

# Right Triangle Applications

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## **I. UNIT OVERVIEW & PURPOSE:**

The unit will be covering properties of right triangles, Pythagorean Theorem, Converse of Pythagorean Theorem, special right triangles, and Trigonometry of right triangles. Pythagorean Theorem is covered in Standards for Algebra 1, Algebra 2, and Geometry. Pythagorean Theorem and Trigonometry is also covered in College Algebra, Pre-calculus, and Calculus. The lesson also can encompass area of triangles as well.

## **II. UNIT AUTHOR:**

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## **III. COURSE:**

Mathematical Modeling: Capstone Course (the course title might change)

## **IV. CONTENT STRAND:**

Problem Solving, Decision Making, and Integration

## **V. OBJECTIVES:**

- Students should be able to recognize a right triangle using the Pythagorean Theorem
- Students should be able to calculate the lengths of sides of a special right triangle given one side
- Students should be able to find the angles or sides of a right triangle given an angle and side or given two sides

## **VI. MATHEMATICS PERFORMANCE EXPECTATION(S):**

MPE.5 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

## **VII. CONTENT:**

The content that will be discussed in this unit deals with using Pythagorean Theorem to solve real world problems covering distances on a map. Using Trigonometry to find angles of elevation, angles of depression, and distances to solve real world problems. Finally I will discuss a recent event (Tornado in Joplin, MO) where students will have to: plan to find the distance to Joplin, Mo from where they live, determine the path of the tornado, how many square feet the tornado affected, the shape of the tornado, and finally the height of the Tornado.

**VIII. REFERENCE/RESOURCE MATERIALS:**

Each student will need a TI -83 or above calculator. Ruler. Protractor. Map of the United States. Worksheets dealing with real world applications. A computer that has internet access for finding data on the Tornado that hit Joplin.

**IX. PRIMARY ASSESSMENT STRATEGIES:**

The assessments will be worksheets that will measure the students understanding of the material. There will also be assessments for real world problems at the end of each class.

**X. EVALUATION CRITERIA:**

For the first two days of the class there will be a seven question worksheets where each question is worth ten points. The final thirty points will be determined by the application question. The final day's lesson will be a project that each student will turn in. The final lesson project will be worth a total of 100 points. All three days of assessments will total 300 points.

**XI. INSTRUCTIONAL TIME:**

The instructional time required for this unit will be three 90-minute class periods.

# Lesson 3 Right Triangle Trigonometry, Pythagorean Theorem and its Converse

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## Strand

Geometry

## Mathematical Objective(s)

Identifying right triangles, Right Triangle Trigonometry. Pythagorean theorem and its converse. In this lesson students will examine real life problems using right triangle trigonometry to determine the size of a F-5 tornado. Students will also use the converse of the Pythagorean theorem to determine if points on a map form a right triangle. Students will develop a travel plan following a right triangle path. In the travel plan students will use their knowledge of right triangles to determine the distance traveled. Students will form the right triangle where two of the vertices will be Joplin, MO and Tuscaloosa, AL. Students will also have to plan a budget according to travel distance, gas prices, hotel rooms, food and drink, and mpg depending on the car they wish to drive.

## Mathematics Performance Expectation(s)

- 1.) The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions. 
- 2.) The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.
- 3.) The student, given a point other than the origin on the terminal side of an angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

## Related SOL

- G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.

## NCTM Standards

- Use trigonometric relationships to determine lengths and angle measures.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

**Additional Objectives for Student Learning (include if relevant; may not be math-related):**

Students will need to know how to read measuring devices. Students will also need to know how to read and use a map.

**Materials/Resources**

- Classroom set of graphing calculators
- Classroom set of tangible road atlas map. Map can be found:  
[http://images.nationmaster.com/images/motw/national\\_atlas\\_1970/ca000012.jpg](http://images.nationmaster.com/images/motw/national_atlas_1970/ca000012.jpg)
- Classroom set of computers

**Assumption of Prior Knowledge**

- The typical student would have already taken Algebra 1 class.
- Student should know how to use the Pythagorean Theorem and its Converse.
- To succeed in this lesson students should know how to use measuring tape.
- Students should know how to solve for a variable that involves using right triangle trigonometry.
- The most common misconception in this problem would be converting different units such as yards to feet and inches to miles.
- The relevant real life context in this problem is determining length, height, angle of depression, and/or angle of depression given limited facts.
- Students should know how to use the Pythagorean Theorem to find the lengths of a missing side in a right triangle.
- The relevant real life context in this problem is budgeting for travel.
- Students should know the interior angle-sum for a triangle.

**Introduction: Setting Up the Mathematical Task**

Understanding Student Comprehension: Before lesson 3 should be conducted students should have completed both lesson 1 and lesson 2 of this unit plan. It is possible to use the lesson without completing the former lessons but the time frames used for the lesson may be longer. The first part of the exercise, students will investigate the natural disaster that affected Joplin, MO. Students will investigate this by using the internet.

Hands on Activity:

- Students will work in groups of two. Students will partner where high-level students are with low-level students. Partnering in this way should limit the amount of time needed for each section.
- Each student will investigate the events that transpired and will have to write down five interesting facts.

- Students will also have to determine the shape of the tornado that hit Joplin. Students will determine the height and width of the tornado and then determine using right triangle trigonometry the all angles formed by the tornado and the hypotenuse of the triangle.
- This hands-on activity should not last longer than 30-minutes

#### Real World Problem:

- Real world problem will begin immediately as the hands-on activity is conducted.
- Students will then be told that they are part of a clean-up crew cleaning up the disaster that has affected Joplin, MO and Tuscaloosa, AL. Students may only travel in a right triangle. Using the same map from lesson 1, students will have to determine the starting city that also makes up a right triangle.
- Students need to plan for everything on the clean-up trip, these are factors students need to think about when creating a budget for the trip.
  1. Distance
  2. Gas Prices
  3. MPG for the car they prefer to take
  4. Hotel cost using a travel website
  5. Food and Beverage
  6. Tourism
- Students will develop a budget using the help of the Internet. Time should not take longer than 30-minutes.
- The last 30-minutes of class students will be allowed to present their scale drawing of the tornado in Joplin along with a building to show the size of the actual tornado. Then the five facts will be presented along with the budget plan. Should take no longer than 5 minutes each.

#### **Student/Teacher Actions:**

To explore and develop this lesson the following are important points of discussions:

- How many feet are in a yard? How many yards make up a football field? How many feet make up a football field?
- Using the map given to each student one inch corresponded to 100 miles
- Since the trip will consist of interstate travel the price of gas will be the national average which was found by google.com to be 3.57
- Students may choose which car they wish to travel in and have to use the internet to find the average miles per gallon highway
- Students may choose which hotel they would wish to stay in but must use the Internet and a travel website.
- Students need to know how to spend their money according to dining. They need to know how much money they need to eat on.
- Tourism is a category in which students must plan sites to visit while on their trip. They can find points of interest by visiting the city's website.

- Students need to know all trigonometric ratios. Students may ask how the trigonometric ratios came about.
- Students need to know that dividing by a trigonometric function is not the same as taking the inverse.
- Make sure calculator is in degree mode. Explain why there is a degree mode and radian mode for trigonometric ratios.
- The impacts of natural disasters such as tornados, tsunamis, and earthquakes.

### **Monitoring Student Responses**

- Students make their mathematical thinking and understanding public by responding to the questions mentioned in their project in their small groups as well as during the class presentation at the end of class. Since there is more than one correct answer for the group project (Example: Not all students will have the same base and hypotenuse for their right triangle on the travel trip because of differing cities) this allows a student to prove their assumption that the sides form a right angle using the converse Pythagorean theorem.
- Students may have a problem converting yards to feet. Make sure all students know how to convert from yards to feet.
- Make sure students have good foundation of Pythagorean theorem and its converse as well as right triangle trigonometry.
- For students who tend to finish more quickly tell them to find five interesting facts about the disaster in Tuscaloosa, AL and present info also in class. Are there any similarities between the two natural disasters?

### **Assessment**

The first assessment will be the hands-on activity that consists of drawing a scale model and five interesting facts.

- **Directions:**
  - Students will find information that will help them draw a scale drawing of the tornado that hit Joplin. To do this they need to know the height, width, and/or angles of the tornado. Make sure students when using the width know to divide by two to form the right triangle. The drawing also needs to include a building that is drawn to scale with the tornado so that the class can see the actual size of the tornado.
  - Students will also need to find five interesting facts about the disaster in Joplin.

The hands-on activity is worth 45 percent of the day's assessment. When grading this assessment determine if the data collected matches the scale drawing. If the student

determined the height to be 20000 ft and the base to be 16000 all the angles should correspond.

The second assessment will be the travel activity.

- **Question**

- Using the cities that the students found formed a right triangle on their own will develop a travel budget for the following trip.
- The budget will consist of gas price, mpg, hotel, travel distance, tourism, and food. Feel free to add or take away from the budget.

The budget will be worth 50 percent and students will have to write these down to present.

Presentation

- The students will make a presentation using the information found with both activities. Students will present no longer than 5 minutes.

The presentation should be worth 5 percent of the day's assessment.

## **Extensions and Connections (for all students)**

- To extend material for students who are ready to move forward introduce the concept of how trigonometric identities are solved by letting them create a similar triangle to their kite triangle and ask the students to find the angles using trigonometric ratios. Let them determine why trigonometric ratios are the values they are.
- To extend material for students who are ready to move forward introduce a different concept with right triangles. Let students try to find 30-60-90 right triangles and 45-45-90 right triangles that have vertices at cities.

## **Strategies for Differentiation**

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Make instruction more concrete, visual, collaborative, and hands-on
- Assign roles to students in collaborative activities. Discover the strengths of EOL students and assign appropriate roles.
- Be aware that there might be some differences in communicating the procedural knowledge of mathematics
- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)
- Language and content should be presented simultaneously
- Seating (near teacher or next to a buddy, native language if possible)
- Write legibly and in print

- Step by step instructions (orally and in writing) Ask students to repeat aloud for the rest of the class.
- Give EOL student more time for questions and answers.
- Let them discuss in pairs first
- Enunciate clearly and slowly without speaking louder.
- Simplify the language used rather than the mathematical concepts taught (use known vocabulary and simple sentence constructions).
- When students speak, focus on their message rather than their grammatical skills and accuracy. Respond using the proper grammatical form rather than overtly correcting their mistakes.
- Give LEP students (especially beginners) alternate ways to participate in whole-class discussions and respond to questions (think/pair/share, flashcards to raise over head, hand and/or body movements, individual chalkboards for solving computations).
- Assess whether LEP students have mastered mathematical concepts rather than their English grammar and fluency.

The accommodations are adopted from the following source.

<http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf>

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