# Transformational Graphing in the Real World

## I. UNIT OVERVIEW & PURPOSE:
This unit specifically addresses the concept of transformational graphing. Absolute value, polynomial, and square root functions will be examined.

## II. UNIT AUTHOR:
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## III. COURSE:
Mathematical Modeling: Capstone Course

## IV. CONTENT STRAND:
Algebra

## V. OBJECTIVES:
- To recognize and translate the graph of an absolute value function.
- To recognize and translate the graph of polynomial (specifically quadratic) functions.
- To establish a pattern for easy recognition of higher order polynomials.
- To recognize and translate the graph of square root function.
- To expose students to possible real world situations involving transformational graphing.

## VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors or these functions.
VII. CONTENT:  
This unit specifically addresses the topic of transformation graphing and general recognition of absolute value, polynomial (specifically quadratic), and exponential functions. All lessons are discussed in the context of a real world application.

VIII. REFERENCE/RESOURCE MATERIALS:  
Graphing calculators will be required. Student Exploration Worksheets and Exit Slip Assessments will be needed for all three lessons. Computers to access GeoGebra would also be beneficial when presenting if you find or make your own animation applets to help students visualize the concepts.

IX. PRIMARY ASSESSMENT STRATEGIES:  
Please see Exit Slip Assessments attached to each lesson. Upon completion of more functions a unit assessment on the transformation of functions will be given.

X. EVALUATION CRITERIA:  
Students will complete a five question exit slip at the end of the lesson. Documents will be attached to each individual lesson. Students will earn a classwork grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group’s participation in the class discussion, and 3 points are for each student’s participation in their group.

XI. INSTRUCTIONAL TIME:  
Three 90-minute class periods.

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**Bacteria – Growth or Decay?**

**Strand**  
Algebra

**Mathematical Objective(s)**  
Functions, Exponential Functions, Transformational Graphing

In this lesson students will discuss transformational graphing by examining the growth and decay of bacteria. Students will explore multiple representations of exponential functions. They will solve given problems by transforming the graph of an exponential function. Tables, graphs, and equations will be used by students to aide in finding the necessary solutions.
Mathematics Performance Expectation(s)

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL

AII.6 The student will recognize the general shape of function (exponential) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

Please note that transformation graphing will be applied in all lessons within the unit. The type of function addressed will change daily. It is intended for students to use basic transformation graphing techniques to help them on a day-to-day basis.

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Interpret representations of functions of two variables.
- Use symbolic algebra to represent and explain mathematical relationships.

Materials/Resources
Classroom set of graphing calculators.
Bacteria: Growth or Decay? – Student Exploration WS #1
Bacteria: Growth or Decay? – Student Exploration WS #2
Bacteria: Growth or Decay? – Exit Slip Assessment

Assumption of Prior Knowledge
- Students should have completed Algebra II.
- Students need to be familiar with the fast growth and/or decay of bacteria.
- Students should have experience using a graphing calculator, specifically finding a specific viewing window.
- Students might find it difficult to realize that horizontal transformations are represented “reversely” from graphic to symbolic representation.
- The relevant real life context in this problem is related to the behavior of bacteria over time.

Introduction: Setting Up the Mathematical Task
In this lesson, students will investigate how vertical and horizontal translations affect the symbolic representation of a quadratic function.

Introduction – 15 minutes
Student Exploration #1 – 25 minutes
Discussion of SE #1 – 10 minutes
Student Exploration #2 – 15 minutes
Discussion of SE #2 – 10 minutes
CW Assignment: Exit Slip Assessment – 10 minutes

- To introduce the task we will discuss the general form of an exponential function, $y = a(b)^x$, and how to tell when it is an example of growth or decay. It is important to note that a minimum of two points are needed to graph an exponential function. The TI-83, 84, or 84+ will give a symbolic representation found through regression that is in standard form.
  - First use the function $y = 2^x$ to discuss exponential growth functions.
  - Then use the function $y = .5^x$ to discuss exponential decay functions.
  - The class will discuss $a$ (the initial value or $y$-intercept) and $b$ (the growth or decay factor).
- Students will work the exploration activities and then the class will discuss their conclusions. Each group will be asked to answer a discussion questions on each exploration worksheet.
and comment on the answers provided by other groups. All students will have the correct answers before the end of class.

- Students are asked to explore transformational graphing of quadratic functions in the two exploration worksheets attached.

**Student Exploration 1:**

*Group Work (groups of 2 or 3)*

**Student/Teacher Actions:**

- Students should use the information from the introduction to complete a worksheet (Bacteria: Growth or Decay? – Student Exploration #1) that asks students to determine if examples (given in symbolic form, word problems, tables, and graphs) of exponential function as examples of growth or decay. Students will be asked to place all of the questions into the other three representations of the function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
- Multiple representations are used to help students who normally struggle see the relationship between the different representations. This should help students be able to convert given information that is confusing into a form that they understand.
- If time allows and you have access to a lab or a classroom set of computers... The dynamic Explore Learning activity gizmo allows students to change $a$, $b$, $h$, and $k$ and see how the graph changes as well. This website requires an initial software download, but is free. [http://www.geogebra.org/en/upload/files/Line/Exponential.html](http://www.geogebra.org/en/upload/files/Line/Exponential.html)

**Monitoring Student Responses**

- Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
- The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

**Student Exploration 2:**

*Group Work (groups of 2 or 3)*

**Student/Teacher Actions:**

- Students should use the information from the introduction and the discussion of the first student exploration to complete a worksheet (Bacteria: Growth or Decay? – Student Exploration #2) that addresses horizontal and vertical stretches and compressions as well as translations of a given exponential function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
Students should realize that a horizontal translation doesn’t always make sense conceptually (in the real world). Moving the graph to the left will technically start when time is negative.

Students should realize that most transformations of exponential functions involve vertical stretches and compressions.

Students will also find it difficult to learn that if \( a < 1 \) and \( b > 1 \) the exponential function is classified as growth.

- If time allows and you have access to a lab or a classroom set of computers... The dynamic Explore Learning activity gizmo allows students to change \( a, b, h, \) and \( k \) and see how the graph changes as well. This website requires an initial software download, but is free.
  

**Monitoring Student Responses**

- Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
- At the end of the second exploration students are asked to make generalizations about how horizontal and vertical shifts (translations) affect the function rule.
- The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

**Assessment**

Students will complete a five question exit slip at the end of the lesson. Please see attached document. Students will earn a CW grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group’s participation in the class discussion, and 3 points are for each student’s participation in their group.

**Extensions and Connections (for all students)**

The concept of vertical and horizontal stretching and compressing was brought up in the last exploration question and will be discussed in a classroom setting. The topic of transformational graphing will continue in subsequent days’ lessons.

**Strategies for Differentiation**

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student’s primary language.
• Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.

• Visual learners will benefit from the graphical representations and the ability to dynamic exploration allowed within Geogebra as well as the graphing calculator.

• Auditory learners will benefit from the classroom and group discussions.

• Kinesthetic learners will benefit from movement from individual work to group work and the involvement in classroom presentation.

• High ability students may start to begin to compare groups for similarities or differences or summarize their findings over the past three lessons about transformational graphing.
Bacteria: Growth or Decay? – Student Exploration #1

Please give three other representations of the following exponential functions.

1. Given the following word problem involving an exponential function determine whether it is a sample of growth or decay. Then given three other representations (graph, table, and function rule) of the exponential function.

The population of a bacteria sample at the start of an experiment is 64. After one hour the population is 16. Three hours into the experiment the bacteria population is 1.

Growth or Decay? Justify your decision.

_____________________________________________________________________

_____________________________________________________________________

Function notation: \( f(x) = \) ________________________________

Hint – You might want to use the graphing calculator to run an exponential regression on the data.

Table:

\[
\begin{array}{c|c}
X & Y \\
\hline
\end{array}
\]

Graph: Be sure the scale on your axes allows all three points found above to be graphed.
2. Given the following three points of an exponential function determine whether it is a sample of growth or decay. Then give three other representations (graph, words, and function rule) of the exponential function.

<table>
<thead>
<tr>
<th>X</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>22.5</td>
</tr>
</tbody>
</table>

Growth or Decay? Justify your decision.

_____________________________________________________________________
_____________________________________________________________________

Function notation: \( f(x) = \) ________________

Hint – You might want to use the graphing calculator to run an exponential regression on the data.

Words: Write a two sentence word problem that uses two of the points given in the table.

_____________________________________________________________________
_____________________________________________________________________

Graph: Be sure the scale on your axes allows all three points found above to be graphed.
3. Given the function notation determine whether this example is growth or decay. Then give three other representations (graph, words, and table) of the exponential function.

Function notation: \( f(x) = 5(3)^x \)

Growth or Decay? Justify your decision.

______________________________________________________________________
______________________________________________________________________

Table:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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</tbody>
</table>

Words: Write a two sentence word problem that uses two of the points given in the table.

______________________________________________________________________
______________________________________________________________________

Graph: Be sure the scale on your axes allows all three points found above to be graphed.
Bacteria: Growth or Decay? – Student Exploration #2

1. Given the parent function, \( y = 3^x \), predict what would happen if the following transformations were done.

   a. \( y = 3^{x+4} \)

   b. \( y = 3^{x-2} \)

   c. \( y = 3^x + 7 \)

   d. \( y = 3^x - 5 \)

   e. \( y = 2(3)^x \)

   f. \( y = .5(3)^x \)

   g. \( y = 4(3)^{x-1} + 6 \)

2. Now using your calculator check to see if your predictions were correct. If not correct them with what you see happening from the graphs of the parent function and the transformation.

   a. 

   b. 

   c. 

   d. 

   e. 

   f. 

   g. 

3. Which of the transformations in question 1 would **NOT** be appropriate to use to represent a bacteria population? Please justify your choices. (Remember that x values represent time and y values represent the number of bacteria present.)

4. Would you describe \( y = -1(2)^{x+3} - 4 \) as exponential growth or decay? Justify your answer.
Bacteria: Growth or Decay? – Exit Slip

1. Describe the transformations of the following exponential function.
   \( y = (4)^{x+6} + 1 \)  
   ____________________________________

2. Describe the transformations of the following exponential function.
   \( y = 5(0.875)^x + 7 \)  
   ____________________________________

3. Describe the transformations of the following exponential function.
   \( y = 0.475 \left( \frac{3}{4} \right)^{x+5} - 3 \)  
   ____________________________________

4. Would the exponential function given in question 3 be appropriate to model a real life situation involving bacteria population? Why or why not?

5. What determines in an exponential function is classified as growth or decay? (Hint – Does it matter what \( a \) is?)