

NAME: _____

ROW: _____
(row farthest to the left = row 1)

QUADRATIC FORMULA

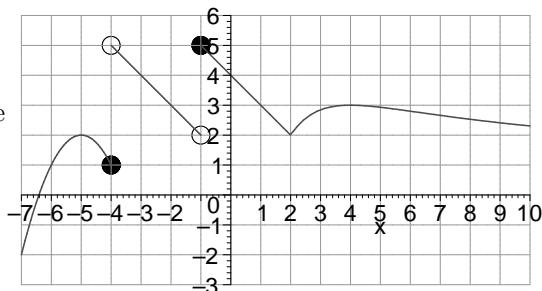
The solutions to the quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**ALL CALCULATED VALUES MUST BE ACCURATE TO
2 DECIMAL PLACES BEYOND THE DECIMAL POINT.**

1. (20 points) Use the given graph of $f(x)$ to answer the following questions. Be as explicit as possible.

(a) Complete the table below with the appropriate symbols for $f'(x)$ and $f''(x)$.

- $\left\{ \begin{array}{l} + \text{ for positive} \\ - \text{ for negative} \\ 0 \text{ for zero} \\ \text{NE for "does not exist"} \end{array} \right.$



x	-6	-5	-4	0	2	4
$f'(x)$						
$f''(x)$						

- (b) Estimate $f'(0) \approx$ _____.
- (c) Circle all of the x -values in the list $\{-6, -5, -4, -1, 2, 4, 6\}$ at which relative maxima occur.
- (d) Circle all of the x -values in the list $\{-6, -5, -4, -1, 2, 4, 6\}$ at which relative minima occur.
- (e) For the function $f(x)$ on $[-6, 6]$ the x -values at which absolute maxima occur are $x =$ _____.
(write NE if none exists)
- (f) For the function $f(x)$ on $[-6, 6]$ the x -values at which absolute minima occur are $x =$ _____.
(write NE if none exists)
- (g) The x -values at which discontinuities occur are $x =$ _____. (write NE if none exists)
- (h) The x -values at which $f(x)$ is NOT differentiable are $x =$ _____. (write NE if none exists)
- (i) Use the graph of $f(x)$ to estimate the following values: (each grid mark = 1 unit)

$$\lim_{x \rightarrow -1^-} f(x) = \underline{\hspace{2cm}} \quad \lim_{x \rightarrow -1^+} f(x) = \underline{\hspace{2cm}} \quad \lim_{x \rightarrow -1} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow -4} f(x) = \underline{\hspace{2cm}} \quad \lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$$

2. (3 points) Find the equation of the line through the points $(-1, 3)$ and $(1, 2)$.

3. (8 points) Use all of the clues you can obtain about domain, asymptotes, extrema, intercepts, etc. in order to match the functions below with the correct figures.

(a) $f(x) = \frac{x}{x^2 + 900}$ matches figure

(b) $g(x) = \frac{x}{x^2 - 900}$ matches figure

(c) $h(x) = \frac{x^2}{x^2 - 900}$ matches figure

(d) $s(x) = \sqrt{900 - x^2}$ matches figure

(e) $t(x) = \sqrt{x^2 - 900}$ matches figure

(f) $u(x) = |x^2 - 900|$ matches figure

FIGURE A

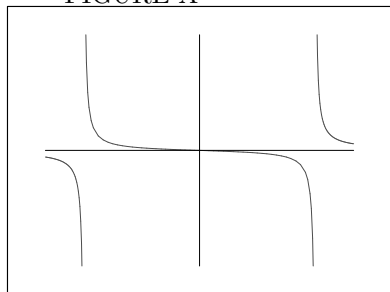


FIGURE B

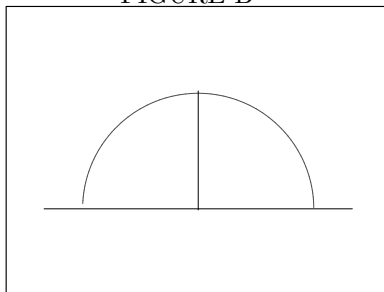


FIGURE C

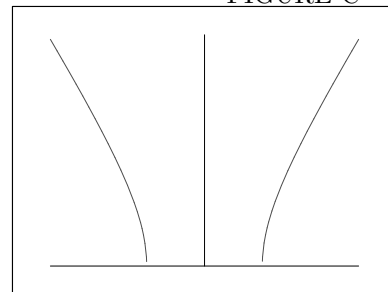


FIGURE D

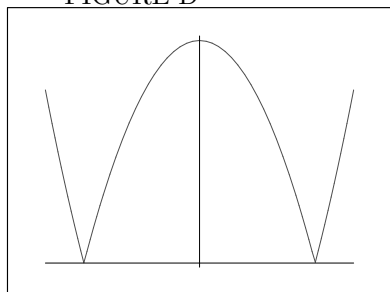


FIGURE E

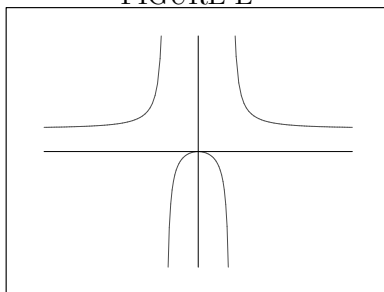
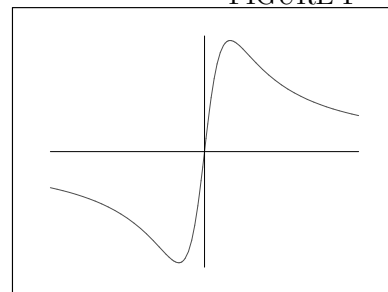


FIGURE F



4. (7 points) Consider the function $g(x) = (x - 3)^2(2x + 5)$.

- (a) Find all critical points: $x =$ _____
 - (b) Classify each critical point as one of $\{relative\ maximum, relative\ minimum, neither\}$. Justify your claim for each conclusion.
 - (c) Find all inflection points: $x =$ _____
 - (d) Find the *absolute* extrema for g on the interval $[-3, 4]$.
-

5. (7 points) Consider the function

$$f(x) = \frac{(x - 2)(2x + 5)}{4(x - 2)(x - 5)}$$

Find the following limits:

- (a) $\lim_{x \rightarrow 0} f(x) =$ _____
 - (b) $\lim_{x \rightarrow 2} f(x) =$ _____
 - (c) $\lim_{x \rightarrow 5} f(x) =$ _____
 - (d) $\lim_{x \rightarrow \infty} f(x) =$ _____
-

6. (3 points) Find the value c that makes the function f continuous.

$$f(x) = \begin{cases} x^2 + c, & \text{for } x \leq 2 \\ 5 - x, & \text{for } x > 2 \end{cases}$$

ANSWER: $c =$ _____

7. (10 points) Find the derivative of each of the following functions (DO NOT SIMPLIFY):

- (a) $f(x) = 3x^3 - \frac{7}{x^2} + 9$
 - (b) $f(x) = 3x\sqrt{7x^2 - 5}$
 - (c) $f(x) = \frac{3x^2 + 5}{4x^3 + 2x}$
 - (d) $f(x) = xe^{3x}$
 - (e) $f(x) = \ln\left(\frac{3}{x}\right)$
-

8. (7 points) Use the DEFINITION of the derivative to find $f'(x)$ for the function $f(x) = 5 - 2x + 3x^2$. Show your work.

9. (7 points) Suppose that the price p per item of a product is related to the sales quantity x (items) by the equation $p = 42e^{-.003x}$. Find the price that will maximize the revenue function R .

10. (3 points) Find the time it takes for a \$1000 deposit to triple in value at an interest rate of 8% compounded continuously. HINT: the formula $A = Pe^{rt}$ was derived to describe the growth of a fund with continuously compounded interest.

11. (3 points) The temperature T in degrees Fahrenheit at which water boils at selected pressures p (in pounds per square inch) can be modeled by the equation

$$T = 87.97 + 34.96 \ln p + 7.91\sqrt{p}$$

Find the rate of change of the temperature when the pressure is 50 pounds per square inch.

ANSWER: _____ (include units)

12. (3 points) The demand, x , for a product depends on its price p (in dollars) according to the formula $x = \frac{800}{\sqrt{p}} - 2$ for $p > 0$. Use differentials to approximate the change in demand if the price changes from \$30 to \$31.

13. (10 points) Choose ONE of the two optimization problems below:

- (a) An apartment complex currently rents apartments for \$850 per month, and has an average occupancy of 200 units. A marketing study shows that for each \$20 increase in monthly rent, the average occupancy drops by one unit. Similarly, a reduction of \$20 in monthly rent results in a rise of one unit in the average occupancy. What rent should be charged to obtain maximal revenue?
- (b) A rectangular window frame must be built to enclose 150 square feet of glass area. The sides and top can be framed with material that costs \$2 per foot, but the material for the bottom edge costs \$4 per foot. Find the dimensions of the window which minimize the cost of the frame.

14. (9 points) The pollution p in particulates per gallon (ppg) in a certain river at time t (in years after January 1, 1998) is modeled by the function

$$p(t) = 4 \frac{5t^2 + 16t + 10919}{t^2 - 26t + 461}$$

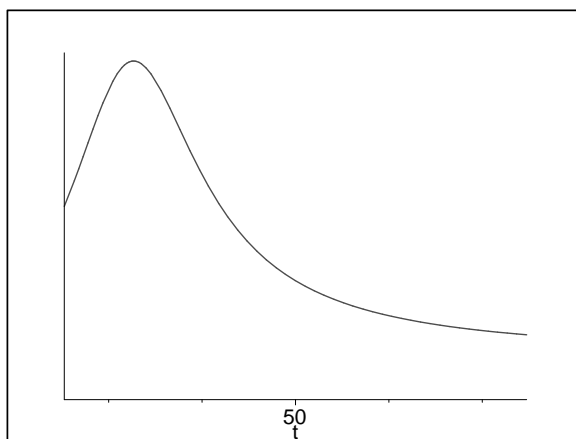
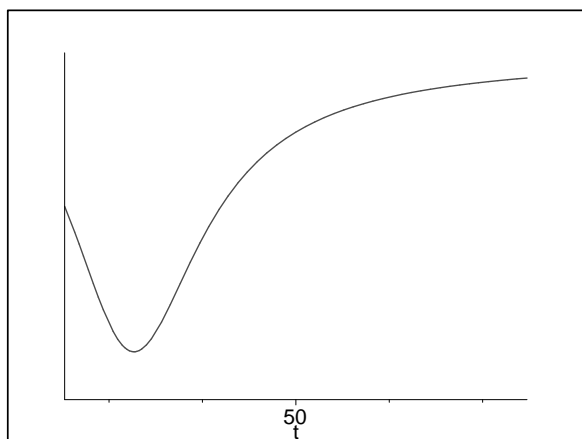
The function $p(t)$ has derivatives

$$p'(t) = -584 \frac{t^2 + 118t - 1995}{(t^2 - 26t + 461)^2}$$

and

$$p''(t) = 1168 \frac{t^3 + 177t^2 - 5985t + 24671}{(t^2 - 26t + 461)^3}$$

- (a) $\lim_{t \rightarrow \infty} p(t) = \underline{\hspace{2cm}}$ (include units)
- (b) Write an *English* sentence (in lanaguage a non-calculus person could understand) which states the fact given in the limit statement above.
- (c) $p(0) = \underline{\hspace{2cm}}$ (include units)
- (d) $p'(0) = \underline{\hspace{2cm}}$ (include units)
- (e) Write an *English* sentence (in lanaguage a non-calculus person could understand) which states the facts given in the **two** statements above at $t = 0$.
- (f) Within the domain $t \geq 0$ find the interval(s) on which $p(t)$ is increasing: $\underline{\hspace{2cm}}$
- (g) Select the correct graph of $p(t)$ below and LABEL (x, y) coordinates of important values.



15. **(BONUS: 10 points)** A consumer considers 3 monthly service proposals for a long-distance telephone carrier. Plan A is a flat rate of 7 cents per minute for long distance service. Plan B is a rate of 3 cents per minute with a service fee of \$5 per month. Plan C is 10 cents per minute for the first 30 minutes and then 5 cents per minute thereafter. The questions below will help you analyze the 3 plans and identify them in the graph below.

- (a) Write a formula $A(t)$ for plan A that produces the fee (in dollars) for a total long-distance time of t (in minutes).
- (b) Write a formula $B(t)$ for plan B that produces the fee (in dollars) for a total long-distance time of t (in minutes).
- (c) Find the break-even time for plan A and plan B.
- (d) Write a recommendation for a consumer choosing between plan A and plan B. Your recommendation should be something like “Choose plan ___ if your long distance time is less then ___, and choose plan ___ otherwise.”

(e) Complete the following table for plan C:

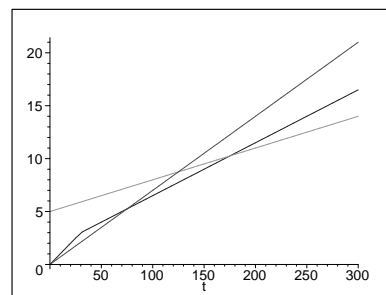
t	15	20	30	40	50
$C(t)$ (\$)					

(f) Write a piecewise formula $C(t)$ for plan C that produces the fee (in dollars) for a total long-distance time of t (in minutes).

(g) Complete the table below, and circle the cheapest plan in each column:

t	10	50	100	200
$A(t)$				
$B(t)$				
$C(t)$				

- (h) Find the break-even time t for plan C and plan A.
- (i) Find the break-even time t for plan C and plan B.
- (j) Identify each plan in the graph below by labeling the appropriate curves with A , B , and C .



(k) Write a recommendation for a consumer choosing between the 3 plans. Your recommendation should read something like “If your long-distance time used is between 0 and ___ choose plan ___, if your long-distance time used is between ___ and ___ choose plan ___, if your long-distance time is over ___, choose plan ___ .”