

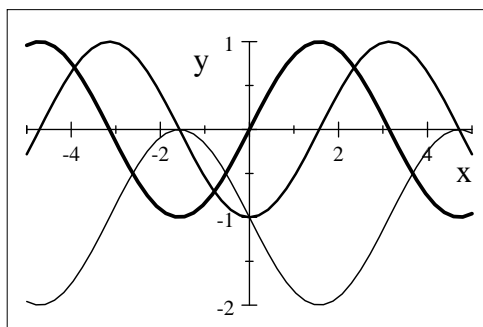
Study guide for test 1 of math 152.

- Describe in steps how you will find the shortest distance between  $O = (0, 0)$  and  $f(x) = (x - 2)^2 + 1$ .
- (a) Derive the Newton's method  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ . (b) Explain why Newton's method fails when applied to the equation  $\sqrt[3]{x} = 0$  with any initial approximation  $x_0$  other than 0. Illustrate your explanation with a sketch.
- Let  $f(x) = 3 \ln(5x + 8)$  be a function which is continuous and differentiable on the interval  $[2, 4]$ . According to the Mean Value Theorem, there exists (at least) a point  $c \in (2, 4)$  such that

$$f'(c) = \frac{f(4) - f(2)}{4 - 2} = 0.6627491.$$

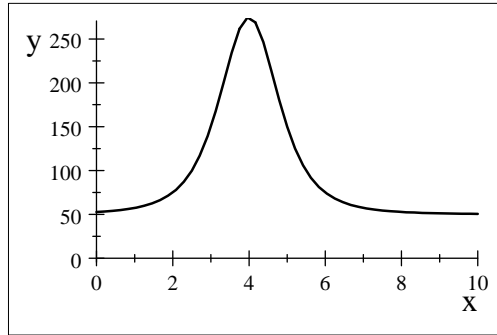
Please find effectively such a  $c$  verifying this condition.

- Find the following limits:
  - $\lim_{x \rightarrow \infty} x^4 e^{-x^2}$ .
  - Find  $\lim_{x \rightarrow 0} (x)^x$ .
- Find the largest area of a rectangle in the first quadrant under the curve  $y = \sqrt{4 - x^2}$ .
- (Maple) Use Newton's method to find the inflection point of the curve  $y = e^{\cos x}$ ,  $0 \leq x \leq \pi$ , correct up to 6 decimal places.
- Use Newton's Rule to estimate the intersection of  $\ln(x)$  and  $e^{-3x}$  correct up to 6 decimal places.
- Prove  $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$ .
- If the graph of  $f$  is given together with its first and second derivatives below. Label the graphs for  $f, f'$  and  $f''$

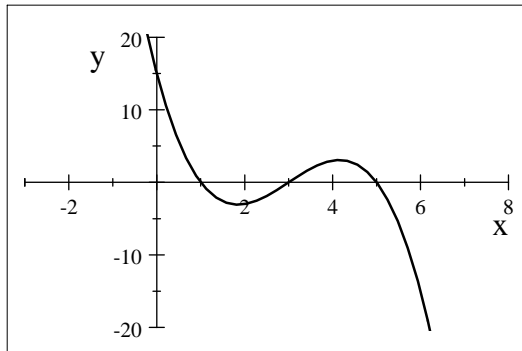


- (Maple) Use Newton's Rule to estimate the zero of  $f(x) = x - \cos x$  correct up to 6 decimal places.
- Find the limit for the followings. Specify whenever L' Hospital rule is used.
  - $\lim_{x \rightarrow 0} (\csc x - \cot x)$
  - $\lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x}}$ .
  - $\lim_{x \rightarrow \infty} x^4 e^{-x^2}$ .
- (Maple) If  $f(x) = 2x \sin x$  and  $g(x) = \sec x - 1$ .
  - Illustrate L'Hospital's Rule by graphing both  $\frac{f(x)}{g(x)}$  and  $\frac{f'(x)}{g'(x)}$  near  $x = 0$  to see that these ratios have the same limit as  $x \rightarrow 0$ .
  - Use Maple "limit" command to confirm that  $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$ .
- Suppose the rate of change for the number  $N$  of bacteria in a culture after  $t$  days is modeled by the

following graph: [Note the following graph represents  $y = N'(t)$ ].



- a. Does the function  $N$  have maximum or minimum during  $t = 0$  and  $t = 10$ .
  - b. When is the rate of change of  $N$  increasing the most?
  - c. Will the number of bacteria be stabilized eventually?
  - d. Approximate when  $N$  will be increasing at its fastest.
14. Suppose you are given the graph of the marginal cost (derivative for cost) for a company. [ The  $x - axis$  represents the number of units in thousands, and the  $y - axis$  represents the cost in millions]. And we assume the  $x - intercepts$  are at  $x = 1, 3$  and  $5$ .



- a. When will the company achieve its largest and lowest cost respectively?
- b. When will the rate of cost increase the most during  $x = 2$  and  $x = 5$ ?
- c. When will the rate of cost decrease the most during  $x = 0$  and  $x = 3$ ?
- d. Explain how you will find where cost function increases the most and decreases the most..