$. True or False?

"EVERY ELEMENT n IN SET SET D IS LESS THAN 20."

$\forall n \in D, (n < 5) \vee (n \text{ is a multiple of } 5)$. True or false?

"EVERY ELEMENT IN SET D IS LESS THAN 5 OR A MULTIPLE OF 5."

Example: D = {-2, -1, 0, 1, 2}.

$\exists n \in D, n \geq -2$. True or False?

"THERE EXISTS ELEMENTS n IN SET D THAT ARE ARE GREATER OR EQUAL TO -2"

$\forall n \in D, n > -2$. True or False?

"EVERY ELEMENTS n IN SET D ARE GREATER THAN -2."

$\forall n \in D, (n > -3) \wedge (n < 3)$. True or False?

$\exists n \in D, (n > 10) \vee (n \leq -2)$. True or False?