Unit: Are you faster than Bacteria?

I. UNIT OVERVIEW & PURPOSE:

This unit is designed to use the concept of bacteria growth to explore exponential equations. The students will analyze data on bacteria growth and determine the equation of the curve of best fit for different bacteria. There are three lessons in this unit. The first lesson focuses on developing an understanding of how bacteria reproduce and how to model this mathematically. Students will use TI Nspire graphing calculators to assist them with this task. In the second lesson, students will use the graphing calculator to determine the equation for bacterial growth from different sets of data. The students would then make predictions based on their findings. In the third lesson the students will examine the complete life cycle of a colony of bacteria and use a polynomial equation for medicine concentration in the blood over time to simulate an antibiotic sure to a sickness

II. UNIT AUTHOR: Rupert Cox

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Algebra, Data Analysis, and Probability

V. OBJECTIVES:

The students will:

- learn how to use the TI Nspire graphing calculator to graph equations
- input data into the calculator and analyze using curve of best fit
- explore changes in the graph that correspond to changes in the equation
- develop an understanding of exponential growth

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

- **12)** Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.
- **14)** Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. Use a transformational approach to graphing. Use graphing calculators as a tool to investigate the shapes and behaviors of these functions.
- **15)** Use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).
- **16)** Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include
  - continuity;
  - local and absolute maxima and minima;

- domain and range, including limited and discontinuous domains and ranges;
- zeros;
- x- and y-intercepts;
- intervals in which a function is increasing or decreasing;
- end behavior;
- inverse of a function;
- finding the values of a function for elements in its domain

VII. CONTENT:

This unit utilizes content from microbiology to give context to the problems. The idea behind using bacteria is multifaceted. The fields of microbiology and biological engineering are potential was to make new technologies such as fuel-producing bacteria that could address global energy demands or plastic-consuming bacteria that could be used to clean up the plastic in our oceans. There are also epidemiological aspects to understanding bacteria. The spread of infectious bacteria is a real concern to the public. This unit will allow the students to discuss and explore these aspects of microbiology.

VIII. REFERENCE/RESOURCE MATERIALS:

- Projector and Nspire emulator
- Class set of Nspire graphing calculators
- Data sets on bacteria growth (provided with each lesson)
- Link to YouTube video showing bacteria growth
- TI Nspire Navigator system
- Petri dish and E. coli sample

IX. PRIMARY ASSESSMENT STRATEGIES:

Lesson 1: lesson reflection
Lesson 2: calculator files, lesson reflection, individual worksheet
Lesson 3: pre-quiz, calculator file
Post Assessment: Unit summary paper
All assessment materials are found in the lessons.

X. EVALUATION CRITERIA:

Lesson reflections/group reflection: The reflections are short written responses to questions about the lesson and is evaluated on students correct response to the questions.

Pre-quiz: This is evaluated based on correct responses

Calculator files: The Nspire calculator has the ability to save student files. These files can be reviewed during class or after class. If a Navigator system is available the files can be collected from the calculators for review.

Quiz: evaluated against correct responses

XI. INSTRUCTIONAL TIME:

Regular schedule: 3-4 days
Block schedule: 2 days

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***Note: This lesson was designed for use with the Nspire graphing calculator and Navigator system. If these are unavailable a TI-83 or TI-84 can be substituted with minor changes to the lesson. The file collection for assessment will not be possible, but the calculators can still be used to find the curves of best fit.**

Lesson 1 Title: How quick can you get sick?

Strand
Algebra

Mathematical Objective(s)
During this lesson the students will develop an understanding of how bacterial growth is exponential by:

- Taking a bacteria sample from a cell phone to grow
- Making scatter plots of data sets to visualize exponential growth
- Calculating the next data value based on a basic exponential function
- Plotting the new data point to see if it fits the model for the data set

The students will develop skills involving the Nspire graphing calculator and creating lists and spread sheets as well as data and statistics plots. The students will also practice using an exponential function to solve for data points. These objectives will be completed in the context of a class experiment to see what bacteria can grow on a cell phone.

Mathematics Performance Expectation(s)

- MPE 12
- MPE 14

Related SOL

- AFDA. 1 c (domain and range), f (intervals), g (end behavior)
- AFDA. 4 (analyze multiple representations of a function)
- AII. 6 (exponential shape & graphing calculator)
- AII. 7 a (domain and range), f (intervals), g (end behavior)

In this unit each of the related SOLs are present in each lesson. Some are included as a reference or referral point. These SOLs will be further developed in subsequent lessons. The main focus of lesson 1 is AFDA. 4 and AII. 6. This first lesson focuses on utilizing the graphing calculator to explore data tables and graphs for students to be able to recognize an exponential function.

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NCTM Standards

- Use symbolic algebra to represent and explain mathematical relationships;
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- Draw reasonable conclusions about a situation being modeled.
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- Interpret representations of functions of two variables
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning

- BIO. 1 d (graphing and calculation for data analysis), e (conclusions formed by data), i (graphing calculator used for analyzing and communicating data)
- BIO. 2 c (germ theory of infectious diseases)
- BIO. 5 e (human health issues)
- BIO. 9 a (limiting factors and growth curves)

Materials/Resources

- Classroom set of TI Nspire graphing calculators
- TI Nspire Navigator system
- Projector
- Optional: YouTube video showing collection of bacteria samples from cell phones http://www.youtube.com/watch?v=4lmwbBzClAc
- YouTube video showing bacteria replication http://www.youtube.com/watch?v=KIpcCyuypzg&feature=related
- Sets of data for three different bacteria growth models (Included at end of lesson)
- Essential vocabulary list (Included at end of lesson)
- Petri Dish and swab, may be available from Biology teacher

Assumption of Prior Knowledge

- The students should have prior knowledge of inputting a list into to Nspire Calculator.
- The students should be able to select the correct variables and create a graph of the data with a date and statistics page on the calculator.
- Students should be familiar with what an exponential function looks like when presented in a table and a graph.
- The actual growth pattern of bacteria is not purely exponential so predetermined data sets are used to align the data to the student's knowledge of exponential functions.
- Make sure that the students are familiar with infectious diseases and cultural issues such as E. coli outbreaks in food.

Introduction: Setting Up the Mathematical Task

- In this lesson you will be exploring and analyzing bacteria growth using the TI Nspire graphing calculators and Navigator system.
- Students will be introduced to bacteria and the concept of bacterial growth with a class discussion of bacteria on a cell phone. (See YouTube video in materials and resources) The teacher will then engage the students by taking a bacterial sample from a volunteer's phone. Use a cotton swab and rub it across the phone and then across a petri dish. Put the petri dish with lid on and upside down in a dark warm place to incubate. This will be revisited at the end of the unit.
- Time frame
    - Introduction/instruction: 10 minutes
    - Calculator activity: 25 minutes
    - Assessment: 10 minutes
- Prompt Questions
    - Have you ever been sick?
    - What normally causes you to get sick?
    - What if you worked for the CDC (center for disease control) and there was a major outbreak of a new bacterium causing many people to get sick?
    - What information might be important about the bacterium?
    - What are some key facts that you know about bacteria?
- Introduce key vocabulary to the students and discuss each of the words briefly with the class. Post a list of the definitions for students to reference.
- Instructional techniques: This activity will be completed in an informal group setting with students investigating the data on their individual calculators. Students can discuss topics with other students in close proximity or they can ask teacher for help. With the Navigator system being used each student needs to be logged into the class and develop an Nspire document file (.tns) file for assessment. Once the investigation is complete the class will discuss their findings and the instructor will retrieve the .tns files from the student calculators.
- To move the students towards the goal of the lesson they will be creating three graphs that will each be close to an exponential function. Students will then make predictions about what that would mean for bacteria growth. Use the YouTube video showing bacteria growth to aid students in seeing the growth.
- The teacher needs to be available to help any student with calculator issues of to answer any questions about the graphs. The teacher should be moving around the room to monitor student work. The teacher can also monitor the calculator activity with the Navigator with the screen capture function.
- The student deliverables will comprise of a .tns file and a lesson reflection in which students will give short answer responses to concept questions

Student Exploration 1:

- Discuss with students about the growth of bacteria and how it is very fast but do not tell them it is exponential.
- Give the students the first set of data and ask them to input the information into their graphing calculators under a lists and spreadsheets page. Instruct them to label the lists Gen1 and Pop1. Let the students know that the units of a bacteria population are measured in **titers**. Once they have completed this they can then create a scatter plot of the data using a data and statistics page. Ask the students which list or variable would be the independent and dependent. This is a good opportunity to ask students about domain and range of the data and what the contextual aspects of this information are.
- After the students have had time to complete this task, use the projector and Navigator software to show them a teacher created scatter plot of the same data. Make sure that the students are able to use the calculator and are able to save their work on the calculator.

Individual or Small Informal Group Work

The students will complete this process for two more sets of data. Instruct the students to use labels Gen2, Pop2 and Time, Pop3. It is important for lesson extension and differentiation for the third data set the have time as the independent variable. Be sure to have the students use a separate data and statistics page for each scatter plot. Once all students have completed the scatter plots, introduce the class to the equation for bacteria population.

$$b = B \bullet 2^n$$

b = new population
B = Starting population
n = number of generations

After explaining this equation have all the students compute the next population value for each of the three sets of data. Once they have finished this instruct them to add this to the lists for each of the data sets and check their scatter plots for see what the new value did. There are sample screen shots of the calculator outputs available at the end of the lesson on pgs. 10-12.

Whole Class Sharing/Discussion

Using the Navigator system, pull the calculator screen view from a student's handheld device and display this for the class discussion to summarize the informal group work. You should pick a different student's handheld for each scatter plot that is displayed. Discuss with the class what is happening in each graph and have the students make predictions about what the bacteria population values would be for the next few generations. The students should have had a problem with the data point they calculated for the data set 3 not fitting the exponential model. A possible student answer is 13696 titers. Discuss with the students all possible

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reasons that this did not match. The actual value should be 12209213.9 titers. This value came from using the time of generation of 25 minutes. So for seven hours of growth divide 420 minutes be the generation time and that will give the number of generations to use in the function provided.

Sample discussion questions:

- What was different about the information in data set 3?
- What effect did using time have on the data instead of generations?
- How could we figure out the next data point from what we know?

Student/Teacher Actions:

- Students should input data sets into their handheld calculator using the same list and spreadsheets page. The students then should create a scatter plot for each set of data. After a discussion about the calculator outputs the students will then calculate the next value in each data set using the provided equation.
- The teacher needs to move around the room monitoring student discussions and aiding in calculator use. The teacher will also facilitate learning by displaying information using the Navigator software.
- Possible calculator problems include dimension mismatch, this is cause by a student omitting a data point from one of the lists. Not all data points will show up in the scatter plot. The lists must have that same amount of data in each list. If this happens have the students recheck their data lists. Another problem that may occur is that they cannot find the variable box for the y-axis after they have selected the variable for the x-axis. The box in no longer highlighted but it is still there. If a student cannot find the box help them by scrolling the cursor to where it should be.

Monitoring Student Responses

- Students should be able to complete the scatter plots with the Nspire calculator
- Students will complete reflection questions as the finish the investigation. These reflections should highlight the exponential growth of bacteria with the students' ability to make contextual predictions bases on their understanding.
- Encourage students to answer each other's questions that may arise during the informal group work. A grade could be given for class participation based on student level and classroom level discussions.
- Remind students about the petri dish and that the class will examine the dish in a few days. Ask them to make predictions about what will be on the dish and if they can see anything if that would make sense with the fact the bacteria is microscopic.

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- Be sure to instruct the students to save their calculator work with the file name "**bacteria1**". Collect the students files and save the to the class portfolio using the Navigator software. These files will be used to assess the students understanding and calculator ability.
- Hold a class discussion about what they have learned from the lesson that day right before the students start the reflections. If the reflections are not finished in class they may be given as homework.

Assessment

- **Scatter Plots and Calculated Data Assessment**
    - The .tns file "**bacteria1**" can be reviewed to see what the students have completed on the calculator. This work can be evaluated based on the sample screen shots at the end of this lesson.
- **Reflections**
    - **Questions**
        - Question 1: Why does a bacteria's population grow so fast?
        - Question 2: What do the variables mean in the equation $b = B \bullet 2^n$
        - Question 3: Find the population value of the 72nd generation of bacteria with an initial bacteria count or titer of 23.
        - Question 4: According to our model from class, bacteria grow exponentially. Do you think that this is always true? Explain your answer.

Extensions and Connections

- This lesson can be extended into solving logarithmic functions by solving for the generation time. The equation for this is $\log b = \log B + n \log 2$
- Another extension to the lesson is having the students investigate other life forms that reproduce exponentially.
- The concept of scientific notation can be included in the lesson with the extreme sizes of the bacterial populations.
- There is a large connection to Biology with the contextual focus on bacteria. The students are increasing their understanding of microorganisms and studying the mathematic that models the live cycle of a bacteria population.

Strategies for Differentiation

- This lesson is a technology based lesson so the ability to address the needs of a diverse population of students is built into the lesson. The technology allows the teacher to have more time to help students who have difficulties or to give students opportunities to progress at faster rates.
- Kinesthetic learners can be engaged by having them take swabs of the cell phones and if resources are sufficient having more than one petri dish sample. Students can also be allowed to move around the room or stand during the informal group work.

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- Auditory learners can benefit from the informal group discussions. Encourage the students to facilitate the communication. The teacher should scaffold the discussion only.
- Visual learners will be engaged with the use of the technology. The graphing calculator will display the data in a visually appealing way that will convey to the student the basic understanding of exponential functions.
- English language learners need to have special considerations for this unit. If there are ELL students in the room have a copy of the instructions for each section printed in plain English for them. Also the ELL students should be given more time to complete the reflections. The language that the Nspire calculator is in can be changed to meet the needs of the student and there are different languages for the Texas Instruments web site at http://education.ti.com/educationportal/preference/selectCountry.do?cid=US
- Students with processing, memory, and motor issues would benefit from having the data sets printed so that students these can feel comfortable entering it into the calculator. These students can also be helped with the calculator investigation by other students in the classroom.
- High-ability students can be asked investigative questions during the informal group work. Ask the students if the think this model for bacteria growth is completely realistic. Have these students write a reflection on how strong of a model the exponential equation is and if they could introduce a better model.

Essential Vocabulary

Growth Factor: The base of the power in the equation $b = B \bullet 2^n$ in this case the number 2.

Generation: Also known as procreation, in biological sciences it is the act of reproducing.

Germ Theory: The theory that proposes that microorganisms are the cause of many diseases.

Nutrient Agar: sugar and plant gelatin substitute use in a petri dish to grow bacteria.

Titer: A way of expressing of concentration, in this context it is the concentration of bacteria cells in a sample or test solution.

Sets of Data

Generation 1	0	1	2	3	4	5	6	7	8	9
Population 1	98	203	381	789	1555	3097	6123	12944	25273	50953

Generation 2	50	51	52	53	54	55
Population 2	1.1259×10^{15}	2.2518×10^{15}	4.5036×10^{15}	9.0073×10^{15}	1.8014×10^{16}	3.6029×10^{16}

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Time	0	1	2	3	4	5	6
Population 3	107	564.7	2980.8	15732.6	83037	438272	2313213.5

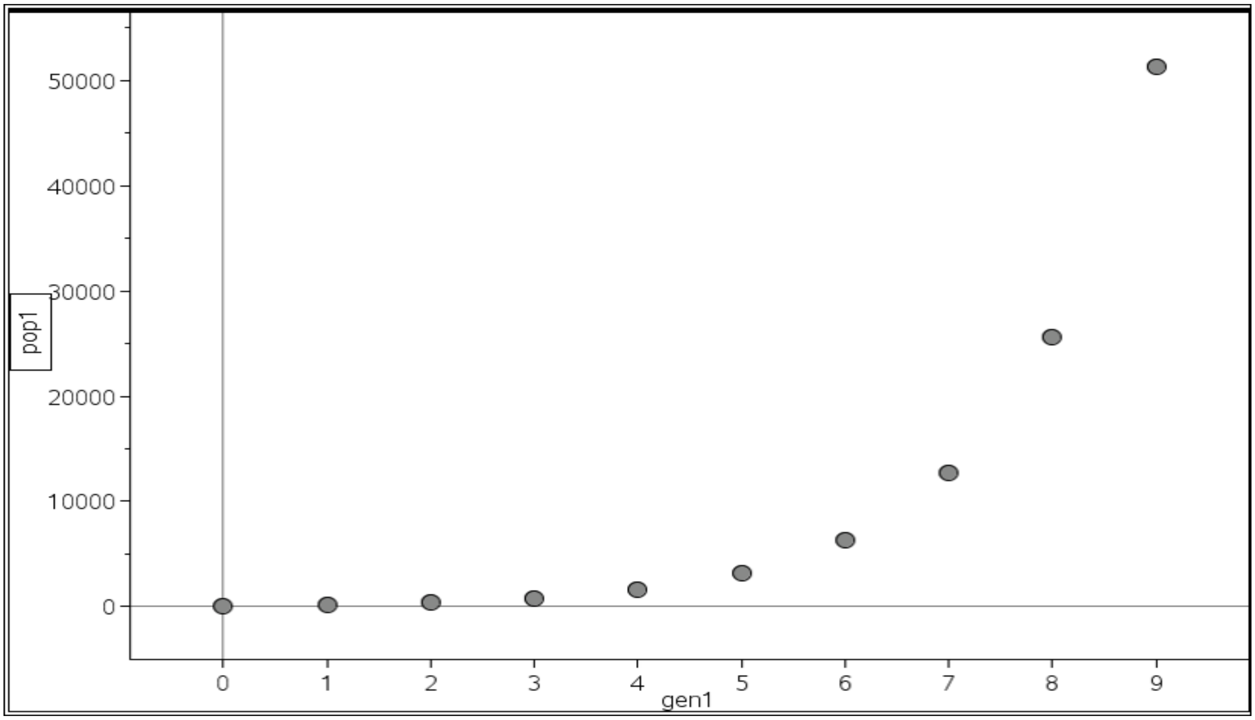
Sample Screen Shots of Nspire Work

The following screen shot is of the lists that that students should have inputted into the calculator. The data points that the students are supposed to calculate are included in these lists. The incorrect value for pop3 that students would get using the basic equation has been used.

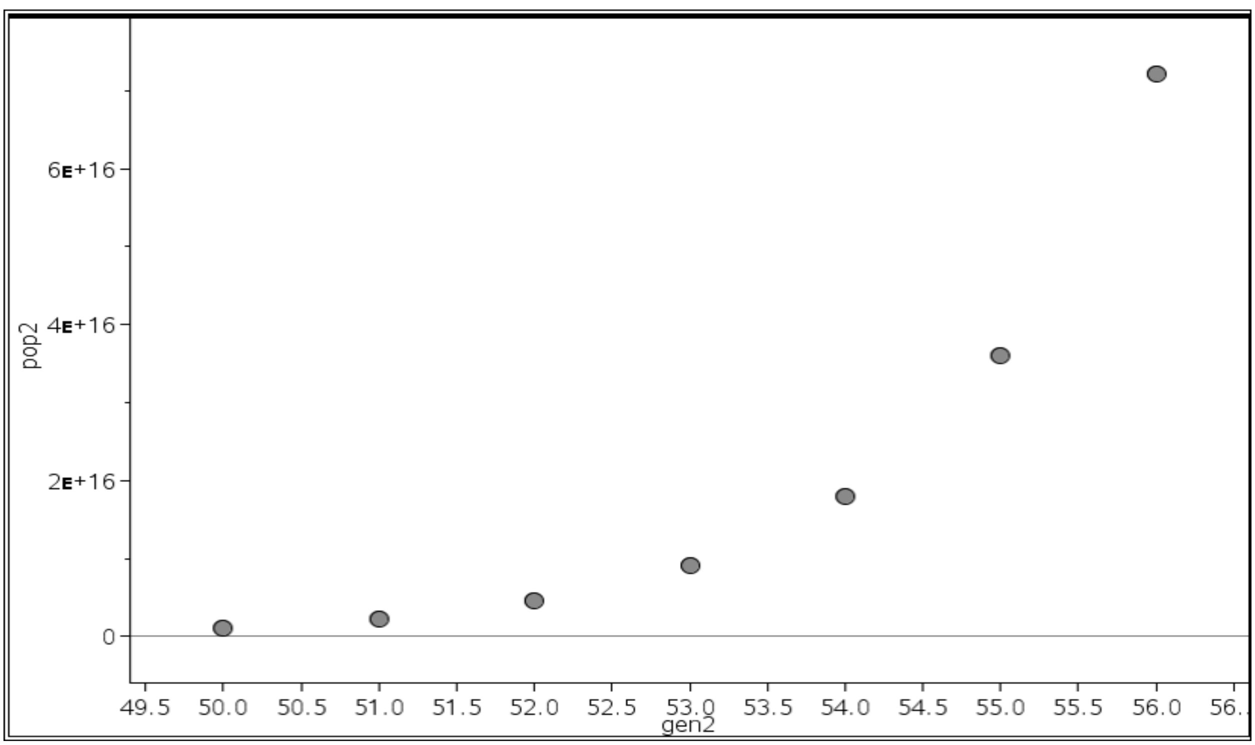
	A gen1	B pop1	C gen2	D pop2	E time	F pop3	G	H	I	J
◆										
1	0	98	50	1.1259*1...	1	564.7				
2	1	203	51	2.2518*1...	2	2980.8				
3	2	391	52	4.5036*1...	3	15732.6				
4	3	789	53	9.007*10...	4	83037				
5	4	1605	54	1.8014*1...	5	438272				
6	5	3197	55	3.6029*1...	6	2.31321...				
7	6	6354	56	7.2058*1...	7	13696				
8	7	12744								
9	8	25673								
10	9	51251								
11	10	100352								
12										
13										
14										
15										
16										
17										

D14

The next screen shot is of the scatter plot for Generation 1 and Population 1



This is the scatter plot for Generation 2 and Population 2.



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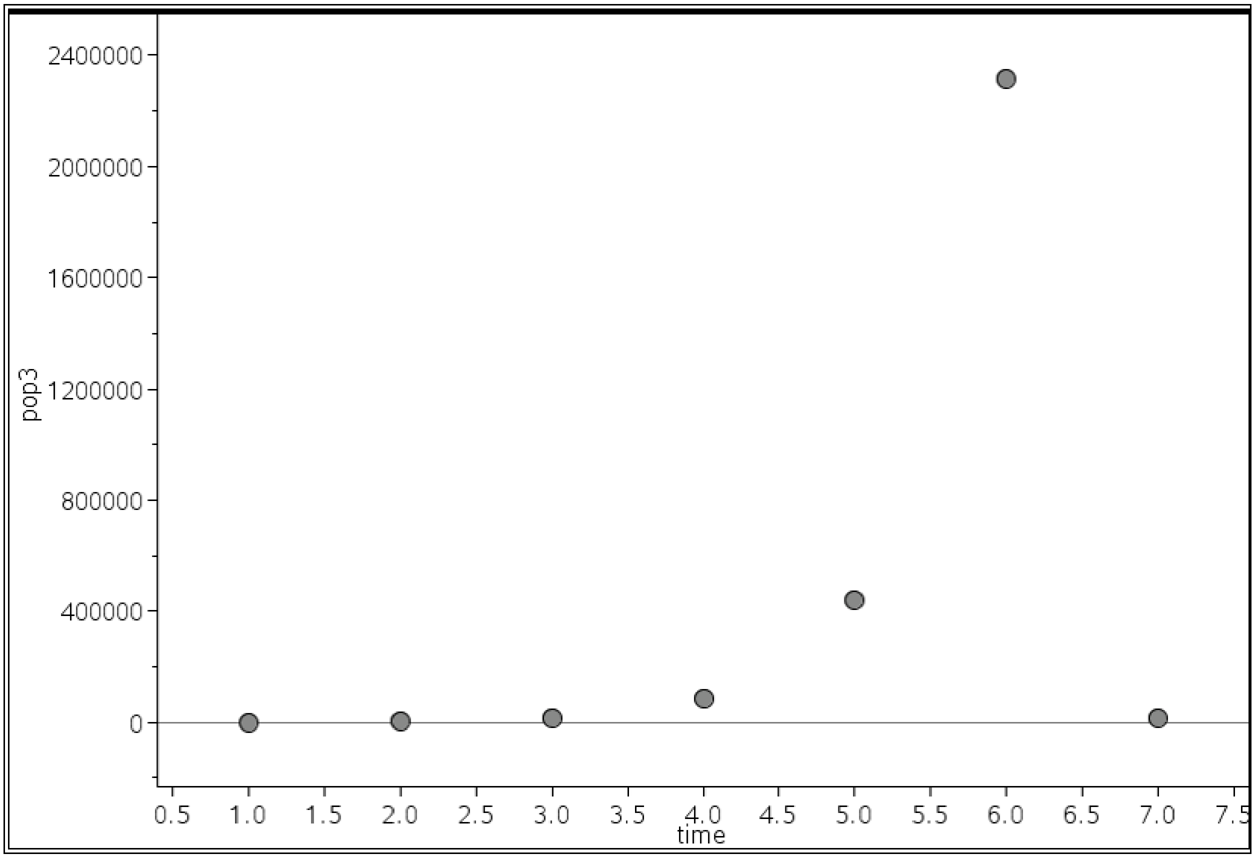
This is the screen shot for Time and Population 3. Notice that the last data point does not fit the model that the provided data creates. This is because time is used instead of generation number.



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Lesson 2 Title: What if it KEEPS GROWING!!!!

Strand
Algebra

Mathematical Objective(s)
During this lesson the students will develop an understanding of the curve of best fit for exponential functions and make predictions by:

- Use the Nspire calculator to find the curve of best fit
- Use the equation of the curve of best fit to find data points
- Determine that an exponential model does not work for the whole life cycle of the bacteria population

The students will develop skills involving the Nspire graphing calculator by using the regression capabilities. The students will use the equations for the curve of best fit that they find for each data set from lesson 1 to predict bacteria populations. These objectives will be completed in continuing context of the class experiment to see what bacteria can grow on a cell phone.

Mathematics Performance Expectation(s)

- MPE 12
- MPE 14

Related SOL

- A11 (curve of best fit)
- AFDA. 3 (curve of best fit)
- AII. 6 (exponential shape & graphing calculator)
- AII. 9 (curve of best fit)

The main focus of lesson 2 is A11, AFDA.3 and AII.9. This lesson focuses on utilizing the graphing calculator to explore scatter plots of data and finding the curve of best fit.

NCTM Standards

- Use symbolic algebra to represent and explain mathematical relationships;
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- Draw reasonable conclusions about a situation being modeled.
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- Interpret representations of functions of two variables

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- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning

- BIO. 1 d (graphing and calculation for data analysis), e (conclusions formed by data), i (graphing calculator used for analyzing and communicating data)
- BIO. 2 c (germ theory of infectious diseases)
- BIO. 5 e (human health issues)
- BIO. 9 a (limiting factors and growth curves)

Materials/Resources

- Classroom set of TI Nspire graphing calculators
- TI Nspire Navigator system
- Projector
- YouTube video showing bacteria replication http://www.youtube.com/watch?v=KIpcCyuypzg&feature=related
- Sets of data for three different bacteria growth models (provided in lesson 1)
- Saved student .tns files from previous lesson.
- Printed copies of lesson worksheet for individual work

Assumption of Prior Knowledge

- The students should have completed lesson 1 and have created the scatter plots.
- Students should have completed Algebra Functions and Data Analysis or Algebra 2. The students should also be operating on Level 2. Abstraction on the Van Hiele scale with respect to exponential curves.
- Students should be using vocabulary terms such as titer and growth factor. (Definition included in Lesson 1)
- Students might have difficulties with the abstraction of the large size of bacteria populations.
- The concept of an exponential function should have been explored in Lesson 1 and students should have an understanding of what a general exponential function looks like.
- Make sure that the students are familiar with infectious diseases and cultural issues such as E. coli outbreaks in food.

Introduction: Setting Up the Mathematical Task

- In this lesson, you will investigate further the aspects of bacterial growth by finding the curves of best fit.

- Students will revisit the concept of bacterial growth with a class discussion of bacteria on a cell phone. The students will then be prompted with the question “What if it keeps growing?”
- Time frame
    - Introduction/instruction: 5 minutes
    - Calculator activity/Group work: 25 minutes
    - Assessment: 15 minutes
- Prompt Questions
    - What if it keeps growing?
    - How big would a population get?
    - What are some key facts that you know about bacteria?
- Instructional techniques: This activity will be completed in an informal group setting with students finding and using the curve of best fit equations. Students can discuss topics with other students in close proximity or they can ask teacher for help. With the Navigator system being used each student needs to be logged into the class and continue to develop the Nspire document file (.tns) file that was started in lesson 1. Once the investigation is complete the class will discuss their findings and the instructor will retrieve the .tns files from the student calculators.
- To move the students towards the goal of the lesson they will be finding the curve of best fit for each of the three graphs from lesson 1. Students will then make predictions using the equations of the curve of best fit to calculate new data point. Use the YouTube video showing bacteria growth to aid students in seeing the growth.
- The teacher needs to be available to help any student with calculator issues of to answer any questions about using the regression menu. The teacher should be moving around the room to monitor student work. The teacher can also monitor the calculator activity with the Navigator with the screen capture function.
- The student deliverables will comprise of a .tns file, a lesson reflection in which students will give short answer responses to concept questions and completing a work sheet. (Attached at the end of the lesson)

Student Exploration 1:

- Have the students exploring how to find the curve of best fit for the scatter plots using the graphing calculator. Start this exploration with the following question.
    - “The famous bacteria E. Coli produces a new generation every 17 minutes when everything is optimal. If a population starting with one cell was to grow exponentially for 48 hours non-stop, the population would have more mass than the planet earth. **Do you believe this?**
- Have the students try to figure out ways to prove or disprove the E. Coli example.

- If using an equation to find this out does not come out of the class discussion then guide the students to this point.
- Instruct the students on how to the regression menu on the calculator and give them the task to find an equation for each data set.
- They should see similar equations for the first two data sets with a growth factor of approximately 2. The third data set will have a different growth factor. This is due to the use of time instead of number of generations.
- Once the students have completed this task, ask the whole class to figure out the value of the 1000th generation for the first two data sets. For the third data set have the students calculate the population after 48 hours.

Small Group Work (5-10 minutes)

- In the small group work the students will split up into groups of two to verify the weight of E. Coli.
- Students should record their work on the same piece of paper they use for the reflections. Have them label this work as group work.
- Give the students the following information.
    - mass of one E. Coli cell = 2.5×10^{-12} grams
    - mass of the planet Earth = 5.9742×10^{27} grams
    - generation time of E. Coli = 17 minutes

Individual Work

- After the students have finished the group work have them work individually on the worksheet. (Attached at the end of Lesson 2)

Student/Teacher Actions:

- Students should complete the calculator work in finding the curves of best fit for the sets of data.
- Students will use the basic function and the calculator to complete the group work and check the validity of the E. Coli statement.
- The teacher needs to move around the room monitoring student discussions and aiding in calculator use. The teacher will also facilitate learning by displaying information using the Navigator software.
- The solution for the group exploration is as follows:
    - $48hrs \bullet 60\min = 2880\min$
      $2880\min/17\min = 169.412\, generations$
      $b = B \bullet 2^n \rightarrow b = 1 \bullet 2^{169.412} = 9.9562\times10^{50}$

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$$9.9562\times10^{50}\bullet2.5\times10^{-12}\,grams=2.48905\times10^{39}\,grams$$

Monitoring Student Responses

- Students should be able to be able to find the curve of best fit with the Nspire calculator
- Students will complete a worksheet as individual work.
- Students will complete reflection questions as they finish the investigation. These reflections should demonstrate an increased understanding of exponential functions and the curve of best fit.
- Encourage students to answer each other's questions that may arise during the informal group work.
- Remind students about the petri dish. Examine the dish and see if anything has started. Inform the students that the class will look again at the end of lesson 3.
- Be sure to instruct the students to save their calculator work from the group activity with the file name "**bacteria1**". Collect the students files and save the to the class portfolio using the Navigator software. These files will be used to assess the students understanding and calculator ability.
- The students should save their individual work from the worksheet in a separate .tns file named "**bacteria worksheet".**
- Hold a class discussion about what they have learned from the lesson that day right before the students start the reflections. If the reflections are not finished in class they may be given as homework.

Assessment

- **Worksheet:** Students can complete the lesson worksheet attached on pg. 19
- **Curves of Best Fit Assessment**
    - The .tns file "**bacteria1**" can be reviewed to see what the students have completed on the calculator. This work can be evaluated based on the sample screen shots at the end of this lesson.
    - The .tns file "**bacteria worksheet"** can be reviewed to see if the students are mastering the calculator use on their own.
- **Reflections**
    - **Questions**
        - Question 1: Will the curve of best fit always fit all the data points?
        - Question 2: List some of the uses of a curve of best fit.
        - Question 3: Do you think information like a curve of best fit would be important to the Center for Disease Control? Why or why not?

- Question 4: Remember from lesson 1 and todays lesson that we have modeled exponential growth for bacteria. Do you think that this is always true? Explain your answer in detail whether or not it changed from lesson one.

Extensions and Connections

- This lesson can be extended into solving logarithmic functions by solving for the generation time. The equation for this is $\log b = \log B + n \log 2$
- The students can verify the mass of E. Coli and the planet Earth.
- There is a large connection to Biology with the contextual focus on bacteria. The students are increasing their understanding of microorganisms and studying the mathematic that models the live cycle of a bacteria population.

Strategies for Differentiation

- This lesson is a technology based lesson so the ability to address the needs of a diverse population of students is built into the lesson. The technology allows the teacher to have more time to help students who have difficulties or to give students opportunities to progress at faster rates.
- Kinesthetic learners can be engaged by having them check the petri dish sample. Remind them not to touch it. Students can also be allowed to move around the room or stand during the informal group work.
- Auditory learners can benefit from the informal group discussions. Encourage the students to facilitate the communication. The teacher should scaffold the discussion only.
- Visual learners will be engaged with the use of the technology. The graphing calculator will display the data in a visually appealing way that will convey to the student the basic understanding of exponential functions.
- English language learners need to have special considerations for this unit. If there are ELL students in the room have a copy of the instructions for each section printed in plain English for them. Also the ELL students should be given more time to complete the reflections. The language that the Nspire calculator is in can be changed to meet the needs of the student and there are different languages for the Texas Instruments web site at http://education.ti.com/educationportal/preference/selectCountry.do?cid=US
- Students with processing, memory, and motor issues would benefit from having the data sets printed so that students these can feel comfortable entering it into the calculator. These students can also be helped with the calculator investigation by other students in the classroom.
- High-ability students can be asked investigative questions during the informal group work. Ask the students if the think this model for bacteria growth is completely realistic. Have these students write a reflection on how strong of a model the exponential equation is and if they could introduce a better model.

Lesson 2 Individual Worksheet Name___________________________

Perform the following tasks on your graphing calculator. Be sure that you save all the calculator work in a file named "bacteria worksheet".

1. Input the following data set into your calculator and find the curve of best fit.

Domain	1	2	3	4	5	6	7	8	9
Range	126	378	1134	3402	10206	30618	91854	275562	826686

Curve of best fit = ____________________________

2. Input the following data set into your calculator and find the curve of best fit.

x	1	2	3	4	5
y	163.41	178.019	193.934	211.271	230.159

Curve of best fit = ____________________________

Identify the growth factor = ______________________

Identify the initial value = _______________________

3. Earning compounded interest is also an exponential model. Using the equation for compound interest $A = P(1+r)^t$ where A is the new balance, P is the principal deposit, r is the annual interest rate, and t is the number of years. Suppose you buy a house at 4.875% interest compounded annually for 30 years. The cost of the house is $220,000. How much will you pay by the end of the 30 years?

Answer = _______________________________

4. Use your calculator to complete the table with the following function $y = 5 \bullet 0.235^x$

x						
y						

What do you notice about the points in the table?

Is this an exponential function?

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Lesson 2 Individual Worksheet Name: Key

Perform the following tasks on your graphing calculator. Be sure that you save all the calculator work in a file named "bacteria worksheet".

5. Input the following data set into your calculator and find the curve of best fit.

Domain	1	2	3	4	5	6	7	8	9
Range	126	378	1134	3402	10206	30618	91854	275562	826686

Curve of best fit = $y = 42 \bullet 3^x$

6. Input the following data set into your calculator and find the curve of best fit.

x	1	2	3	4	5
y	163.41	178.019	193.934	211.271	230.159

Curve of best fit = $y = 150 \bullet 1.0894^x$

Identify the growth factor = 1.0894

Identify the initial value = 150

7. Earning compounded interest is also an exponential model. Using the equation for compound interest $A = P(1+r)^t$ where A is the new balance, P is the principal deposit, r is the annual interest rate, and t is the number of years. Suppose you buy a house at 4.875% interest compounded annually for 30 years. The cost of the house is $220,000. How much will you pay by the end of the 30 years?

Answer = 917449

8. Use your calculator to complete the table with the following function $y = 5 \bullet 0.235^x$

x	1	2	3	4	5	6
y	1.175	0.276125	0.064889	0.015249	0.003584	0.000842

What do you notice about the points in the table?
The data points are getting smaller and the rate of change is decreasing

Is this an exponential function? Yes

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Lesson 3 Title: The Complete Story

Strand
Algebra

Mathematical Objective(s)
During this lesson the students will develop an understanding of mathematical model of the life cycle of a bacteria population by:

- Checking a bacteria sample taken from a cell phone
- Examining exponential decay with the graphing function on the Nspire Calculator

The students will develop skills involving the Nspire graphing calculator the graphing aspect of the calculator to examine the death of a bacteria population. These objectives will be completed in the context of a class experiment to see what bacteria can grow on a cell phone.

Mathematics Performance Expectation(s)
- MPE 12
- MPE 14

Related SOL
- A11 (curve of best fit)
- AFDA. 1 c (domain and range), f (intervals), g (end behavior)
- AFDA. 3 (curve of best fit)
- AFDA. 4 (analyze multiple representations of a function)
- AII. 6 (exponential shape & graphing calculator)
- AII. 7 a (domain and range), f (intervals), g (end behavior)
- AII. 9 (curve of best fit)

In this lesson the main focus in on AFDA.4 and AII.6 with the exploration of exponential decay.

NCTM Standards
- Use symbolic algebra to represent and explain mathematical relationships;
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- Draw reasonable conclusions about a situation being modeled.
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- Interpret representations of functions of two variables

- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning

- BIO. 1 d (graphing and calculation for data analysis), e (conclusions formed by data), i (graphing calculator used for analyzing and communicating data)
- BIO. 2 c (germ theory of infectious diseases)
- BIO. 5 e (human health issues)
- BIO. 9 a (limiting factors and growth curves)

Materials/Resources

- Classroom set of TI Nspire graphing calculators
- TI Nspire Navigator system
- Projector
- Pre-quiz printed out or distributes as a .tns file using the Navigator system
- YouTube video showing bacteria population life cycle http://www.youtube.com/watch?v=SuvGpMevLPU
- Sets of data for three different bacteria growth models (Included at end of lesson 1)

Assumption of Prior Knowledge

- The students should have completed lesson 1 and lesson 2 and have created the scatter plots found the curves of best fit.
- Students should have completed Algebra Functions and Data Analysis or Algebra 2. The students should also be operating on Level 3. Deduction on the Van Hiele scale with respect to exponential curves.
- Students might have difficulties with the abstraction of the large size of bacteria populations or the mass of the planet Earth.
- The concept of an exponential function should have been explored in lesson 1 and students should have an understanding of what a general exponential function looks like.
- The concept of curves of best fit should have been explored in lesson 2 and students should have an understanding how to find and use a curve of best fit with the Nspire graphing calculator
- Make sure that the students are familiar with infectious diseases and cultural issues such as E. coli outbreaks in food.

Introduction: Setting Up the Mathematical Task

- In this lesson, you will investigate further the aspects of bacterial growth by studying exponential decay.

- Students will revisit the concept of bacterial growth with a class discussion of bacteria on a cell phone. The students should then watch the YouTube video on the life cycle of bacteria.
- Time frame
  - Pre-quiz: 5 minutes
  - Introduction/instruction: 10 minutes
  - Calculator activity/Group work: 15 minutes
  - Assessment: 15 minutes
- Prompt Questions
  - How fast do the bacteria die?
  - How can we model the death phase of a bacteria population
  - What are some key facts that you know about bacteria?
- Instructional techniques: This activity will be completed in an informal group setting with students discussing different ways to show the death phase of a bacteria population. Students can discuss topics with other students in close proximity or they can ask teacher for help. With the Navigator system being used each student needs to be logged into the class and continue to develop the Nspire document file (.tns) file that was started in lesson 1. Once the investigation is complete the class will discuss their findings and the instructor will retrieve the .tns files from the student calculators.
- To move the students towards the goal of the lesson they will graph the curve of best fit on a graphs page on the calculator for each of the three graphs from lesson 1. Students will then explore changing values of the growth factor to change the graphs. The students should use the $f_2(x)$ function to make the changes to the growth factor so they can compare it to the original function.
- The teacher needs to be available to help any student with calculator issues of to answer any questions about using the regression menu. The teacher should be moving around the room to monitor student work. The teacher can also monitor the calculator activity with the Navigator with the screen capture function.
- The student deliverables will comprise of a pre-quiz, a .tns file, and a unit summary paper.

Student Exploration 1:

- The students should start the lesson with a short pre-quiz. This can be given paper pencil or as a calculator file. If the file is used it should be collected using the Navigator system. (attached on pg.27)
- Have the students explore exponential decay by using the graphing calculator. Start this exploration with the following question.
  - How can we use the exponential model that we have made to show the death phase of bacteria?
  - Do you think it would have to deal with the growth factor?

- The students should add a graphs page to the "**bacteria 1"** .tns file. In this page they will graph the curve of best fit for the first data set using the function bar at the bottom of the page. A sample screen shot is included after the lesson.
- Have the students use the $f_2(x)$ function to perform the calculator exploration.
- Instruct the students to make changes to the growth factor until they come up with a way to see the population go down.
- Once the students have found a way to model an exponential decay, ask them to try and create one that the graph will be the opposite (inverse) of the original growth function.
- The students should arrive at using the inverse of the growth factor to find the appropriate graph.
- When the students have completed this task have them find a decay equation for the other two data sets from lesson 1.
- To bring the lesson and unit to closure, conduct a class discussion on the following topics.
    - Are there ways to speed up the death cycle of a bacteria population?
    - How would we see this in the decay function?
    - Why does a bacterium not grow as fast inside the human body?
    - How would we see this in the growth function?
    - Can you compare the life cycle of bacteria with the way a person gets sick?
- At the end of the unit summary the students will write a short letter to a health official discussing how students should not swap phones. Give the students a scenario in which there is an outbreak of a particular bacterium. The health official wants to slow the spread of the bacteria. Have them write a convincing letter for a public warning about cell phone swapping to be issued based on their understanding of the unit.

Individual Work

- Once the students have finished the exploration portion of this lesson have them begin writing the summary of the unit.

Student/Teacher Actions:

- Students will take a pre-quiz to demonstrate mathematical ability.
- Students will use the basic function and the calculator to determine a decay factor.
- Students will complete the unit summary at the end of the lesson; this will be finished as a homework assignment.
- The teacher needs to move around the room monitoring student discussions and aiding in calculator use. The teacher will also facilitate learning by displaying information using the Navigator software.

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Monitoring Student Responses

- Students should be able to be able to find approximate decay factors with the Nspire calculator
- Students will complete unit summaries when the finish the unit. This summary should demonstrate an increased understanding of exponential functions and the curve of best fit as well as the contextual understanding of bacterial populations.
- Encourage students to answer each other's questions that may arise during the informal group work.
- Remind students about the petri dish. Examine the dish and see if anything has started. There should be colonies of bacteria visible at this time. Have them reflect upon this in the unit summary. **Do not open the petri dish, it may contain a contagion.** Dispose of the dish properly by soaking it in bleach before discarded.
- Be sure to instruct the students to save their calculator work from the group activity with the file name "**bacteria1**". Collect the students files and save the to the class portfolio using the Navigator software. These files will be used to assess the students understanding and calculator ability.
- Hold a class discussion about what they have learned from the unit right before the students start the unit summary. The summaries should be finished as a homework assignment. Students should type the final summary.

Assessment

- **Pre-quiz:** The pre quiz is included at the end of the lesson on pg. 27
- **Exponential Decay Assessment**
    - The .tns file "**bacteria1**" can be reviewed to see what the students have completed on the calculator. This work can be evaluated based on the sample screen shots at the end of this lesson.
- **Unit Summary:** The unit summary will be a typed document that summarizes the unit and contains the letter to health official. The summary and letter should include the following
    - A description of the life cycle of bacterium with emphasis on the exponential growth and decay functions.
    - A description of using the graphing calculator for this unit.
    - A description of the petri dish experiment
    - A letter to a health official communicating the knowledge of growth and decay functions. The letter should also include reference to the petri dish experiment.

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Extensions and Connections (for all students)

- This lesson can be extended into solving logarithmic functions by solving for the generation time. The equation for this is $\log b = \log B + n \log 2$
- There is a large connection to Biology with the contextual focus on bacteria. The students are increasing their understanding of microorganisms and studying the mathematic that models the live cycle of a bacteria population. Students can be encouraged to look for other used for bacteria model such as fuel production or food production

Strategies for Differentiation

- This lesson is a technology based lesson so the ability to address the needs of a diverse population of students is built into the lesson. The technology allows the teacher to have more time to help students who have difficulties or to give students opportunities to progress at faster rates.
- Kinesthetic learners can be engaged by having them act out the conversation with the health official.
- Auditory learners can benefit from the class discussions of the functions and contextual implications.
- Visual learners will be engaged with the use of the technology. The use of the YouTube videos as well as the graphics from the graphing calculator will be advantageous to them.
- English language learners need to have special considerations for this unit. If there are ELL students in the room have a copy of the instructions for each section printed in plain English for them. Also the ELL students should be given more time to complete the unit summary. The language that the Nspire calculator is in can be changed to meet the needs of the student and there are different languages for the Texas Instruments web site at http://education.ti.com/educationportal/preference/selectCountry.do?cid=US
- Students with processing, memory, and motor issues would benefit from having extra time to complete the unit summary. Help should be provided for using a word processing software as needed.
- High-ability students can be given the task of writing a letter that contains graphs and data to back up any claims that they would make.

Pre-Quiz Lesson 3 Name______________________

Complete each of the following questions.

1. In the function $f(x) = 2203 \bullet 2.3154^x$ what is the initial value and growth factor?

2. Input the following data into your calculator and give the curve of best fit equation for it?

15 year periods	0	1	2	3	4	5	6
Fox Population	75	293	1154	4762	19198	76853	307123

Curve of best fit = ___________________________

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Pre-Quiz Lesson 3 Name______________________

Complete each of the following questions.

1. In the function $f(x) = 2203 \bullet 2.3154^x$ what is the initial value and growth factor?

   Initial value = 2203
   Growth factor = 2.3154

2. Input the following data into your calculator and give the curve of best fit equation for it?

15 year periods	0	1	2	3	4	5	6
Fox Population	75	293	1154	4762	19198	76853	307123

   Curve of best fit = $y = 73.5629 \bullet 4.01242^x$

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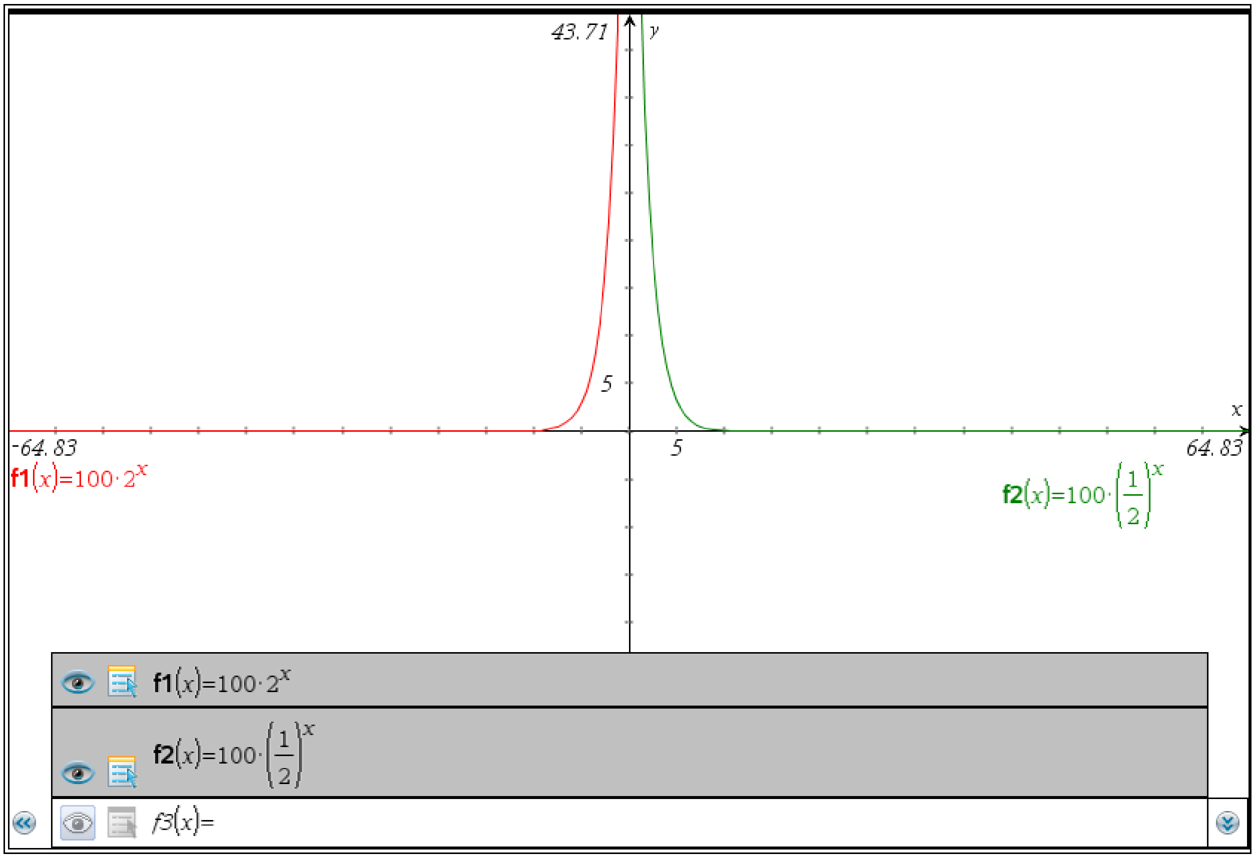
This is a screen shot of the graphs page for the growth and decay functions for data set 1.



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