Unit: Being Green – Minimizing the Surface Area of a Soda Can

I. UNIT OVERVIEW & PURPOSE:

This unit is designed for high school students to understand the relationship between surface area and volume through a social justice application. Students will work in teams as they are introduced to the calculus topic of optimization to minimize the surface area of a cylinder using the volume as a constraint. First, students will measure a soda can and calculate the volume and surface area. Then they will use an Excel spreadsheet to test new dimensions and choose the one which provides the minimum surface area. Students will design a model using their chosen dimensions. Finally, students will prepare an argument for why or why not soda companies should consider their new design.

II. UNIT AUTHOR:

Lauren LaVenture
Lord Botetourt High School
Botetourt County Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

The student will:

- calculate surface area and volume of a cylinder
- use technology (Excel spreadsheet) to test and judge optimized dimensions
- create a net and three dimensional model
- construct an argument for or against changing the dimensions of a soda can

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE 2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE 6: The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

MPE 7:The student will use similar geometric objects in two- or three-dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

VII. CONTENT:

Through this unit students will see how surface area and volume are related but can be manipulated independently. Students are introduced to a calculus concept of optimization while minimizing the surface area in an effort to reduce waste and live greener. Students will make an argument, targeted to the soda company, to persuade them to change the dimensions of the soda can or maintain the current dimensions.

VIII. REFERENCE/RESOURCE MATERIALS:

- Unit handout and rubric for students (attached in the lesson)
- Soda cans
- Rulers
- Calculators
- Microsoft Excel (example spreadsheet attached in the lesson)
- Construction paper
- Scissors
- Tape
- Compasses
- Internet
- Participation Rubric (Source: HA Program, Auburn U. Mark Burns, Instructor)
- Worksheets created by Kuta Infinite Geometry Software
- Website: Keep America Beautiful www.kab.org

IX. PRIMARY ASSESSMENT STRATEGIES:

Assessments will be in the form of:

- Mathematical accuracy
- Oral presentation of argument
- Participation
- Rubric for overall unit performance (attached)

All specific questions for the assessments are attached in the respective lessons

X. EVALUATION CRITERIA:

- Mathematical accuracy is graded on correctness for the following:
  Surface area of soda can
  Volume of soda can
  Minimized surface area dimensions (Excel)
  Construction of model
  *keys provided for each in the lessons
- The group argument will be assessed on preparation and persuasive strength. There is no right or wrong side.
- Participation of each individual group member will be evaluated by each member and the instructor using a rubric.
- Each student will receive a final unit grade using a rubric which contains all of the points mentioned above. The rubric can be seen in the following lessons.

XI. INSTRUCTIONAL TIME:

Regular Schedule: about 5-6 days
Block Schedule: about 3-4 days

Lesson 1: Finding Surface Area and Volume

Objectives: Students will calculate surface area and volume of a soda can (cylinder)

Standards:

MPE 6 and G.13: The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

NCTM Geometry: analyze properties and determine attributes of two- and three-dimensional objects

NCTM Measurement: analyze precision, accuracy, and approximate error in measurement situations; understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

Materials:

Unit handout with rubric (appendix 1)
Website: Keep America Beautiful www.kab.org
Surface area and volume worksheet (appendix 2, optional)
Soda cans – 1 per group
Rulers – 1 per group
Calculators

Process:

1. Introduce the unit: class discussion on waste and recycling, brainstorm methods to reduce waste, ask students about frequently used items that are often thrown away. According to Keep America Beautiful, 14.5% of roadside litter is in the form of beverage containers. See the website, Keep America Beautiful www.kab.org ,for other possible discussion topics.
2. Divide class into groups of about 4 and pass out the unit handout (appendix 1)
3. Discuss the situation: each team is to determine if a more earth friendly version of the soda can exists. Discuss what aspect(s) of the soda can would need to change to reduce waste and what aspect(s) would need to remain the same. Explain that as a team, each group will come to a conclusion about changing or not changing the soda can and must present an argument targeted at a soda company.
4. Optional: (as determined by the teacher or a pre-test if desired)
   provide students who are not familiar with surface area and volume formulas the practice worksheet (appendix 2 with answer key) and review the formulas below:
   Prisms: SA = sum of the area of all surfaces V = area of the base x height
   Cylinders: $SA = 2\pi r^2 + 2\pi rh$ $V = \pi r^2 h$ Note: r = radius, h = height
5. Calculate both the surface area and volume of the soda can by measuring the dimensions of radius (find diameter first) and height. This unit uses inches but this can be changed. Students may give their dimensions in decimal or fraction (as determined by the teacher). It will be easier to eventually use decimals and multiply through by 3.14159
6. Check each group's results before moving on
   Answer Key: The following are approximates due to possible measurement error
   h = 4.875 in, r = 1.0625 in, V = 17.29 in^3, SA = 39.64 in^2

Lesson 2: Minimizing the Surface Area

Objectives: Students will use technology (Excel spreadsheet) to test and judge optimized dimensions

Standards:

MPE 2 and AII.9: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE 7 and G.14: The student will use similar geometric objects in two- or three-dimensions to

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;

c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and

d) solve real-world problems about similar geometric objects.

NCTM Geometry: analyze properties and determine attributes of two- and three-dimensional objects;

Materials:

Microsoft Excel spreadsheet – at least 1 per group

Process:

1. Before class starts create the Excel spreadsheet as shown below and send the file to each group or each student
2. Before students begin review what they learned from the previous lesson: the volume of the soda can needs to remain 17.3 in^3 but we want to minimize the surface area by adjusting the dimensions of radius and height. Explain the height column of the spreadsheet is a formula for height given a radius value and the constant volume. Optional: students can derive the height formula and compare to the spreadsheet or students can create the spreadsheet as a team.

	A	B	C	D	E	F
1	Find the minimal surface area by entering a value for the radius. The height will be calculated for you so the volume remains constant at 17.29 cubic inches. Highlight the line that gives the minimal surface area.					
2		Radius (in)	Height (in)	Volume (in^3)	Surface Area (in^2)	
3	Standard soda can:	1.0625	4.875145816	17.29	39.63900352	
4			#DIV/0!	17.29	#DIV/0!	
5			#DIV/0!	17.29	#DIV/0!	
6			#DIV/0!	17.29	#DIV/0!	
7			#DIV/0!	17.29	#DIV/0!	
8			#DIV/0!	17.29	#DIV/0!	
9			#DIV/0!	17.29	#DIV/0!	
10			#DIV/0!	17.29	#DIV/0!	
11			#DIV/0!	17.29	#DIV/0!	
12						

Formulas: Height column = (17.29/(3.14159*(B3)^2))

Surface Area column = =(2*3.14159*B3^2)+(2*3.14159*B3*C3)

3. Students will type a "test" radius value and the spreadsheet will fill in the correct values for height and surface area. Students will continue to test radius values until they are confident they have found the minimal surface area to the nearest tenth. Below is an example of a team's spreadsheet.

	A	B	C	D	E	F
1	Find the minimal surface area by entering a value for the radius. The height will be calculated for you so the volume remains constant at 17.29 cubic inches. Highlight the line that gives the minimal surface area.					
2		**Radius (in)**	**Height (in)**	**Volume (in^3)**	**Surface Area (in^2)**	
3	Standard soda can:	1.0625	4.875145816	17.29	39.63900352	
4		1.1	4.548415356	17.29	39.03901144	
5		1.2	3.821932348	17.29	37.86444587	
6		1.3	3.25655774	17.29	37.2185742	
7		1.4	2.807950296	17.29	37.0150328	
8		1.5	2.446036703	17.29	37.19048833	
9		1.38	2.889929942	17.29	37.02365901	
10		1.41	2.768262452	17.29	37.01641285	
11		1.405	2.788000446	17.29	37.01525404	
12						

4. Students should create a scatterplot to compare radius values to surface area to verify their minimal dimensions.
5. Students will have Excel calculate the curve of best-fit and students should answer the following questions (answers provided)

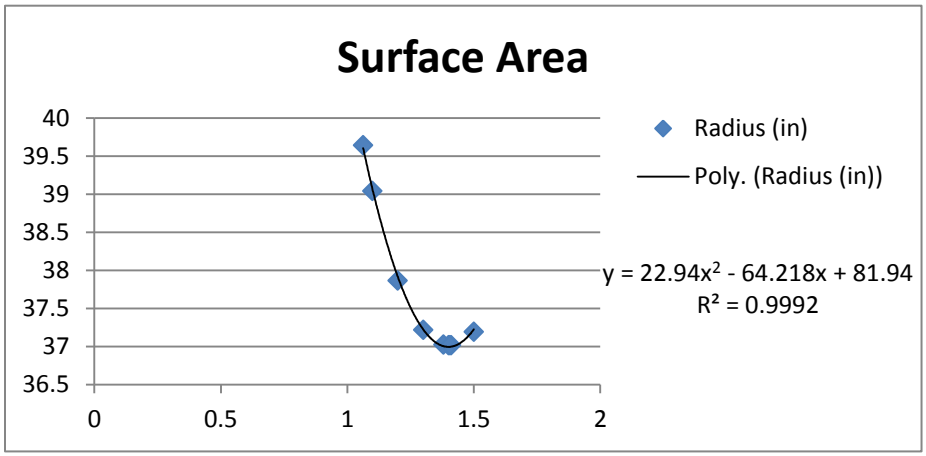
Is the equation of best-fit a line? No

What type of equation is the best-fit? Quadratic

What is the equation? $y = 22.94x^2 - 64.218x + 81.94$

Locate your minimal surface area, what is this point in respect to the quadratic? Minimum

Explain, in your own words, the relationship between the graph of the quadratic and finding the optimized surface area of a soda can.



Lesson 3: Constructing a Model

Objectives: Students will create a net and a three dimensional model

Standards:

MPE 7 and G.14: The student will use similar geometric objects in two- or three-dimensions to

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;

c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and

d) solve real-world problems about similar geometric objects.

NCTM Geometry: analyze properties and determine attributes of two- and three-dimensional objects;

draw and construct representations of two- and three-dimensional geometric objects using a variety of tools;

visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections;

NCTM Measurement: analyze precision, accuracy, and approximate error in measurement situations;

Materials:

Net worksheet (appendix 3, optional for prerequisite practice)
Construction paper
Rulers
Compasses
Scissors
Tape

Process:

1. Optional: If you students are not familiar with constructing 3D models from a net, they should first complete the attached worksheet (appendix 3 with answer key)
2. Using the optimized dimensions found in the previous lesson, students will construct a model from a net. Students will measure and sketch the net on construction paper. Only one construction per team is necessary.
3. Students should measure the height ($h = 2.8\ in$) as the width of the rectangle for the net.
4. Students need to find the circumference ($C = 2\pi r = 8.8\ in$) of the circular base. The length of the circumference is the length of the rectangle for the net.
5. At each end of the rectangle, students need to construct the circular bases using the radius measurement. The compass should be set to $r = 1.4\ in$.
6. Students will cut out the net, fold it up and tape it to hold together. See figures 1 and 2 on the next page for an example of the model construction process.

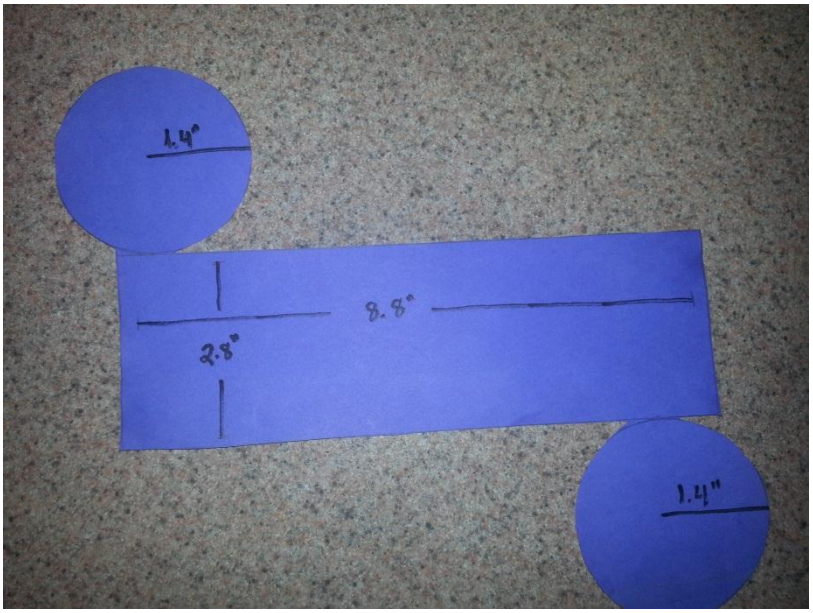


Figure 1: This is the net of the model with the specific lengths labeled

Figure 2: This is the net folded and taped up to represent the optimized dimensions of the soda can

Lesson 4: Choose a Side and Present an Argument

Objectives: Students will construct an argument for or against changing the dimensions of a soda can

Standards:

MPE 7 and G.14: The student will use similar geometric objects in two- or three-dimensions to

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;

c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and

d) solve real-world problems about similar geometric objects.

NCTM Geometry: Analyze properties and determine attributes of two- and three-dimensional objects;

Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections;

Use geometric models to gain insights into, and answer questions in, other areas of mathematics;

Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Materials:

Soda cans
Models
Internet for research
Presentation software such as Microsoft Power Point
Participation rubric – each student will need one for each group member including themselves (appendix 4)
Unit rubric 1 for each student (appendix 5)

Process:

1. Students should first compare and contrast their model with the soda can. This list can include appearance, marketing, shelf space, packaging, ease of use, material cost, material waste and any other characteristics students notice. See figure 3 on the next page for a visual comparison.
2. As a team, students must decide if soda companies should change the dimensions of the soda can or keep the current dimensions.
3. As a team, students will prepare an argument for their decision. The argument should be targeted to the soda company and so should appeal to the concerns of a business (cost, productivity, marketing). Students should have access to the internet to conduct any necessary research. Students should make use of mathematics and the difference in material cost between the two options or in the shelf space difference between the two options. For example: the optimized dimensions reduce the surface area and likewise the material to make the soda can by 2.6 in^2 per can. Also, the optimized soda can will increase the shelf space by 2.7

in^2 per can.

Suggested websites for research:

www.kab.org Keep America Beautiful: cost analysis of litter cleanup and suggestions for waste reduction

www.epa.gov Environmental Protection Agency: steps for businesses to reduce waste

http://www.aluminum.org The Aluminum Association: history of aluminum cans

4. Teams will present their arguments orally to be shared with the class. These presentations could take the form of a debate if there are enough teams on either side. Students may want to use presentation software to construct their arguments.
5. Upon completion of the unit, student team members will critique themselves and each other on their contributions to the group. Attached is a rubric students can use for this critique (appendix 4).
6. Each student will receive a final unit grade based on the teacher's evaluation using the attached unit rubric (appendix 5)

Figure 3: Comparison of the optimized model (left) and standard soda can (right)

Appendix 1: Student Unit Handout

Being Green: Minimizing the Surface Area of a Soda Can

You and your teammates are concerned citizens. You see so many soda cans being thrown away or tossed on the side of the road and you want to do something about it. Sure you could promote a recycling program (which is a great idea) but you want to make a difference that will help the cans carelessly strewn along the roadway. You and your teammates will test if there is another design for the soda can that is still cylindrical and still holds 12 fluid ounces, but reduces waste. Once the new design is made, you and your team will need to construct a convincing argument for the soda company to either alter the current soda can or keep it the same.

1. Find the dimensions of the current soda can and calculate the volume and surface area
2. Use an Excel spreadsheet to find the optimal dimensions that will hold the current volume constant but minimize the surface area
3. Construct a model using the optimized dimensions – you will first construct a net.
4. Compare and contrast the soda can and the model. Consider the following: appearance, marketing, shelf space, packaging, ease of use, material cost, material waste and any other characteristics
5. Decide, as a team, if you should convince the soda company to use the new dimensions or keep the current dimensions
7. Construct an argument targeted to the soda company. Consider cost, productivity and marketing. You will have class time to conduct research. Your argument should use the shelf space difference between the two options.
6. Present your argument as a team to the class

	Acceptable	
Mathematical Accuracy	Accurately measured the soda can Accurately calculated the volume Accurately calculated the surface area Accurately found the optimized dimensions Accurately constructed the model	Acceptable: 10 points Borderline: 8 points Inadequate: 5 points Unacceptable: 2 points
Argument	Created a list of at least 6 comparisons Argument has at least 3 statements At least one statement is mathematically based beyond the project Argument was designed to persuade a soda company	Acceptable: 15 points Borderline: 12 points Inadequate: 8 points Unacceptable: 4 points
Presentation	Each member had a role in the presentation Presentation was clear and persuasive	Acceptable: 10 points Borderline: 8 points Inadequate: 5 points Unacceptable: 2 points
Participation	Determined by the average ratings given by yourself, teammates, and your teacher in the following categories: contribution, attitude, encouragement of others, listening to others, making decisions, knowledge, organization	Acceptable: 5 points Borderline: 4 points Inadequate: 3 points Unacceptable: 2 points

Appendix 2: Kuta Infinite Geometry Software Surface Area and Volume Worksheet

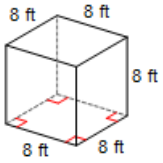
ID: 1

Geometry Name_____

Surface Area and Volume: prisms and cylinders Date_____ Period____

Find the volume of each figure. Round your answers to the nearest hundredth, if necessary.

1)



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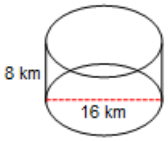

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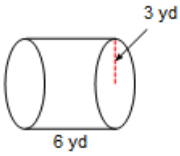

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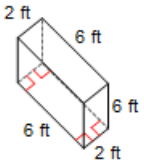
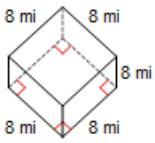


Find the surface area of each figure. Round your answers to the nearest hundredth, if necessary.

5)



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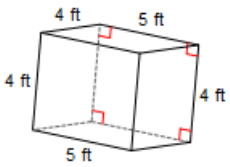

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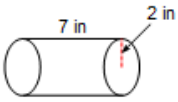

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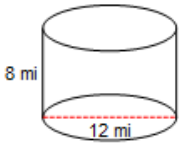


Answer Key:

Answers to Surface Area and Volume: prisms and cylinders (ID: 1)

1) 512 ft^3
2) 1608.5 km^3
3) 169.65 yd^3
4) 72 ft^3
5) 384 mi^2
6) 112 ft^2
7) 113.1 in^2
8) 527.79 mi^2

Appendix 3: Kuta Infinite Geometry Nets of Solids Worksheet

ID: 1

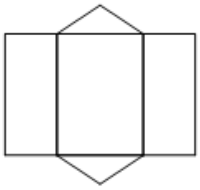
Geometry Name_____

Nets of Solids: prisms and cylinders Date_____ Period____

Identify each solid given its net.

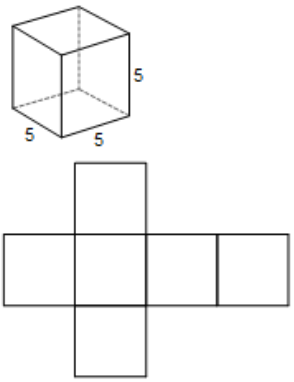
1)

2)



Copy the measurements given onto the net of each solid.

3)



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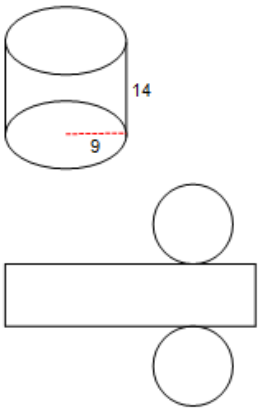
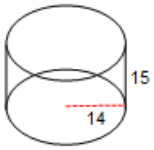


Sketch the net of each solid. Label the measurements given.

5)



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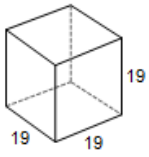
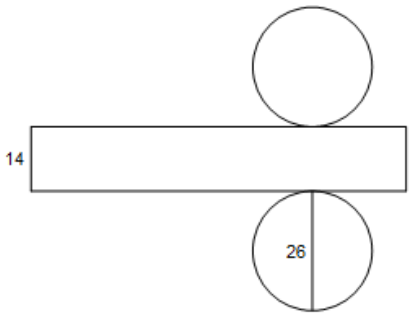


Sketch the solid that can be created from each net. Label the measurements given.

7)



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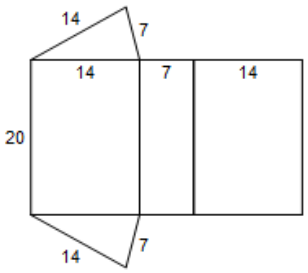


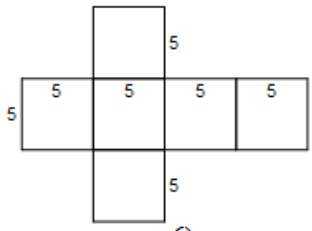
Answer Key:

Answers to Nets of Solids: prisms and cylinders (ID: 1)

1) cylinder

2) triangular prism

3)



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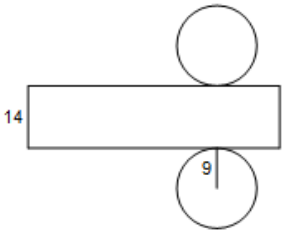

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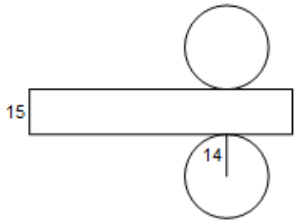

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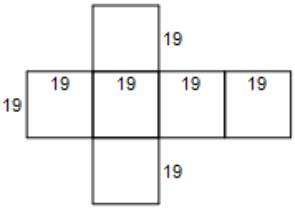

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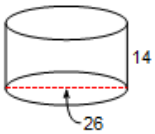

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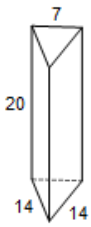

Appendix 4: Participation Rubric Source: HA Program, Auburn U. Mark Burns, Instructor

Partner Being Evaluated: _____ Name of Evaluator: _____
Quantitative Evaluation (Circle your response in each category.)

Contributed at the meetings	5	4	3	2	1	Criticized with no solutions offered
Initiated ideas	5	4	3	2	1	Sat back & let others do the hard work
Accepted responsibility	5	4	3	2	1	Kept quiet, hoping to avoid tasks
Delivered on promises	5	4	3	2	1	Late, or didn't get work done
Positive attitude	5	4	3	2	1	Complained a lot
Organized	5	4	3	2	1	Scattered and unproductive
Prepared & helped make decisions	5	4	3	2	1	Unprepared and disruptive
Knowledgeable	5	4	3	2	1	Weak conceptual/practical Background
Encouraged everyone's participation	5	4	3	2	1	Tried to dominate group discussions

Appendix 5: Final Project Rubric

	Acceptable	
Mathematical Accuracy	Accurately measured the soda can Accurately calculated the volume Accurately calculated the surface area Accurately found the optimized dimensions Accurately constructed the model	Acceptable: 10 points Borderline: 8 points Inadequate: 5 points Unacceptable: 2 points
Argument	Created a list of at least 6 comparisons Argument has at least 3 statements At least one statement is mathematically based beyond the project Argument was designed to persuade a soda company	Acceptable: 15 points Borderline: 12 points Inadequate: 8 points Unacceptable: 4 points
Presentation	Each member had a role in the presentation Presentation was clear and persuasive	Acceptable: 10 points Borderline: 8 points Inadequate: 5 points Unacceptable: 2 points
Participation	Determined by the average ratings on the Participation Rubric	Acceptable: 5 points Borderline: 4 points Inadequate: 3 points Unacceptable: 2 points