Parallel Lines Cut By a Transversal

I. UNIT OVERVIEW & PURPOSE:

The goal of this unit is for students to understand the angles and the properties related to parallel lines. Students will learn multiple methods for verifying that lines are parallel. They will also understand the relationship of parallel lines to transversal lines.

It is important for students to see that mathematical concepts serve as useful means to solving problems that affect our everyday lives. Parallel lines are important to understand not only for a mathematics course, but also in everyday life such as in the design of airports, railways, bridges, buildings and many more geometric components of the real world.

In this five-lesson thematic unit, students will engage with the concept of parallel lines cut by a transversal line as they design plans for various elements of a fictitious city. Lessons 1 – 4 can be seen as "building block" lessons through which students acquire and practice the skills needed to create the culminating city project. Lesson 5 is a showcase lesson where students present their projects to the class and recap all that they have learned throughout this thematic unit. Students will complete the city project in groups of three or four. We believe that together, they can negotiate meaning and deepen understanding as they work together to apply geometric concepts to each task. Students will be given several criteria, which must be met in the design of their city, and they will be asked to discuss how geometric methods can assist them in creating optimal design for various elements of the city. For example, students will apply the angle theorems to prove lines parallel, practice geometric proofs, constructions and discover the connections to other topics. At the end of this project, students will be asked to articulate their learning in written and oral summaries of their work to be presented to their classmates in the final class lesson.

II. UNIT AUTHORS:

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III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

At the end of this unit:

1. Students will understand the definition of parallel lines;
2. Students will understand what it means for parallel lines to be cut by a transversal line;
3. Students will be able to construct parallel lines;
4. Students will be able to verify that lines are parallel using:
    a. Algebraic methods;
    b. Coordinate methods;
    c. Deductive proofs;
5. Students will make connections between the geometric material included in the lesson and real-life examples of parallel lines;
6. Students will demonstrate knowledge of ratios and proportions and the ability to transfer these previously learned concepts to a real world problem.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

Key Focus: MPE.32 – The student will use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

Additional: MPE.1 – The student will solve practical problems involving rational numbers (including numbers in scientific notation), percentages, ratios, and proportions.

VII. CONTENT:

Students will use prior knowledge of parallel lines cut by a transversal and geometric constructions to create plans for the construction of fictitious towns. This project will

encourage students to work cooperatively and to see how these concepts are used in the real world.

Lesson One: Parallel lines and Constructions

Students will review the concepts of parallel lines and several angles. Students will use Geogebra or constructions by hand to visualize the properties of parallel lines, transversals and angles formed. This will be an interactive lesson based in the use of technology. Students will be able to manipulate lines and angles so that they learn more about variation and relationships amongst these elements.

In this lesson, students will also receive the rubric for the final project so that they are aware of the learning objectives that will be incorporated into the final project and presentation.

Key Concepts – constructions, parallel lines, interior angles, alternate interior angles, same side interior angles, exterior angles, alternate exterior angles, vertical angles, and corresponding angles.

Lesson Two: Street Design

Students will create a street map for their city, and the street map must include parallel lines cut by transversals. They will be required to place various buildings or elements of the city at certain points around the city. These points will be indicated by types of angles. Therefore, students must understand the properties of these angles and how to create and identify them.

Key concepts –parallel lines, transversal lines, congruence, alternate interior angles

Lesson Three: Designing a Parking Lot

Students will design a parking lot for a sports complex in their city. They will incorporate a formerly learned concept of ratios in order to calculate the number of parking spots needed. They will be given an estimated number of people who visit the complex and asked to assume that approximately 1/3 of those people will be driving their own vehicle. Therefore, they should show how they calculate the number of spaces needed.

For this lesson, students will use geometric concepts and algebraic methods to create a model of a parking lot and to verify that lines are parallel. They will need to construct parking spaces designated by parallel lines. They will learn how to do so on the coordinate plane. These lines will be transversals intersecting vertical lines in the coordinate plane.

These will completed as an activity that will be started in class and then finished as homework. The handout/worksheet has a total of 15 questions and 3 extension questions. The first 15 will be marked as either 0 (incomplete), 1 (attempted but incorrect), 2 (approaching target) or 2 (target). Feedback will be provided on top of the mark. The three extension questions are worth 1 extra credit point each

Key concepts – construction, parallel lines, perpendicular lines, area of a parallelogram, congruence

Lesson Four: Designing an Airport

Any thriving city should have an airport in order to promote tourism and business traffic. A safe and operational airport is characterized by design rooted in geometric properties. Students will utilize the properties of parallel lines to create a model of an airport with a runway, taxi routes, and gates. Students will use construction methods learned in lesson two, but they will now also be asked to verify that their lines are parallel using postulates and theorems.

Key concepts – parallel lines, transversal lines, construction by hand or by computer program, angles of a circle.

Lesson Five: Final project presentation

Students will have the final project assignment during the class following lesson four and will use that lesson to get started. They will then have a week to work on the project on their own outside of class. The in-class instructional time will move on to the next scheduled topic in the curriculum. In this lesson, students will present their projects to the class and give feedback to one another (peer assessment).

Written reflection: Individually, students will complete a written reflection of their learning through the entire unit, summarizing new and reviewed knowledge. In this reflection, each student must include at least two deductive proofs, demonstrating how they can verify that particular lines in their city are parallel.

Oral group presentation: Each group will present their city and its properties to the entire class. This presentation should be done in an electronic presentation format. Each group member should speak during the presentation. The group should engage the class in a mathematical discussion about at least one element of their city.

Key concepts – ALL concepts from the unit, critical thinking and communication skills

VIII. REFERENCE/RESOURCE MATERIALS:

SMART Board, Geogebra software, Computer lab, document camera, or LCD projector for modeling, markers, calculators, rulers, compasses, poster boards, and handouts (attached to each lesson).

IX. PRIMARY ASSESSMENT STRATEGIES:

FORMATIVE METHODS

Observation: The teacher will observe students as they work on tasks to see if and where any additional help may be needed. Teacher will monitor, observe, and communicate with students as they work in groups.

Class discussion and participation: The teacher will engage students in the discussion during the instructive portion of each lesson, monitoring for understanding.

Homework: Homework assignments will be given after each lesson. The homework will be related to the unit plan task and discussed with the class. Handouts will be given to students during class with activities related to the lesson's learning objectives, and unfinished problems can be done for homework. Doing such activities each day will help both students and the teacher check for understanding throughout the unit. Groups will also be expected to work together outside of class on the final city plan project.

Reflective journal: Students record their journals for the last five minutes of class. Entries should include key concepts learned each day as well as any remaining questions that they may want to bring up with the teacher outside of class or in the next class. They may also use this journal to draw connections between content of multiple lessons.

SUMMATIVE METHODS

Written assessment: Students will be incorporating concepts learned into the final city plan project and presentation. Concepts learned will also appear on our chapter test.

Application: Completed project model with all materials.

Culminating project presentation: Students will synthesize their learning in a written reflection and an oral presentation.

X. EVALUATION CRITERIA:

Students will be observed as they work during class. Whole class discussions will also help the instructor to determine student knowledge. Presentations by the students and feedback from their peers will also serve as an evaluation tool. The final assessment will be the completed project but all geometric concepts learned will also appear on the written chapter test.

XI. INSTRUCTIONAL TIME:

We assume block scheduling (90 minute class sessions)

This entire unit should be completed in six class sessions. There will be one class session between lessons four and five reserved for groups to work together on the completion of their city projects.

Lesson 1: Parallel lines and Constructions

Strand: Geometry

Mathematical Objective(s)
Students will be able to

- Use construction tools to construct parallel lines and transversals.
- Create parallel lines, transversals, and different angles and visually see how the different angle postulates are represented.
- Define the following:
    - Interior angles
    - Alternate interior angles
    - Same side interior angles
    - Exterior angles
    - Alternate exterior angles
    - Vertical angles
    - Corresponding angles

Mathematics Performance Expectation(s)
MPE 32.a The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel.

Virginia SOL
G. 2a (The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel.)

NCTM Standards

Mathematics as Problem Solving
Students will demonstrate the ability to use problem-solving approaches to investigate parallel lines and transversals and the angles made by them.

Mathematics as Communication
Students will communicate mathematical ideas about angle relationships made by parallel lines and transversals. The final lab report gives the opportunity for students to reflect and clarify what they have learned.

Mathematics as Reasoning
Students will reinforce logical reasoning skills by comparing and contrasting different angle relationships.

Mathematical Connections
Students will use and value connections between mathematical topics and other disciplines.

Materials/Resources

- Compass
- straight edge
- paper
- Geogebra
- Computer lab
- Construction instructions – to be used with the constructions using paper and pencil.
- Smart Camera
- Smart projector
- Construction steps if using Geogebra : reproducible at http://www.geogebra.org/book/intro-en.pdf
- Vocabulary worksheet
- Final project and rubric

Assumption of Prior Knowledge

This is a construction exercise that will be built upon in later projects, therefore, the following is important to know and be able to build upon:

- Students should be able to construct a pair of parallel lines.
- Students should be able to measure angles.

Introduction: Setting Up the Mathematical Task

In this lesson, our goal is to have students use Geogebra or construction tools to visualize the properties of parallel lines and transversals in terms of the angles. The goal is to provide an interactive, technology based activity that allows students to manipulate angles and lines more in depth than otherwise possible. By manipulating the lines, students can see what will happen to the angles helping them to see their relationships.

Student/Teacher Actions

Introduction: Review construction of parallel lines and measuring angles.

A. Pass out tools needed for the project.
B. Have students construct parallel lines with a transversal and measure the angles made. Teacher will verify that the students can read the protractor and measure angles correctly.
C. Explain to the class they will be working on constructing parallel lines cut by a transversal.

Construction: Use either Geogebra or constructions tools to do the following:

A. Have the students construct parallel lines and a transversal. (Students will be given a clean sheet of paper with the hope that the angle measurements when compared will be different measurements. This will prove it works for any measured angle.)
B. Teacher will also construct the lines using the Smart Camera and Smart Projector.
C. Have the students mark the angles 1-8.

D. Using the protractor, students will measure all of the 8 angles made by the parallel lines and transversals.

Group discovery: Have the students get into groups to compare their constructions by asking the following questions: (see handout)

A. What relationship is there among the interior angles?
    1. The two interior angles that line on the same side of the transversal sum to 180°. (They are supplementary)
    2. The interior angles that are on opposite sides of the transversal and do not form a linear pair are the same measure. (They are the alternate interior angles)
B. What relationship is there among the exterior angles?
    1. The two exterior angles that line on the same side of the straight line sum to 180 degrees. (Same side exterior)
    2. The two exterior angles on opposite sides of the straight line are congruent. (They are alternate exterior angles)
C. Teacher should use vocabulary to have the students come up with the names of the angles during discussion. This helps students see where the names came from.(Alternate interior, alternate exterior, same side interior, same side exterior)

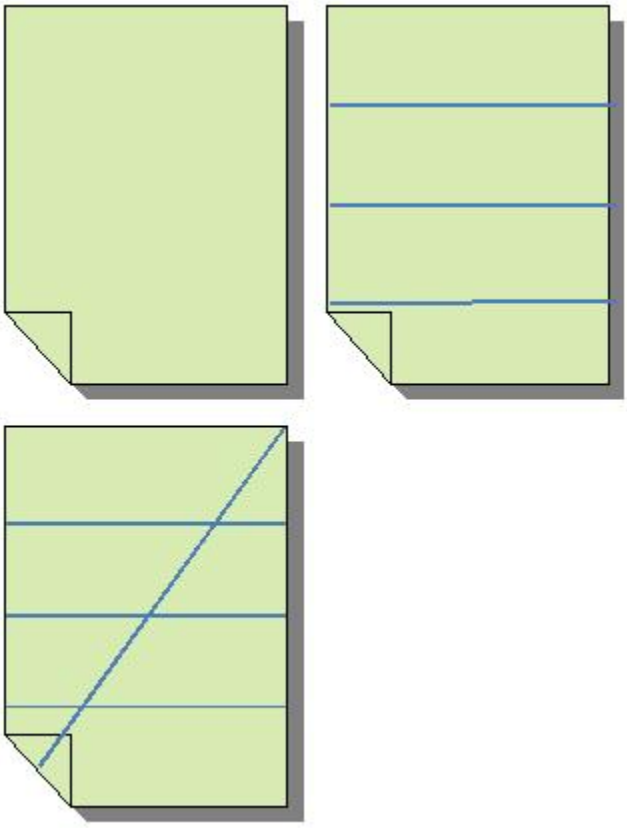
Homework and Extensions

Vocabulary worksheet is included to check for understanding and to be used with future assignments. The final product should be presented in an electronic presentation. During the next 5 days, time will be needed in the computer lab or at home to start working on the PowerPoint. (See handout and rubric)

Note: Teacher should assign or allow students to choose groups for final project during lesson one.

Strategies for Differentiation

A. Some students have difficulty using a compass and protractor. Students can be paired with stronger students who will help, or a worksheet with the correct construction may be used to trace. Also, students may use folding techniques to accomplish the same objective. For example: fold a piece of regular sized paper in half and in half again. Then open it to fold it from one corner to the other. This will give 3 parallel lines with one transversal. (See below)



B. Vocabulary may be difficult. After the discovery is through, have a handout with the vocabulary and angles listed. (See extension vocabulary worksheet)

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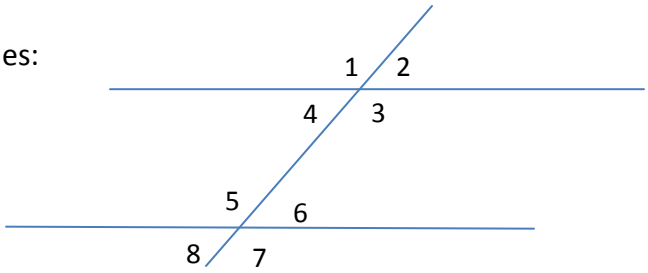
Vocabulary for angles made by transversals.

Use the figure to identify the following angles:



1. Alternate interior angles_____
2. Alternate exterior angles _____
3. Same Side interior angles _____
4. Same Side exterior angles _____
5. Corresponding angles _____

Which angles are congruent?

Which angles are supplementary?

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Group Discovery Handout

A. What relationship is there among the interior angles?
   1. The two interior angles that line on the same side of the transversal sum to 180°. (They are supplementary)
   2. The interior angles that are on opposite sides of the transversal and do not form a linear pair are the same measure. (They are the alternate interior angles)

B. What relationship is there among the exterior angles?
   1. The two exterior angles that line on the same side of the straight line sum to 180 degrees. (Same side exterior)
   2. The two exterior angles on opposite sides of the straight line are congruent. (They are alternate exterior angles)

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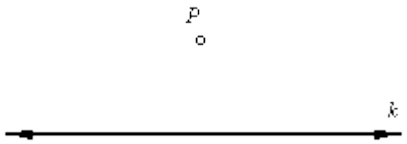
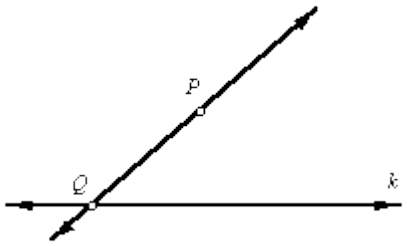
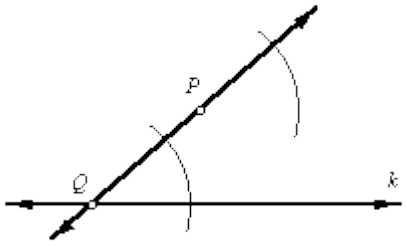
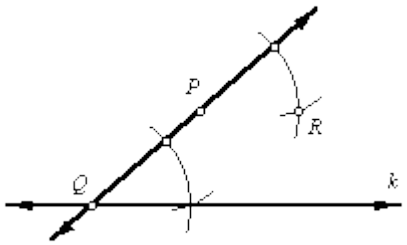
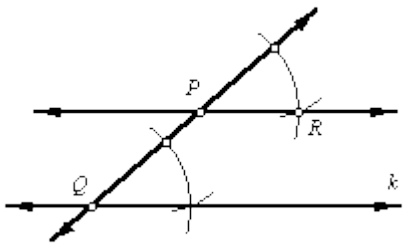
The following instructions were found at http://whistleralley.com/construction/reference.htm

. Begin with point *P* and line *k*.	
2. Draw an arbitrary line through point *P*, intersecting line *k*. Call the intersection point *Q*. Now the task is to construct an angle with vertex *P*, congruent to the angle of intersection.	
3. Center the compass at point *Q* and draw an arc intersecting both lines. Without changing the radius of the compass, center it at point *P* and draw another arc.	
4. Set the compass radius to the distance between the two intersection points of the first arc. Now center the compass at the point where the second arc intersects line *PQ*. Mark the arc intersection point *R*.	
5. Line *PR* is parallel to line *k*.	

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Handout: CITY PROJECT

In this group project, your group will have the opportunity to create a fictitious city. You will be working on various components of your city over the next three lessons. We will have one class period when you can work on the assignment together in class, but you will also need to work together to complete the project outside of class. The final product should be presented in an electronic presentation, such as PPT or Prezi. Your presentation may include text, pictures of example city layouts, and pictures or models of what you have created for your city. If you are unable to put everything into the presentation, it is okay to supply some things in written form or to write things on the board. The following components should be included in your final project:

I. Introduction
   a. Name of city
   b. Description of city

II. Street Design
   a. A layout of the streets
   b. Indication of the properties of the lines and angles
   c. Contains the following buildings:

III. Parking Lot
   a. What is the parking lot for?
   b. How many spaces will there be in the parking lot and how did you decide on the number of spaces? (show your calculations)
   c. A mini-model of one or two rows in the lot
   d. A description of how the coordinate plane helped plan the design of these spaces/rows

IV. Airport
   a. Label the runways correctly
   b. How many terminals will you use?

V. Overall presentation
   a. Clear communication
   b. Visuals are good quality and easy to understand
   c. The group engages the rest of the class in mathematical discussion by asking questions and giving feedback to students on their answers

VI. Mathematical Criteria (to be covered at least once at some point in the final project)
   a. Demonstration or description of how you used a construction
   b. Demonstration or description of how you used the coordinate plane
   c. Demonstration or description of how proved lines were parallel in lesson 4.

VII. Individual Written reflection (30 points)

a. Each student will write a 1-2 page reflection answering the following questions:
   1. What did you learn in this unit? Explain the key geometric properties you worked with. (10 points)
   2. How did you see geometry applied to the real world in this unit? (5 points)
   3. Can you name some real world situations, other than planning a city, where geometry might be needed? Please name at least five. (5 points)
   4. What did you like about working in groups? (5 points)
   5. What was difficult about working in groups? (5 points)

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Grading Rubric for Reflection

Name: _____

Criterion	Standard
Question 1 *Score / 10*	10 – clearly addresses the topic and responds effectively to all aspects of the assignment; 8 – clearly address the topic, but may respond to some aspects of the assignment more effectively than others 6 – addresses the topic, but may slight some aspects of the topic 4 – indicates confusion about the topic or neglects important aspects of the assignment 2 – suggests an inability to comprehend the assignment or to respond meaningfully to the topic
Question 2 *Score / 5*	5 – explores the issues showing thorough comprehension of the text; goes beyond the obvious or class discussion 4 – shows some depth and complexity of thought 2 – Has an idea, but gives answers that does not apply 0 – Does not have an answer
Question 3 *Score / 5*	5 – explores the issues showing thorough comprehension of the text; goes beyond the obvious or class discussion 4 – shows some depth and complexity of thought 2 – Has an idea, but gives answers that does not apply 0 – Does not have an answer
Question 4 *Score / 5*	5 – Gives a detailed list of the good things about working in a group 3 – Addressed the question but does not explain why it is good 0 – No opinion
Question 5 *Score /5*	5 – Gives a detailed list of the bad things about working in a group and how to fix it 3 – Addressed the question but does not explain why it was difficult 0 – No opinion
Total *Score /30*	

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Scoring Rubric for City Project

Team Members: _____

	0	**1**	**2**	**3**	**Peer Score**	**Teacher Score**
Title, description of city, rationale	Not included	Contains some but not all of these items	Contains all of these items, but description or rationale is weak	Contains all three and provides detailed description and rationale		
Comments:						
Street design	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Parking lot	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Airport	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Other element of the city	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Overall presentation	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Mathematical Criteria	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						

Overall comments:

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Lesson 2: Parallel Lines Project

Strand: Geometry

Mathematical Objective(s)

1. Students will demonstrate their knowledge of parallel lines with a transversal.
2. Students will show when angles are congruent or supplementary given parallel lines and a transversal.

Mathematics Performance Expectation(s)

MPE 32.a The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel.

Virginia SOL

G. 2a (The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel.)

NCTM Standards

Mathematics as Problem Solving

Students will demonstrate the ability to use problem-solving approaches to investigate parallel lines and transversals and the angles made by them.

Mathematics as Communication

Students will communicate mathematical ideas about angle relationships made by parallel lines and transversals. The final lab report gives the opportunity for students to reflect and clarify what they have learned.

Mathematics as Reasoning

Students will reinforce logical reasoning skills by comparing and contrasting different angle relationships.

Mathematical Connections

Students will use and value connections between mathematical topics and other disciplines.

Materials/Resources

- Pencil
- Colored pencils or markers
- Ruler
- Paper (graph paper, if desired)

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Assumption of Prior Knowledge

This is a construction exercise that will be built upon in later projects, therefore, the following is helpful to know and be able to build upon although it can be learned quickly:

- Students should be able to construct a pair of parallel lines.
- Students should be able to measure angles with a protractor.

Introduction: Setting Up the Mathematical Task

This activity would be good for students in a Capstone or Geometry class because they are basically just making a map of a city, which would most likely be fun for them. But while they are doing this, they are also thinking about where each building should be. They need to know which angles are congruent and supplementary since the instructions of the placement of the buildings use these terms. Number 4 under instructions is where they really have to think. They have to realize that in order to have non-congruent alternate interior angles; they must turn the map and use the transversals as their 2 lines and one of the parallel lines as their transversal.

Overview

For this project, each group of 2 students will make a street map for a fictional city (you must name your city). This city will consist of:

1. Six (6) streets that are parallel to each other. Each street should be constructed and named for reference.
2. Two (2) transversal streets. (i.e., Two or more streets that intersect all six of the above streets). These should be named as well. **Do *not* make the transversals parallel to each other; however, one may be perpendicular to the parallel lines!!!**
3. Traffic lights or stop signs at four (4) different intersections.
4. The following buildings, represented in your city:
    a. Post office
    b. Bank
    c. Fire Department
    d. Police Station
    e. Gas Station
    f. School
    g. Restaurant
    h. Grocery Store
    i. Sports Complex

j. Your own house

Instructions (see handout)

The point of this project is not to place these buildings anywhere, but to demonstrate your understanding of different angles, as well as to understand when they are supplementary or congruent. You can still be creative in doing so, but please place the buildings in the following locations.

1. Your house and the school at congruent alternate interior angles.
2. The post office and the bank at same side interior angles.
3. The fire department and police station at congruent alternate exterior angles.
4. The restaurant and sports complex at *non-congruent* alternate interior angles.
5. The gas station and grocery store at congruent corresponding angles.

Remember to be creative. You may be as artistic as you would like in drawing the buildings/roads, but be sure to label each one. (Creativity will earn extra points).

Strategies for Differentiation

- Some students have difficulty using a compass and protractor. Students can be paired with stronger students who will help, or a worksheet with the correct construction may be used to as a guide.
- Vocabulary in the instructions of the project may be difficult. Group students who struggle with reading with a stronger reader.

Instructions for the City Project

The point of this project is not to place these buildings anywhere, but to demonstrate your understanding of different angles, as well as to understand when they are supplementary or congruent. You can still be creative in doing so, but please place the buildings in the following locations.

1. Your house and the school at congruent alternate interior angles.
2. The post office and the bank at same side interior angles.
3. The fire department and police station at congruent alternate exterior angles.
4. The restaurant and sports complex at *non-congruent* alternate interior angles.
5. The gas station and grocery store at congruent corresponding angles.

Remember to be creative. You may be as artistic as you would like in drawing the buildings/roads, but be sure to label each one. (Creativity will earn extra points).

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Rubric for Project: Parallel lines (Turn in with Project)

Number of Points	Category	Points Earned	Comments
2	**Name of City**		
6	**Name of Each Street**		
4	**Two Transversal Streets (named)**		
4	**Traffic lights or stop signs at four (4) different intersections**		
2	**Your house and the school at congruent alternate interior angles.**		
2	**The post office and the bank at same side interior angles.**		
2	**The fire department and police station at congruent alternate exterior angles.**		
2	**The restaurant and courthouse at *non-congruent* alternate interior angles.**		
2	**The gas station and grocery store at congruent corresponding angles.**		
4	**Neatness**		
10	**Creativity**		
Total Points: **40**		**Points Earned:**	

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Lesson 3: Lines in a Parking Lot

Strand: Geometry

Mathematical Objective(s)
Students will be able to

- Use algebraic methods to calculate ratios and proportions.
- Use coordinate methods to create parallel lines cut by a transversal
- Measure angles using a protractor
- Use the coordinate plane to verify that lines are parallel
- To identify the following for lines on a coordinate plane:
    - Interior angles
    - Alternate interior angles
    - Same side interior angles
    - Exterior angles
    - Alternate exterior angles
    - Vertical angles
    - Corresponding angles

Mathematics Performance Expectation(s)
MPE 1 The student will solve practical problems involving rational numbers (including numbers in scientific notation), percentages, ratios, and proportions.

MPE 21. The student will use the relationships between angles formed by two lines cut by a transversal to:

a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

MPE 19. The student will graph linear equations and linear inequalities in two variables, including

a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and
b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

Virginia SOL
G.2 a, b & c (Key areas of focus in this lesson are the use of coordinate methods (b) and the application of these mathematical concepts to solve a real world problem (c))

This concept is essential for architectural design or city planning. In this case, the design of a parking lot with aisles and spaces that will accommodate all vehicles rests on being able to use mathematical methods to ensure that all the lines are parallel and evenly spaced. Geometry is useful in verifying this. In this lesson, students see how coordinate methods, a method with which they already have practice, can be applied to a real world problem.

G.3 b (A key area of focus is applying slope to determine that lines are parallel).

This is important because it allows students to see the connection between prior learning about slope and geometry. They should see through using a coordinate plane that parallel lines in a plane will have the same slope.

NCTM Standards

Mathematics as Problem Solving

Students will demonstrate the ability to use problem-solving approaches to investigate parallel lines and transversals and the angles made by them.

Mathematics as Communication

Students will communicate mathematical ideas about angle relationships made by parallel lines and transversals. The final lab report gives the opportunity for students to reflect and clarify what they have learned.

Mathematics as Reasoning

Students will reinforce logical reasoning skills by comparing and contrasting different angle relationships.

Mathematical Connections

Students will use and value connections between mathematical topics and other disciplines.

Materials/Resources

- Compass
- straight edge
- graph paper
- Worksheet: “Graphing Parallel Lines”

Assumption of Prior Knowledge

- Students should be able to plot points in a coordinate plane.
- When given a line on a coordinate plane, students should be able to find the slope of the line.

The above two skills are necessary for the activity involving using a coordinate map in order to construct a model for a parking lot.

- Students should know the definitions of parallel and transversal lines
- Students should know the properties of vertical, supplementary and complementary angles.

The above two skills will also be incorporated into the parking lot model activity, because students will be asked to label these parts of the model. Knowing the properties of these elements and how to identify them is useful in real world planning. This saves planners from having to perform the same calculations on all elements of a model when they can recognize with distances and angles are the same or have relationships of similarity.

Introduction: Setting Up the Mathematical Task

Teacher says: "In this lesson, you will see how a coordinate plane can be useful when developing real world examples of parallel lines cut by transversals. Plotting such lines on a coordinate plane will also deepen your comprehension of the properties of parallel lines, transversals, and the angles they form. We will be utilizing your prior knowledge of the coordinate plane from your Algebra classes such as plotting points and finding slope to describe geometric properties of lines."

Time outline (90 minute block period)

Introduction 10 minutes
Warm-up activity: 15 minutes
Student exploration 1: 25 minutes
Student exploration 2: 35 minutes
Wrap/up and going over HW: 5 minutes

<u>Warm-up</u>

As students enter the room, the following warm-up problem will be on the board

How many parking spaces are needed? (See model at the end of this lesson)

Objective: To review the concept of ratios and proportions in the context of a real world problem.

Today, we will be designing a parking lot for a sports complex in your town. *The teacher should project the following questions or distribute as a handout*.

1. What sport(s) are played in this complex? _____

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2. What is the name of the sports complex? _____
3. The sports complex has a maximum capacity of 6500 people. When visiting the stadium, people usually come with more than one person in the car. There may also be groups visiting the stadium in large vehicles. Therefore, you are going to estimate that one parking space is needed for every three people. Calculate the minimum number of parking spaces needed in the lot. Show your work and explain your answer in words.

Student/Teacher actions: Students write in their warm-up books as they work individually. They may raise hands if they have difficulty remembering how to solve this problem. Students who realize that 3 does not divide 6500 easily may become confused and ask the teacher for assistance. The teacher should pose questions to help the student come to the correct answer independently.

Student Exploration 1: Review Activity

Review key concepts learned in lessons one and two.

Students are placed in groups of 4 to 5 (Small Group Work).

- Each group is given a card with one of the geometric concepts covered in the previous lessons (parallel lines, perpendicular lines, transversal, interior angles, alternate interior angles, same side interior angles, exterior angles, alternate exterior angles, vertical angles, or corresponding angles).
- The group is also given a picture of a real world structure (see attached)
- The task is for the group to: 1. Define the geometric element in your own words and 2. Identify an example of the element in the picture.

Student/Teacher Actions

- Teacher circulates the room to offer guidance where needed
- Students may have difficulty putting the definition into their own words without looking at their previous notes or in the text. The teacher should encourage them that it may even help to identify lines and/or angles in the picture first and then try to describe them.

Monitoring Student Responses

- The teacher should pop in and out of small groups to monitor how independently students are able to think on these topics.

- The teacher projects each picture and also gives these pictures to groups as a handout. The teacher invites one member of the group to write the definition of their property on the board while another member identifies an example in the picture.
    - Then ask the group, “Do you see any other geometric elements represented in the picture?”

- Open a class discussion (Whole Class sharing) at the end of the activity on the following question: “What real world tasks and/or occupations may benefit from understanding the properties of lines and angles?”

- Student/Teacher Actions:
- Allow as many students to participate during the whole class discussion as possible
- Students may need the teacher to call up on them individually
- Monitoring Student Responses:
- During the sharing time, ask questions of students from other groups to check that students are catching onto concepts other than those addressed by their particular groups
- HW (individual): Students should complete a reflection in their journals on this topic, recounting things that stood out from this discussion.

Student Exploration 2

- Distribute handout “Graphing Parallel Lines” and ask students to take out a sheet of graph paper and a pencil.
- Complete steps 1 – 10 in class. Students work first as partners. The teacher then brings the class together to go over the activity.
- Allow students the opportunity to offer suggestions for how to complete these steps and model these on the smart board.
- Students will complete the activity for homework.
- More advanced students will be able to work through the challenge questions.

Student/Teacher Actions

- Students will probably seek to simply copy what is being done on the board/overhead. The teacher should try to involve as many students in the investigation as possible before modeling how to do each problem
- It may be good to have students attempt problems in partners before going over it as a class

Monitoring Student Responses

- When one student offers a solution, ask the rest of the class if they agree. Why or why not? Explain.
- Such discussions involve more students and help the teacher to monitor progress for a larger percentage of the class

Homework and Extensions

Complete steps 11 – 18.

Assessment

A. Utilize various strategies of formative assessment throughout the lesson.
    a. During the warm-up: check for understanding by walking around the room as students complete the warm-up. Also, call upon those students you did not get to check with during the walk-around when you are going over the warm-up as a class.
    b. Review activity: Visit each group while they complete the activity. Students' answers on the board demonstrate levels of understanding. Also, involve various students in dialogue about the definitions on the board.
    c. Teaching activity: Allow students to offer input for the activity rather than demonstrating how to do the steps for them. Give students time to try the steps on their own BEFORE you go over them as a class. Allow students to share their work with the person next to them before you go over the correct approach as a class.
B. Journal/writing prompts
    a. For homework, ask students to summarize the discussion on the question: "What real world tasks and/or occupations may benefit from understanding the properties of lines and angles?" They should also add any additional thoughts.

    b. A conclusive journal entry: What did you learn about how the coordinate plane can be applied to real world problems? Can you think of an example other than a parking lot when a coordinate plane could be used?

C. Summative assessment:
    a. Homework assignment to be handed in the following day
    b. The final project

Extensions and Connections

A. Lesson extensions/follow-up: The next lesson on airports provides an additional real world application of parallel and transversal lines. The final lesson in this unit allows students to draw connections between these concepts independently
B. Connections to other mathematics content within and among grade levels: This lesson reviews previously learned concepts of ratios/proportions, and plotting points and finding slopes of lines in a coordinate plane
C. Connections to content in other subject areas: This lesson relates to the field of architecture and demonstrates the usefulness of mathematics to those in that career field.

Strategies for Differentiation

A. Circulate through the room during the warm-up to assist any students who have difficulty remembering how to solve the ratio problem.

B. During the group review activity, students will assist one another and those who feel most comfortable with oral communication will report to the class.

C. It will be helpful for ELLs and will also cater to various learning styles to pair visual and auditory material when going over the activities in class on the board or projector/screen.

D. Some students have difficulty using a compass and protractor. Students can be paired with stronger students who will help, or a worksheet with the correct construction may be used to trace.

E. Before distributing the final activity, check that students recall how to find the area of a parallelogram. If they do not, help them individually when students begin working on the activity, or pull a small group of students together to review this together.

F. Students may struggle with the coordinate plane. Ask a lot of questions, engaging as many students as possible in order to check for understanding. For example, you may want to ask some review questions like how to plot points, how to find slope and then draw connections to how they can use this information to identify geometric relationships of lines on a coordinate plane such as parallelism or symmetry.

G. High ability students: Advanced students will have the ability to excel on the challenge question.

Student organization ideas helpful to particular groups of learners:

- Be sure all students have access to graph paper
- Allow students who are struggling to work with a peer helper

Images for Student Exploration 1

Figure 1: Eiffel Tower, Retrieved from: www.neatorama.com

Figure 2: Baseball Diamond, Retrieved from: http://www.topendsports.com/testing/tests/sprint-first-base.htm

Figure 3: Arc de Triomphe, Retrieved from: http://www.visitingdc.com/paris/arc-de-triomphe-picture.asp

Figure 4: House, Retrieved from http://autokeylocksmithin.com/Coralsprings.html

Figure 5: Parking Lot, Retried from http://www.techprosecurity.com/security-articles/cctv-security-surveillance-articles/parking-lot-cameras/

Figure 6: School Bus, Retrieved from: http://cusd.schoolfusion.us/modules/cms/pages.phtml?pageid=38488&SID

Figure 7: Window, Retrieved from: http://www.cornerstone-exteriors.com/windows/easy-steps-to-refinish-your-wood-window-frames/

Name: _____ Date: _____ Class Period: _____

Handout: Graphing Parallel Lines

An important part of designing any city in our culture requires the allotment of parking spaces. While a city planner would have to consider many parking lots, this project will focus on one sample—a parking lot for a sports complex. Using a parking lot as an example, this activity will demonstrate how the use of a coordinate plane can be helpful for planning the construction of real world things with geometry.

1. Use a ruler to create a coordinate plane on a piece of graph paper.
2. Plot and label the following points A (0,0), B (4,2), C (0,4), D (4,6), E (0,8), and F (4,10)
3. Form lines by darkening the Y axis and then connecting A and B, C and D, and E and F to form three line segments
4. Use a protractor to measure and label the angles of each parking lot space.
5. What do you notice about the angles? Which angles are congruent?

6. Which angles are complimentary and which are supplementary?

7. From what you have learned in the previous lessons, what can you assume about lines AB, CD and EF?

8. What is the relationship of these lines to the y axis?

9. Now, find the slopes of AB, CD and EF. Explain how you found these slopes.

10. What does this tell you about the slopes of parallel lines?

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HOMEWORK:

11. How do you think this method could be useful to someone creating a plan for painting lines in a parking lot?

12. Your parking lot will need many more spaces than what will fit on your graph. This will serve as a model for how your rows of spaces will look. Can you use what you discovered about the lines that you have already made to assist you in drawing at least 4 more parking spaces? At least one space should be in a different row. Label all points that you create with new letters so that you can discuss them.
13. What things are the same about the lines you have made? What things are different?

14. On your graph, identify the following:

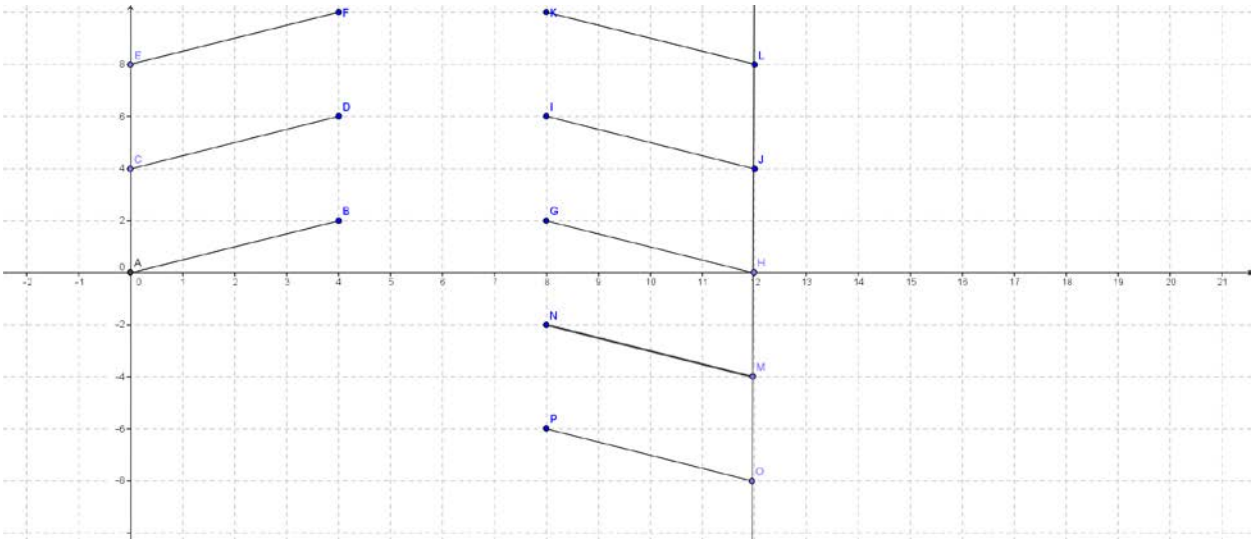
    a. Interior angles
    b. Alternate interior angles
    c. Same side interior angles
    d. Exterior angles
    e. Alternate exterior angles
    f. Vertical angles
    g. Corresponding angles

15. Pay attention to at least one parking lot that you are in between now and our next class and write down some similarities and differences between the actual parking lot and the model that you have made here.

Challenge:

16. On a new sheet of graph paper, can you create three parking spaces of your own with different slopes than those on the original worksheet?
17. Why do you think in some parking lots there may be spaces of different sizes or shapes?

18. Can you calculate the area of each parking space? Please show your calculations. Remember that this is a model, so if each parking space will suit any car, truck or SUV, what do you think the scale is? In other words, how many feet squared does each unit represent? Therefore, what would be the area of an actual parking space? Look up the size of a few cars online and verify that this will fit most vehicles.



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Lesson 4: Designing an Airport

Strand: Geometry

Mathematical Objective(s)
Students will be able to:

- To review and practice concepts regarding parallel lines cut by a transversal.
- To determine whether two lines are parallel.
- To work cooperatively in small groups.
- Recognize properties of parallel lines in real world examples.
- To apply concept of interior angles of circle to real world example.

Mathematics Performance Expectation(s)
MPE 32. The student will use the relationships between angles formed by two lines cut by a transversal to:

a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

MPE 11. The student will use angles, arcs, chords, tangents, and secants to:

b) solve real-world problems involving properties of circles.

Virginia SOL

G.2 a, b, c, G.11 a, b (The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel, verify the parallelism using the corresponding angles postulate and parallel lines postulate, and investigate a real-world example of parallel lines cut by a transversal. The student will also use angles to solve a real-world problem involving the properties of a circle.)

NCTM Standards

Mathematics as Problem Solving
Solve problems that arise in mathematics and in other contexts.

Mathematical Reasoning and Proof
Select and use various types of reasoning and methods of proof.

Mathematics as Communication
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Mathematical Connections
Recognize and apply mathematics in contexts outside of mathematics.

Mathematical Representation
Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Resources

- Rulers
- Protractors
- Highlighters or colored pencils
- SMART Board or Document Camera
- Handouts: "Airport Runway Pictures", "Airport Runway Design"

Assumption of Prior Knowledge

- Students should understand the concept of parallel lines and the angles formed when cut by a transversal.
- Though students may have difficulty recalling the proper vocabulary, (i.e. alternate interior, alternate exterior, etc.), they should recognize their location and relationship.
  The two skills above will be used when highlighting angles in Student Exploration 1 to verify parallelism of given runway pictures.
- The student should know that a circle is comprised of 360 degrees, and that a semi-circle is 180 degrees.
  The above skill will be used when labeling runways in Student Exploration 2.

Introduction: Setting Up the Mathematical Task

In this lesson, you will investigate the relationship between the angles created when parallel lines are intersected by a transversal. The use of parallel lines as airport runways will be examined to give students a real world example of geometry as it is used in airport design. The students will begin by examining aerial photographs to see the parallel orientation of runways. They will then verify the parallelism using known geometrical postulates and theorems. The students will then examine how airport runways are named and labeled for aviation and recognize the relationship between the nomenclature and mathematics. In small groups, students will diagram and label their own airport runway to be included in the unit on the creation of a fictional city. The description of the airport will be given to the class in their final presentation.

Student Exploration 1

The teacher should begin by displaying the results of a Google image search "airport runway design". Allow the students to make observations about the images. The teacher may prompt the students with questions such as:

Do you see any examples of parallel lines that have been discussed in class up to this point?

Do you see any transversals?

Do you see any numbers or letters marking the runways? (Have students hold on to this observation for Student Exploration 2.)

After the students have had time to make observations about the images, give them the handout "Airport Runway Pictures". (Teachers have the option of allowing students to search and print their own images should they not wish to use the images provided.) Direct student attention to the lines marking parallel runways and transversal taxi ways. Working independently, have students use a ruler to highlight or color the straight lines down the middle of the runways in the pictures. In a different color, have the students use the ruler to highlight or color the transversals. Model this on the document camera if students are having difficulty. In a third color (different colors may be used for all angles or the same color may be used in each photograph if resources are limited), have the students highlight the angles in the photographs as described below:

Photograph 1: Any pair of corresponding angles

Photograph 2: Any pair of alternate interior angles

Photograph 3: Any pair of alternate exterior

Photograph 4: Any pair of same side interior

- Using Photo 1, write the Converse of the Corresponding Angles Postulate on the board. (If two coplanar lines are cut by a transversal so that two corresponding angles have the same measure, then those lines are **parallel**.) Bold words may be left out if the teacher wishes to have the students complete the postulate. Have the students measure their highlighted corresponding angles with a protractor to see if they get equal measures verifying parallelism.
- For Photo 2, write the Alternate Interior Angle Theorem on the board. (If two coplanar parallel lines are cut by a transversal, then each pair of alternate interior angles is **congruent**.) Have students measure the alternate interior angles to verify parallelism.
- For Photo 3, write the Alternate Exterior Angle Theorem on the board. (If two coplanar parallel lines are cut by a transversal, then each pair of alternate exterior angles is **congruent**.) Have students measure the alternate exterior angles to verify parallelism.
- For Photo 4, **ask students what they notice about the measures of the same side interior angles?** They should notice that they are right angles. **Ask what types of lines create right angles**? (Perpendicular) Write the Perpendicular Transversal Theorem on

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the board. (In a plane, if two lines are perpendicular to the same line, then they are **parallel** to each other). **Ask students how the right angles verify parallelism in this case?**

Student Exploration 2

The teacher should begin by asking students to recall any observations of numbers and letters marking runways that they may have noticed in the Google image search from Student Exploration 1. The teacher may show the images again if necessary. Have the students read the handout "Airport Runway Design" independently and answer any questions they can. Then allow students to work together in their cooperative groups to compare answers and share their thoughts about each question. After sharing, go over the handout with the students and facilitate class discussion.

After class discussion, put students back into their project groups to begin design of their airport. The design should include:

- At least two parallel runways marked with both number and letter on both ends of the runways.
- A picture of a compass similar to the one in "Airport Runway Design" showing arrows on the directions of your runways.
- At least two transversals that serve as taxi-ways from your runways back to the main terminal of the airport.
- An airport name

Assessment

A. Utilize various strategies of formative assessment throughout the lesson.

- All activities: check for understanding by walking around the room as students complete the activities and facilitate discussion to help students grasp the difficult concepts.
- Student exploration 1: Visit each group while they complete the activity. Allow groups to share answers on the board or document camera and explain what angles were chosen and how the postulate or theorem was used to prove lines were parallel. Also, involve various students in dialogue about the runway images and angles.
- Student exploration 2: Allow students time to read through handout alone attempting to answer questions. Then facilitate guided reading and use various questioning techniques to gauge understanding. Visit groups during airport design to gauge understanding.

B. Journal/writing prompts

- Student Exploration 1: For homework, ask students to summarize the discussion of the question: “What types of angles do you think would be most beneficial in airport design when it comes to planes taxiing to and away from the runway? Explain your reasoning. Also summarize how you can prove two lines are parallel when you know the measures of angles created by a transversal.”

- Student Exploration 2: Have students share their thoughts on “How are geometry and math used in the design of airport runways? What other areas of science or design might include geometry?”

C. Summative assessment:
    - Airport design project
    - The final project
    - Standards covered in this lesson will also be assessed on the next chapter exam

Extensions and Connections (for all students)

A. For the homework assignment, ask students to research the five largest airports in the United States and give a rough sketch of their runway design. Students may also work together outside of class on their airport design.
B. Extra credit offered for 3-D model of the city and airport.

Strategies for Differentiation

A. Circulate through the room during the portion of Student Exploration 1 to assist any students having difficulty identifying parallel lines and transversals.
B. While verifying parallelism, it may be necessary to distribute a copy of the postulates and theorems for visually impaired or those who struggle with copying notes from the board.
C. While verifying parallelism, it may be necessary to assist ELL and ESL students with vocabulary.
D. During Student Exploration group-work, students will assist one another and those who feel most comfortable with oral communication will report to the class.
E. Some students have difficulty using a protractor. Students can be paired with stronger students who will help.

Scoring Rubric for Airport Design

Team Members: _____

	0	1	2	3	4	Peer Score	Teacher Score
Runways **Must include:** **a. two parallel runways,** **b. marked by number,** **c. marked by letter,** **d. marked on both ends**	None of a, b, c, or d included	One of a, b, c, or d included	2 of a ,b, c, or d included	3 of a, b, c, or d included	All of a, b, c, and d included		
Comments:							
Compass: **a. picture of compass included** **b. labeled by direction,** **c. labeled by degrees,** **d. direction of runway in project highlighted**	None of a, b, c, or d included	One of a, b, c, or d included	2 of a, b, c, or d included	3 of a, b, c, or d included	All of a, b, c, and d included		
Comments:							
Taxiways: **a. at least two,** **b. lead back to main gate,** **c. are truly transversals,** **d. designated as taxi ways**	None of a, b, c, or d included	One of a, b, c, or d included	2 of a, b, c, or d included	3 of a, b, c, or d included	All of a, b, c, and d included		
Comments:							
Airport name	Not included	included					
Comments:							

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Airport Runway Pictures

Figure 1. Massachusetts Institute of Technology, 2008. Airport Diagram of Boston's Logan International Airport. Retrieved from www.ll.mit.edu/news/runwaystatuslights.html

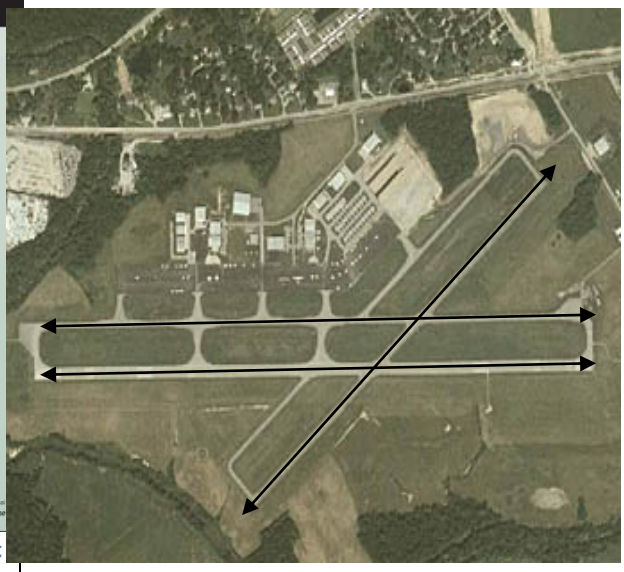


Figure 3. Jacobi, Toombs, and Lanz, n.d. [Photograph of Clark Regional Airport] Retrieved from www.jtleng.com/clark_regional_airport.htm

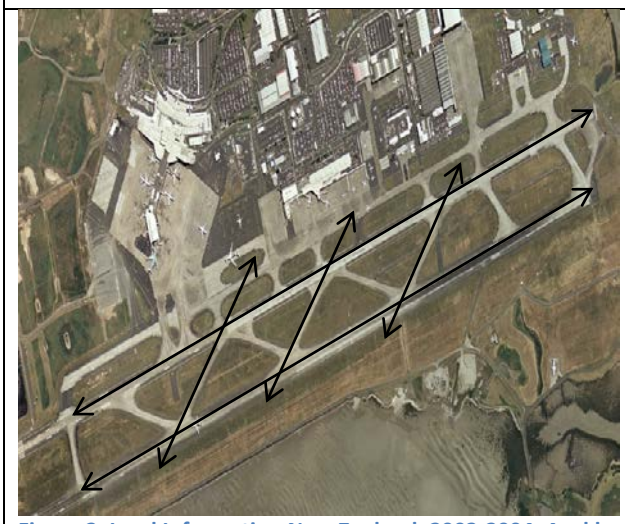


Figure 2. Land Information New Zealand, 2003-2004. Auckland International Airport Aerial Photograph. Retrieved from http://commons.wikimedia.org/wiki/File:Auckland_Int_Airport_aerial_photo.jpg

Figure 4. Fly Away Simulation, n.d. [Overhead airport photograph] Retrieved from www.flyawaysimulation.com

Airport Runway Design

There are many different configurations for airport runways. Many small airports have single runways. Single runways sometimes make takeoff and landing difficult if there is a major crosswind blowing perpendicular to the runway. For this reason, airports began designing runways that intersected so that planes could take-off and land in different directions based on the direction of the wind. As time passed, advances in aircraft made winds less of a factor and major airports began going to parallel runways so that they could maximize the number of flights by having planes take-off and land in opposite directions. The largest airport in the United States as far as passengers served is Hartsfield-Jackson Atlanta International Airport. It has four parallel runways and averages 252,000 passengers per day.

In this exploration, you will learn how airport runways are labeled by number and letter. You will also learn how the naming system is related to Geometry. After the exploration, you will design your own two-dimensional airport for your fictional city project.

Examine the set of parallel runways in the picture.

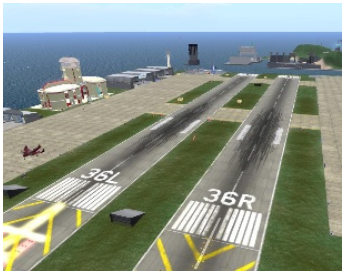
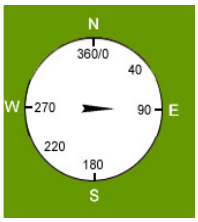


Figure 1. Daniel Voyager, n.d. [Image of simulation from Hollywood Airport from Second Life] Retrieved from http://danielvoyager.wordpress.com/2011/07/30/top-10-airports-in-second-life-2011/

Notice the "36R" and "36L" designations. The large number indicates the runways compass direction. Study the picture of the compass below:



Notice that the compass has a total of 360^{o} like any circle. A runway's number is not written in degrees. Instead, it is given a shorthand format. In the first picture, both runways are numbered with a "36". This means the runways are heading due north or 360^{o}. A runway with a marking of "9" would be close to, if not directly, due east or 90^{o}. A runway with a marking of "18" would be heading due south or 180^{o}.

What would be the direction of a runway with a marking of "27"?

Because both runways in the first picture are parallel, both are labeled "36". This means both takeoffs and landings in this direction are occurring at an orientation of 360 degrees, or due north.

What do you think the "R" and "L" mean in the first photograph?

If you said "right and left", you are correct. Take a moment to convince yourself that planes could land and takeoff in the same direction and that the "R" and "L" would correspond to the right and left of a pilot's line of sight.

Take a moment to examine the next picture. The runway markings are magnified so that you can see them from the aerial view.

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Figure 2. Associated Newspapers ltd., 2011 [Photograph of Tampa International Airport] Retrieved from http://www.dailymail.co.uk/sciencetech/article-1344899/Shift-magnetic-north-pole-affects--Tampa-airport.html

Notice the parallel runways "1R" and "1L". What direction on the compass do you think the runways are heading?

Now notice that "1R" and "1L" are labeled "19L" and "19R" respectively in the other direction. The same runways have different markings on both ends. What is the difference between 19 and 1? (19-1)

How many degrees does 18 correspond to on the compass?

What do you think the 180^{o} means with regards to the way the runway can be used?

A runway can normally be used in both directions and is named for each direction separately. "Runway 1" in one direction is "Runway 19" in the other direction. The two numbers always differ by 18 (180^{o}).

Now take a look at the first picture again below.

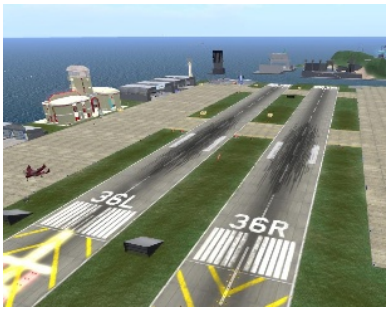

Figure 3. See Figure 1 above.

What do you think the markings would be on the other end of the runway?

"36R" would be _____ in the other direction.

"36L" would be_____ in the other direction.

Now it is time to construct your own airport for your city. The design will be a two-dimensional aerial view of your airport. Your airport design must contain the following:

- At least two parallel runways marked with both number and letter on both ends of the runways.
- A picture of a compass similar to the one in this document showing arrows on the directions of your runways.
- At least two transversals that serve as taxi-ways from your runways back to the main terminal of the airport.
- An airport name

Lesson 5: Final Project Presentation

Strand: Geometry

Mathematical Objective(s)
Students will be able to

- Communicate both verbally and in written form about mathematical concepts learned through this unit project and how these concepts are useful for real world situations.
- Describe the properties of the following as well as identify them in real world examples:
    - Interior angles
    - Alternate interior angles
    - Same side interior angles
    - Exterior angles
    - Alternate exterior angles
    - Vertical angles
    - Corresponding angles
- Explain how to use construction tools to create parallel lines cut by a transversal
- Explain how to use coordinate methods to create parallel lines cut by a transversal
- Explain how to measure angles using a protractor and when this is a useful method
- Explain how the concept of interior angles of a circle relates to objects in the real world, such as in the example of an airport runway.

Mathematics Performance Expectation(s)
MPE 32 The student will use the relationships between angles formed by two lines cut by a transversal to:

a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

MPE 7 The student will use similar geometric objects in two- or three-dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

Virginia SOL:
G.2 a, b, c, G.3 b
All standards addressed throughout this unit project are reviewed in lesson 5 since students will be presenting their projects which incorporate all standards covered. They will also listen to and

participate in the presentations of others. This lesson is important, because it gives students the opportunity to engage more deeply with this standard as they communicate their learning to others.

NCTM Standards

Mathematics as Problem Solving

Students will demonstrate the ability to use problem-solving approaches to investigate parallel lines and transversals and the angles made by them.

Mathematics as Communication

Students will communicate mathematical ideas about angle relationships made by parallel lines and transversals. The final lab report gives the opportunity for students to reflect and clarify what they have learned.

Mathematics as Reasoning

Students will reinforce logical reasoning skills by comparing and contrasting different angle relationships.

Mathematical Connections

Students will use and value connections between mathematical topics and other disciplines.

Materials/Resources

- Presentation software (PPT, Prezi or other)
- Computer lab
- Laptop computer
- Screen and projector
- Journals
- White board, white board markers
- Rubrics/scoring guides

Assumption of Prior Knowledge

- Students should know the properties of parallel lines cut by a transversal taught in the four previous lessons.
- Students should understand how the coordinate plane can help with the construction and verification of parallel lines.
- Students should know how to construct parallel lines using a compass and ruler.
- Students should know the properties of vertical, supplementary and complementary angles.

Introduction: Setting Up the Mathematical Task

The goal of this lesson is to deepen students' understanding of all concepts learned during this unit on parallel lines cut by transversals. The city project would have been assigned in lesson one, and students have been working toward a final product throughout the lessons. Between lesson four and this lesson, students had a whole class period to work on their group projects. They also will have spent time outside of class completing the project and putting it into a power point presentation.

Timing: Note that the presentations may need to be continued during a subsequent lesson, depending on the size of the class.

Warm-up

As students enter the room, the following warm-up questions will be on the board:

1. Reflect on this entire unit on parallel lines. What has been the most difficult concept to master, or which task or lesson was the most difficult for you to complete? Do you still struggle with this concept or skill? Explain.
2. Do you have any questions about parallel lines that you feel have not been answered in this unit?

Student/teacher actions: Students write in their journals while the teacher rotates around the room to monitor progress and answer any questions. Encourage students to think deeply and to be honest about any remaining questions.

- Students may hesitate to share their shortcomings. A teacher may decide to ask students to share which concepts they think would be most difficult for "someone" rather than to write what they themselves found most difficult. This might alleviate some anxiety related with sharing shortcomings.
- The teacher should encourage students to be honest. They are not being graded on how well they understand concepts learned. Instead, the teacher wants to see authentic reflection.
- These questions can be good sources of feedback to the teacher when warm-up books are collected and can influence review before the unit test.

Student Activity 1: Power point presentations

- Each group has had time over the past week to prepare a power point presentation that describes the city they create for this learning unit.
- Individual work: Each student was asked to write a separate, written reflection.
- During class, groups will present their power point presentations. Each student must play a verbal role in the presentation.

Student/teacher actions:

- Students should be encouraged to interact during their peers' presentations by asking questions and commenting.
- The teacher may need to serve as a catalyst for class discussions by reminding students of the key concepts completed in the unit including the concept of parallel lines as used in the city layout, parking lots, and airport design.

Student Activity 2: Peer Assessment

- Each group evaluates another group using the rubric below.

Student/teacher actions:

- The teacher should walk students through the meaning of the rubric and ratings.
- The teacher should make the peer assessment process very clear.
- Students should be encouraged to be honest and fair.
- Ideally, the class should fill out a sample rubric as a whole.

Strategies for differentiation

- In lesson one, students should have been informed that they can use the school computer lab or library to create the power point, if needed.
- Circulate through the room during the group project to assist any groups having difficulty with the requirements for the project.
- During the group presentation, students will all be expected to participate. The teacher may need to engage students who seem hesitant to orally present.

Name: _____ Date: _____ Class Period: _____

Handout: CITY PROJECT

In this group project, your group will have the opportunity to create a fictitious city. You will be working on various components of your city over the next three lessons. We will have one class period when you can work on the assignment together in class, but you will also need to work together to complete the project outside of class. The final product should be presented using an electronic presentation tool. Your presentation may include text, pictures of example city layouts and pictures or models of what you have created for your city. If you are unable to put everything into the presentation, it is okay to supply some things in written form or to write things on the board. The following components should be included in your final project:

3. Introduction
    a. Name of city
    b. Description of city
    c. Why did you choose to create this city? What is positive about your city?
4. Street Design
    a. A layout of the streets
    b. Indication of the properties of the lines and angles
    c. Contains the following buildings:
5. Parking Lot
    a. What is the parking lot for?
    b. How many spaces will there be in the parking lot and how did you decide on the number of spaces? (show your calculations)
    c. A mini-model of one or two rows in the lot
    d. A description of how the coordinate plane helped plan the design of these spaces/rows
6. Airport
7. Other element of city
    a. Select another element of the city that you would want to design
    b. Explain in words how geometry will help create this
8. Overall presentation
    a. Clear communication
    b. Visuals are good quality and easy to understand
    c. The group engages the rest of the class in mathematical discussion by asking questions and giving feedback to students on their answers
9. Mathematical Criteria (to be covered at least once at some point in the final project)
    a. Demonstration or description of how you used a construction
    b. Demonstration or description of how you used the coordinate plane
    c. Demonstration or description of how proved lines were parallel in lesson 4.
10. Individual Written reflection (30 points)
    a. Each student will write a 1-2 page reflection answering the following questions:

        1. What did you learn in this unit? Explain the key geometric properties you worked with. (10 points)
        2. How did you see geometry applied to the real world in this unit? (5 points)
        3. Can you name some real world situations, other than planning a city, where geometry might be needed? Please name at least five. (5 points)
        4. What did you like about working in groups? (5 points)
        5. What was difficult about working in groups? (5 points)

Developed by Dr. Agida Manizade & Dr. Laura Jacobsen, Radford University MSP project in collaboration with
Mr. Michael Bolling, Virginia Department of Education

Scoring Rubric for City Project

Team Members: _____

	0	**1**	**2**	**3**	**Peer Score**	**Teacher Score**
Title, description of city, rationale	Not included	Contains some but not all of these items	Contains all of these items, but description or rationale is weak	Contains all three and provides detailed description and rationale		
Comments:						
Street design	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Parking lot	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Airport	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Other element of the city	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Overall presentation	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						
Mathematical Criteria	Not included	Meets some criteria	Meets most criteria	Meets all criteria		
Comments:						

Overall comments:

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