

Percentages

I. UNIT OVERVIEW & PURPOSE:

The unit has the purpose of students learning about percentages and its applications to personal finance. Students will be calculating net salaries, possible car payments, and developing and analyzing a personal budget.

II. UNIT AUTHOR:

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III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Number and Operations

V. OBJECTIVES:

Students will learn applications of percentages and will be applying this to personal financial situations. Students should see the real-world applications of percentages and how budgets will play a critical role in their future.

VI. MATHEMATICS PERFORMANCE EXPECTATION(S):

MPE 1.

The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE 26.

The student will solve, algebraically and graphically,

- a) absolute value equations and inequalities;
- b) quadratic equations over the set of complex numbers;
- c) equations containing rational algebraic expressions; and
- d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions.

VII. CONTENT:

Students will be applying mathematical concepts to other content areas outside mathematics including economics and budgeting. Students will be provided with practical problems and will be asked to choose a method to solving them. Many of the problems can be solved by setting up an algebraic equation and solving for the unknown.

VIII. REFERENCE/RESOURCE MATERIALS:

Calculators, Internet access via classroom laptops or school computer lab.

IX. PRIMARY ASSESSMENT STRATEGIES:

Each lesson (3 lessons) will have an assessment collected in the form of worksheets.

These assessments will be completed by the students as individuals or in pairs (it is up to the students and to the teacher).

The last day, there will be a final assessment and it will be completed individually.

X. EVALUATION CRITERIA:

For grading the whole unit, it is suggested that each of the three lesson assessments count as 20% each and the final assessment count as 40%. Possible correct solutions are included in this document. There may be variations in the answers. Teachers should determine how to distribute the points (some points given for accuracy of the answer, neatness in presentation, clarity in explanations, etc.). On the final assessment, students

will be asked to complete two Excel spreadsheets. It is suggested that these each count for 50% of the final assessment grade.

XI. INSTRUCTIONAL TIME:

Four-45 minutes classes (including the final assessment).

Lesson 2--Percentages

Strand

Number and Operations

Mathematical Objective(s)

Percentages. In this lesson students will develop ways to apply percent concepts to calculating net pay and car payments.

Mathematics Performance Expectation(s)

MPE 1.

The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE 26.

The student will solve, algebraically and graphically,

- a) absolute value equations and inequalities;
- b) quadratic equations over the set of complex numbers;
- c) equations containing rational algebraic expressions; and
- d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions.

Related SOL

- A.4 The student will solve multistep linear and quadratic equations in two variables, including
- a) solving literal equations (formulas) for a given variable;
 - b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
 - c) solving quadratic equations algebraically and graphically;
 - d) solving multistep linear equations algebraically and graphically;
 - e) solving systems of two linear equations in two variables algebraically and graphically; and
 - f) solving real-world problems involving equations and systems of equations.
- Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

Additional Objectives for Student Learning (include if relevant; may not be math-related):

Students will learn how to budget for car payments.

Materials/Resources

- Classroom set of graphing calculators.
- Access to a classroom set of laptops (or the class will need to take place in a computer lab).
- Internet access.

Assumption of Prior Knowledge

- Students should already have the basic concept of percents (out of 100) and how to find percent of a number. Students should also know how to calculate a percent.
- Students should have already completed Algebra 1.
- Students should also have prior knowledge of equations. This prior knowledge includes setting up (modeling) an equation given a practical problem and then solving it.
- Students should understand the basic idea behind gross versus net pay and should have prior knowledge of what income tax is and how to calculate it.
- The relevant real life context in this problem involves salaries, income tax, gross and net pay, and monthly car payments.
- Students should also be familiar with the idea of interest and should know what each part of the equation $I = prt$ stands for.

Introduction: Setting Up the Mathematical Task

- “In this lesson, you will investigate the applications of percents when calculating salaries and calculating car payments.”

Begin with a review of calculating interest, using the equation $i = prt$. Remind students that time is represented in this equation in years. After a few problems reviewing this concept, begin the exploration.

Student Exploration 1 and Assessment:

Give students a copy of the following questions. It is suggested that students work together (no more than 2 students). Teachers should circulate around the room and provide hints and ask leading questions. This is to be collected at the end of the class and is the assessment for the class period. Require students to show their work and write down their calculations. Simply giving an answer should not be acceptable. Encourage them to explain their reasoning. Also, if the teacher prefers, one could start the class out as a whole group and have a discussion of methods, what the unknowns are, etc.

1) The average American spends 6.5% (source: <http://financemymoney.com/wp-content/uploads/2010/05/wheredidthemoneygo.jpg>) of their net income on purchasing a car or on car payments. If your income is currently \$29,000 and you are offered an interest rate of 5.5% for 36 months, how expensive of a car can you buy? (Assume the interest rate is a simple interest rate, use the equation $i=prt$. Also assume that over the course of 3 years, you received no raise).

One method of finding a solution:

$29,000 - 0.1 * 850 - (29,000 - 8,500) * 0.15 = 25,075$ (these are the steps to finding the net pay)

6.5% of 25,075 = 1,629.88/year (how much money a year allotted for car payments)

1,629.88 x 3 (3 years) = 4,889.63 = i + p (over 3 years, this is how much money is to be spent on the car payments—this includes the principal and interest amounts).

4889.63 – p = p*0.055*3 (this equation is the I = prt equation.)

4,197.11 = p

2) What will your monthly payments be?

$\$4889.63/36 = \135.82

3) Please research, on the internet, and find what kind of car you could possibly buy for this amount (using the principal amount).

Extensions and Connections (for all students)

- The following extension could be used as a differentiation activity, extra-credit activity, homework assignment, or for an after-school session

1) Find your dream car online. How much does it cost?

2) Assuming that you will receive an interest rate of 5.5% for 60 months and you are only allotted 6.5% of your net pay to spend on car payments, how much money do you need to GROSS in order to pay for this car?

Example response: My dream car is \$25,000.

Using I = prt, shows that I will have to spend a total of \$31,875 on this car, or \$6,375 a year.

$6,375 = 0.065x$ (6,375 is 6.5% of what?)

$x = \$98,077$ (this is the NET pay).

Now to find the gross pay, one could use this equation:

$x - 0.1*8,500 - 0.15*(34,500 - 8,500) - 0.25*(83,600 - 34,500) - 0.28*(x - 83,600) = \$98,077$
 $\$127,352$

3) Please respond in a journal entry describing if the needed salary seems feasible for you to obtain. What kind of job do you need to get to pay for this car? If this cost of this car warrants a substantial income, would you be more willing to find the job (include getting the education, training, etc.) it takes to pay for this car, or more willing to settle for a less expensive car? Or would you be willing to spend a higher percentage of your income on the car payments?

Strategies for Differentiation

- Lower ability students: Be willing to spend longer on reviewing $i=prt$ for some students.
- Higher ability students: See Extensions.

Lesson 3--Percentages

Strand

Number and Operations

Mathematical Objective(s)

Percentages. In this lesson students will develop ways to apply percent concepts to calculating net pay and analyzing a budget.

• **Mathematics Performance Expectation(s)**

- MPE 1.
- The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
- MPE 26.
- The student will solve, algebraically and graphically,
 - a) absolute value equations and inequalities;
 - b) quadratic equations over the set of complex numbers;
 - c) equations containing rational algebraic expressions; and
 - d) equations containing radical expressions.
- Graphing calculators will be used for solving and for confirming the algebraic solutions.

• **Related SOL**

- A.4
The student will solve multistep linear and quadratic equations in two variables, including
 - a) solving literal equations (formulas) for a given variable;
 - b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
 - c) solving quadratic equations algebraically and graphically;
 - d) solving multistep linear equations algebraically and graphically;
 - e) solving systems of two linear equations in two variables algebraically and graphically; and
 - f) solving real-world problems involving equations and systems of equations.
- Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

Additional Objectives for Student Learning (include if relevant; may not be math-related):

Students will learn how to develop a budget in Microsoft Excel.

Materials/Resources

- Classroom set of graphing calculators.
- Access to a classroom set of laptops (or the class will need to take place in a computer lab).
- Internet access.

Assumption of Prior Knowledge

- Students should already have the basic concept of percents (out of 100) and how to find percent of a number. Students should also know how to calculate a percent.
- Students should have already completed Algebra 1.
- Students should also have prior knowledge of equations. This prior knowledge includes setting up (modeling) an equation given a practical problem and then solving it.
- Students should understand the basic idea behind gross versus net pay and should have prior knowledge of what income tax is and how to calculate it.
- The relevant real life context in this problem involves salaries, income tax, gross and net pay, and budgeting.
- Students should be familiar with Microsoft Excel.
- Students should understand the concept of the median of a set of data.

Introduction: Setting Up the Mathematical Task

- “In this lesson, you will investigate the applications of percents when calculating salaries and by developing and analyzing a budget.”

Explaining the importance a budget might also be an appropriate way to lead into the lesson.

Student Exploration 1 and Assessment:

Give students a copy of the following instructions. It is suggested that students work together (no more than 2 students). Teachers should circulate around the room and provide hints and ask leading questions. This is to be collected at the end of the class and is the assessment for the class period. Require students to show their work and write down their calculations. Simply giving an answer should not be acceptable. Encourage them to explain their reasoning. Also, if the teacher prefers, one could start the class out as a whole group and have a discussion of methods, what the unknowns are, etc.

1) The median salary for Americans in 2008 with a 4-year college degree is \$55,700. The median salary for Americans with a high school diploma is \$33,800.

(Source: http://trends.collegeboard.org/downloads/Education_Pays_2010.pdf (pg. 12)).

2) In 2009, Americans averaged spending their net income in the following ways:

(Source: <http://financemymoney.com/wp-content/uploads/2010/05/wheredidthemoneygo.jpg>)

17.6% of net income on transportation (car payments, gasoline, and maintenance).

12.4% of net income on food

34.1% of net income on housing (shelter, furnishings, maintenance)

16.5% of net income on insurance and healthcare

5.0% of net income on clothing and cosmetics

The remaining portion is left for various expenditures (i.e. education, entertainment, charity, and vacations).

3) In Microsoft Excel, create two budgets on two different worksheets (a monthly budget!). One budget should be for an average high school graduate and one for an average college graduate. List the above categories and how much money each one will get to spend in each category. Investigate how much is left for each one to spend on the various expenditures. The spreadsheet should include the gross and net pay of each salary. Require that student use formulas in Excel and do not just simply calculate all of the figures on the calculator and then enter them into Excel. This should be a requirement for the rest of the unit.

Example of College Graduate Budget in Excel:

Median income

(gross pay) : \$55,700

**Net Pay (Federal
Income taxes taken
out):**

\$45,650

Category	Percent	Amount	Monthly Amount
Food	12.40%	\$5,660.60	\$471.72
Transportation	17.60%	\$8,034.40	\$669.53
Housing	34.10%	\$15,566.65	\$1,297.22
Insurance/Healthcare	16.50%	\$7,532.25	\$627.69
Clothing/Cosmetics	5.00%	\$2,282.50	\$190.21
Other	14.40%	\$6,573.60	\$547.80
		\$45,650.00	\$3,804.17

Example of High School Graduate Budget in Excel:

Median income

(gross pay) : \$33,800

**Net Pay (Federal
Income taxes
taken out):**

\$29,155

Category	Percent	Amount	Monthly Amount
Food	12.40%	\$3,615.22	\$301.27
Transportation	17.60%	\$5,131.28	\$427.61
Housing	34.10%	\$9,941.86	\$828.49

Insurance/Healthcare	16.50%	\$4,810.58	\$400.88
Clothing/Cosmetics	5.00%	\$1,457.75	\$121.48
Other	14.40%	\$4,198.32	\$349.86
		\$29,155.00	\$2,429.58

4) Research how much money you would expect to spent on entertainment a year. Use the internet to do this. Look up the cell phone plan you wish to have, the cost of wireless or DSL, the cost of satellite/cable. How much will you spend on Wii games, songs from I-tunes, and movies? Do you plan to buy a new I-Pod or laptop every year? Put these costs into a separate sheet in the same Excel document. Find the average monthly costs of your entertainment spending. How much would you have over per month if you had the salary of a high school graduate? A college graduate?

Example of Entertainment Budget in Excel:

Cell Phone Plan	\$80.00	
Cable/Satellite	\$120.00	
Wireless	\$40.00	
4 Wii Games a Year/12	\$7.00	
1 New Ipod a Year/12	\$20.00	
100 songs from Itunes a Year/12	\$10.00	
	\$277.00	(monthly amount)

Amount left per month for average college graduate: \$270.80
 Amount left per month for average high school graduate: \$72.86

5) Keeping in mind that the category in which entertainment falls, vacations, savings, charity, and education are also included. Consider how much you have left over after you spend money on entertainment. Write what you would do with the remaining money. Save? Take a vacation? Would you cut your entertainment costs to you could do other things with that money? Please record in the journal what adjustments you would make to afford the things most important to you. (Consider also the level of education you anticipate obtaining).

Here are some possible (SHORT) journal entries (the teacher could share these with the students to help get them started):

I plan to finish college and see that I would have some left over for savings. Vacations are not a high priority for me, but giving to charity is. I would save 75% of the remainder of the money and donate 25% of the remaining money to charity.

I plan to complete high school. I see that I would have little left for other things. I would choose to cut my cable/satellite bill so that I could save up money to go to the beach every summer.

Strategies for Differentiation

- Lower ability students and ELL students: The teacher should already have a budget set up in Excel (with the words typed in). Students can still fill in the formulas, but to already have the sheet formatted would be helpful to students who take longer and also for ELL students.

Final Assessment (to be completed individually and students are not allowed to reference the materials or documents used in the past three lessons):

- 1) What career/job do you hope to have after high school or college?
- 2) Please find the average salary (gross pay) of an entry level position in your potential occupation (site the source (website) from where you got the information).
- 3) Please create an Excel document (similar to the one from yesterday) that shows your personal budget. Use the same categories as we did earlier in the unit (see below).

In 2009, Americans averaged spending their net income in the following ways:

(Source: <http://financemymoney.com/wp-content/uploads/2010/05/wheredidthemoneygo.jpg>)

17.6% of net income on transportation (car payments, gasoline, and maintenance).

12.4% of net income on food

34.1% of net income on housing (shelter, furnishings, maintenance)

16.5% of net income on insurance and healthcare

5.0% of net income on clothing and cosmetics

The remaining portion is left for various expenditures (i.e. education, entertainment, charity, and vacations).

Be sure to include what your gross and net pay will be using <http://www.taxbrackets2011.com/>.

Example response:

Median income \$88,000

**(gross pay) for
an Aerospace
Engineer:
Net Pay
(Federal
Income taxes
taken out):**

Category	Percent	Amount	Monthly Amount
	12.40		
Food	%	\$8,648.13	\$720.68
	17.60		
Transportation	%	\$12,274.77	\$1,022.90
	34.10		
Housing	%	\$23,782.36	\$1,981.86
	16.50		
Insurance/Healthcare	%	\$11,507.60	\$958.97
Clothing/Cosmetics	5.00%	\$3,487.15	\$290.60
	14.40		
Other	%	\$10,042.99	\$836.92
		\$69,743.00	\$5,811.92

4) Assume that you received a 4% raise. In the rare case that your cost of living doesn't rise, in which category (or categories) would you put the remaining money. How would this change your spending percentages of each category. Reflect these changes in an Excel sheet.

Example response:

**Median income
(gross pay) for an
Engineer:
Net Pay (Federal
Income taxes
taken out):**

\$91,520
\$72,277

	Category	Percent	Amount	Monthly Amount
Difference in Net Pay:	Food	11.97%	\$8,648.23	\$720.68
\$2,534	Transportation	16.98%	\$12,274.90	\$1,022.90
Difference in Net Pay per Month:	Housing	32.90%	\$23,782.50	\$1,981.86
\$211.20	Insurance/Healthcare	15.92%	\$11,507.01	\$958.91
	Clothing/Cosmetics	6.64%	\$4,800.04	\$400.00

I increased Clothing/Cosmetics by

				\$109.40
Other	15.59%	\$11,264.73	\$938.72	I increased Other
Totals	100.00%	\$72,277.40	\$6,023.07	by \$101.80