Performance Based Learning and Assessment Task

Confetti Task

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
In this task, Geometry students will investigate how surface area and volume is used to estimate the cost of constructing two mega containers used to celebrate the New Year

II. UNIT AUTHOR:
Christina Nowlin, Hampton City Schools

III. COURSE:
Geometry

IV. CONTENT STRAND:
Geometry

V. OBJECTIVES:
Student will be able to: use geometric formulas to find surface area of cylinders, apply geometric concepts to real world situations

VI. REFERENCE/RESOURCE MATERIALS:

- Students will use: Cylinder nets, ruler with inches, tape, calculator, activity Sheet 1 and 2 (attached), assessment, and assessment list, calculator, compass

VII. PRIMARY ASSESSMENT STRATEGIES:
Activity Sheet 1, Activity Sheet 2, Rubrics and Self-Assessment

VIII. EVALUATION CRITERIA:
Self-Assessment and Teacher Assessment Rubric

IX. INSTRUCTIONAL TIME:
One ninety-minute class session

Confetti Task

Strand
Geometry, Three-Dimensional Figures (Cylinders)

Mathematical Objective(s)

1. Students will measure a given net to calculate surface area and volume of a cylinder.
2. Given the radius, height and cost of materials students will be able to estimate the cost of two mega cans by:
    a. Computing the surface area each can
    b. Finding the volume of one can

Related SOL

- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems. (Cylinders only)
    a. Find the total surface area of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.
    b. Calculate the volume of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.
    c. Solve problems, including real-world problems, involving total surface area and volume of cylinders, prisms, pyramids, cones, and spheres as well as combinations of three-dimensional figures.
    d. Calculators may be used to find decimal approximations for results.

NCTM Standards

- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools
- use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;

Materials/Resources

- Cylinder nets, ruler with inches, tape, calculator, activity Sheet 1 and 2 (attached), assessment, and assessment list, calculator, compass

Assumption of Prior Knowledge

Students should:

- know how to use a ruler to measure the length and width of a rectangle and the diameter of a circle.
- Know how to substitute values into a formula and evaluate.
- know how to solve equations.
- be able to calculate the cost of an item when given the unit amount.
- know the area of a circle and rectangle.
- have the basic understand of surface area.
- know how to calculate surface area of a rectangular and triangular prism.
- be able to convert feet to inches.
- be operating on Analysis level on van Hiele scale with respect to solving equations and using formulas.

Introduction: Setting Up the Mathematical Task

- In this task, Geometry students will investigate how surface area and volume is used to estimate the cost of constructing two mega containers used to celebrate the New Year.
- For the students to complete both Activity sheets and the assessment it will take 2 class periods. The activity sheet 1 is to be used to introduce students to cylinders, deriving the cylinder formulas and how to apply the formulas. The activity 2 sheet is to help students connect surface area and volume to the real world using an application problem. The assessment sheet is for the teacher to assess their understanding of the surface area and volume. Question 2 of the assessment sheet could be used as an extension problem instead of part of the assessment.
- For this task students are going to explore how to derive the surface area formula of a cylinder and volume formula of a cylinder. Students will practice using the formulas and how to apply them to a real world application prior to the assessment.
- To introduce the task students will be asked to name objects that are cylinders and to identify the radius and height of a cylinder. Students will compare examples and non-examples of cylinders to make sure the students know the difference of cylinders to other 3D-shapes.
- To help the students with the task, ask the following questions:
    - How does the radius relate to the diameter? How do you measure the diameter of a circle? How do you change feet to inches or inches to feet? What is the area of a circle and a rectangle?
- The students will complete activity 1 using think/pair/share strategy. However, they will think with a partner, pair with another group of two and share with the class as a whole.

Activity 2 will be completed using think/pair/share. The students will start on their own, pair with another student, share with the class. The students will complete the assessment on their own.

Student Exploration

Student/Teacher Actions:

- The students will work with a partner to complete Activity 1. After completing the first four questions students will work with another group to compare answers. After comparing answers the teacher will check the solutions and give guidance for any incorrect answers. Students cannot move on to the remaining questions until the teacher has checked the first 4 questions and they are answered correctly. The students will then work only with their partner to complete the remaining questions on activity 1. Groups will receive different size nets to complete the activity. Different groups will then be asked to present their solutions for the remaining questions. As students present their solutions other groups will be asked to provide the presenting group with their net measurements. As a class the students will use the formulas the group presenting derived to check the solution of group who gave the measurements.
- The Activity 2 sheet is for the students to complete individually. After working on the assignment for 20 minutes the students will share their results with a partner and finish what had not been completed. After completing the task groups will be asked to share their results with the class.
- Once the students have completed activity 2, the students are to complete the assessment on their own.
- The teacher will show the video http://tube.geogebra.org/student/m8875 to help students make the connection between length of the rectangle relate to the circle.
- The teacher needs to walk around and help students with questions they may have. The teacher is to direct the students and not give answers.
- The teacher must check for accuracy for the first 4 questions of activity 1 before the students can continue.

Monitoring Student Responses

- Students will communicate their new knowledge with the class by presenting a question that is assigned
- If students are having difficulties with expressing their thoughts the teacher is to use questions to prompt the students
- After completing activity 2 the teacher will lead the students through a series of questions to pull everything together. The students will then complete the assessment.
    - What two formulas are used to find surface area?
    - How can you use these two formulas to calculate the surface area of a cylinder?

- What measurements are required to find the surface area and volume?
- What is the general idea for idea the volume of a cylinder?

Assessment List and Benchmarks

Assessment List, Rubric and Benchmarks are attached.

Questions

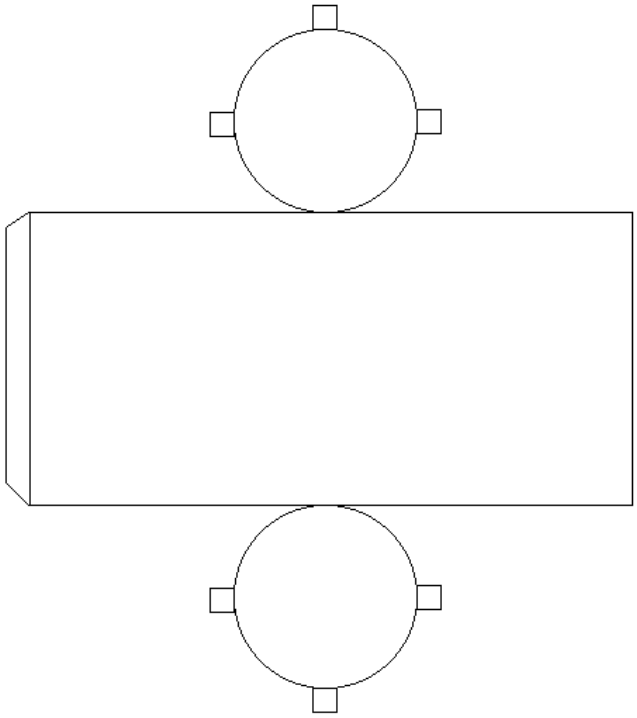
- If the surface area of a soup can is 28π in^2 with a radius of 2 inches, what is the height?
- Is the volume or surface area greater of a cylinder with radius of 9 and height of 4?
- If the volume of a cylinder is 36π in^3 with a radius of 3 inches, what is the height?

Journal/writing prompts

Give another real world example for why surface area and volume are important.

Activity Sheet 1: Introduction to surface area of a cylinder:

1. Label the height, diameter and radius of the can below. Label the height, length, diameter and radius of the net below.



2. Write the formula for the area of a circle. Write the formula for the area of a rectangle in terms of height and length.

3. How does the length of the rectangle relate to the circle? Use the provided net to compare the length with the circle.

4. Using the results from questions 2 and 3 write a formula for the surface area of a can (cylinder). After completing this question ask teacher to check result.

For the following questions all answers of measurement should be in inches. Round results to the nearest tenth. Make sure you include units of measurements with your solutions. Show all work.

5. Using the ruler, measure the dimensions needed of the provided net to calculate the surface area.

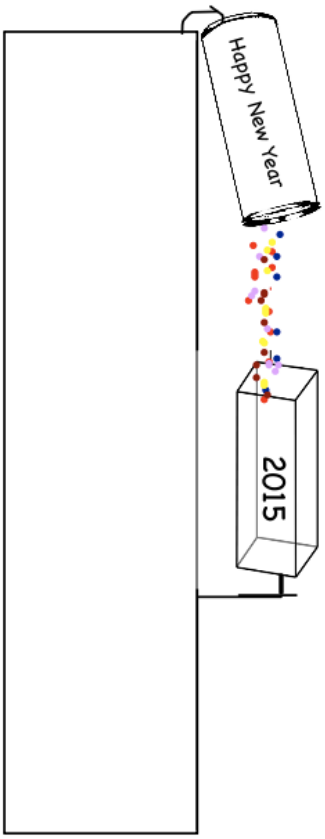
6. Use the surface area formula from question 4 to calculator the surface area of the net. Then tape the net together.

7. If volume is calculated by multiplying the area of the base of the figure times the height, what is the formula for the volume of a cylinder? Explain how you came up with the formula.

8. Use the measurements found in question 5 to find the volume of the net when it is made into a cylinder.

Activity Sheet 2: Calculating Cost using Surface Area and Volume

The City of New York asked a can company to create two mega containers to set up in Time Square. A can will be set up high in Time Square to leak confetti into a rectangular prism below all day of the last day of the year to count down to the new year. At 12:00 am, the rectangular prism will throw all the confetti that is in it into the air to bring in the New Year. The city wants the can dimensions to have a diameter of 7 feet and a height of 14 feet. The rectangular prism would have a height of 14 feet with a width and length of 7 feet. The can would need to be a closed can to keep the confetti from coming out to soon and the rectangular prism would not need a top. The company charges $.008 per square inch to create each container. To fill the can with confetti it will cost $.002 per cubic inch.



1. What are the dimensions of the containers in inches?

2. What is the surface area of the two containers? Round answers to the nearest tenth and include unit of measurements.

3. Which container has the largest volume? Include all calculations as part of your solution. Is the bottom container large enough for the project? Explain.

4. Using your answers from questions 2 and 3, how much will the total cost be to make the containers and fill the can with confetti?

5. If the bottom container was a prism in which the bases were equilateral triangles and the side lengths were 7 feet, which container would have the greatest surface area and which would have the greatest volume? Include all calculations as part of your solution.

Cylinder Surface Area and Volume Assessment:

For the following questions all answers of measurement should be in inches. Round results to the nearest hundredths. Make sure you include units of measurements with your solutions. Show all work that leads to your calculations.

The City of New York asked a can company to create two mega containers to set up in Time Square. A can will be set up high in Time Square to leak confetti into a rectangular prism below all day of the last day of the year to count down to the new year. At 12:00 am, the rectangular prism will throw all the confetti that is in it into the air to bring in the New Year. The city originally wanted the can dimensions to have a diameter of 7 feet and a height of 14 feet. The rectangular prism would have a height of 14 feet with a width and length of 7 feet. The can would need to be a closed can to keep the confetti from coming out to soon and the rectangular prism would not need a top. The company charges $.008 per square inch to create each container. To fill the can with confetti it will cost $.002 per cubic inch.

1. The city decided to invest between $3,500 and $4,000 for this project.

a. Create new dimensions for the can that would be larger than the original can that would stay within the budget. Include units of measurement.

Your creation must include the following:

I. The dimensions of the cylinder and rectangular prism in inches.

II. Surface area of the cylinder and rectangular prism

III. The volume of the container that is to be filled with confetti.

IV. The total amount it would cost the city for the project.

b. On the graph paper provided, draw the net of the can. Provide a scale for each unit of measurement.

2. If the city wanted to max out their spending on the project at $4,000 and they wanted the rectangular prism to have a height of 14.5 feet and the length and width of 8.5 feet. What would be dimensions of the largest cylinder that could be created to not exceed $4,000? The company only uses whole units in inches. Include all calculations as part of your solution. Would the rectangular prism given be large enough to hold the confetti of that cylinder? Explain.

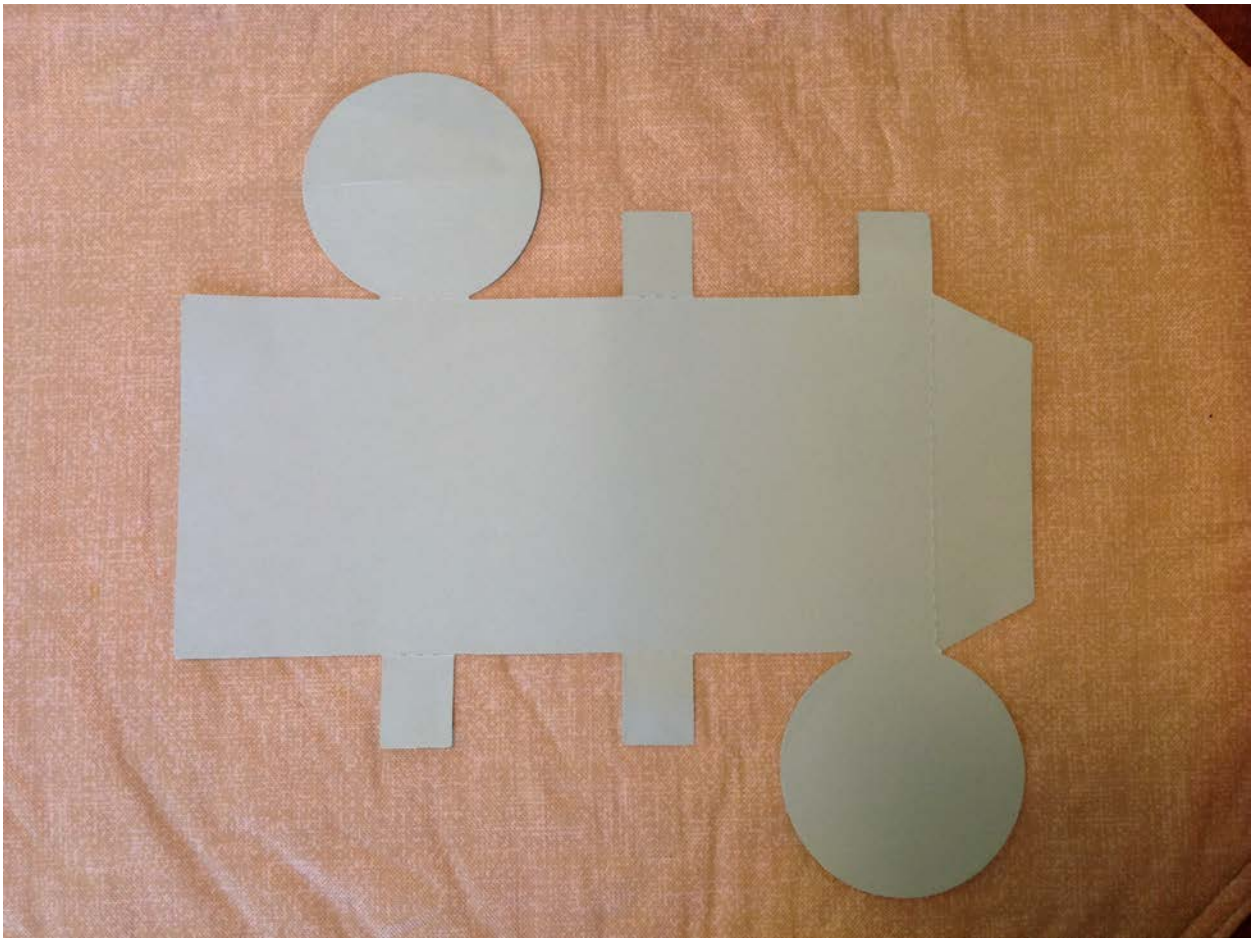
Rubric for Cylinder Surface Area and Volume Assessment

#	2	1	0
1. I.	Gave dimensions in inches that are larger than the original can and correctly converted all measurements to inches. Included units.	Gave dimensions that are larger than the original can and all measurements are not in inches.	Dimensions were not larger than the original cans.
1 II	Calculated total surface area of both containers correctly, rounding to the nearest hundredth with correct unit in inches.	Calculated surface area of one of the two containers.	Did not calculate surface area correctly.
1 III	Calculated volume correctly rounding to the nearest hundredth with correct unit in inches.	Calculated volume correctly without units of measurements.	Did not calculate volume correctly.
1 IV.	Calculated the cost that stayed in the city budget correctly based on the solutions from the above parts.	Calculated the cost correctly based on the solutions from the above parts.	Did not calculate the cost correctly.
1b.	Provided a net of the cylinder with accurate unit of measurements based on the scale they created.	Provided a net not drawn to the scale they provided.	Did not include a net.
2.	Accurately provided at least three calculations to find the largest cylinder. Included correct unit of measurements.	Provided three calculations with some error in solutions.	Provided one calculation for the largest cylinder.
	Correctly calculated the prism' volume and gave a correct explanation of why it is large enough for the project.	Calculated the prism' volume of an explanation.	Calculated the prism' volume.

Assessment List

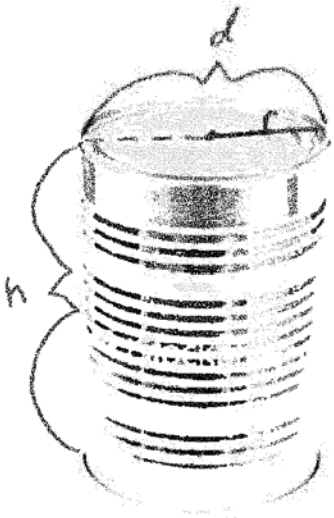
	Assessment points		
Element	Points Possible	Earned Assessment	
		Self	Teacher
1. Appropriate dimensions were given with units of measurement.	2		
2. Correct surface areas with units of measurement.	2		
3. Correct volume with units of measurement.	2		
4. Cost calculated based on surface area and volume that was found.	2		
5. The net was scaled appropriately based on the dimensions.	2		
6. At least three calculations determining the largest cylinder.	2		
7. Calculations with explanation to determine if the prism is large enough.	2		

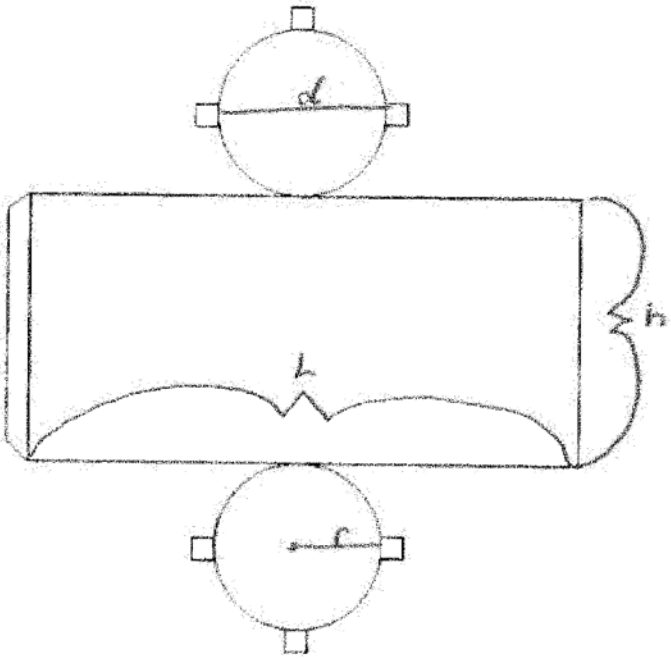
Picture of a net needed for Activity 1. There should be at least three different size nets for the students to complete this activity.



Activity Sheet 1: Introduction to surface area of a cylinder:

1. Label the height, diameter and radius of the can below. Label the height, length, diameter and radius of the net below.





2. Write the formula for the area of a circle. Write the area of a rectangle in terms of height and length.

Circle Area

$A = \pi r^2$

Area of a rectangle

$A = lh$

3. How does the length of the rectangle relate to the circle? Use the provided net to compare the length with the circle.

The length is the circumference of the circle.

$l = 2\pi r$

4. Using the results from questions 2 and 3 write a formula for the surface area of a can (cylinder). After completing this question ask teacher to check result.

$S.A. = \pi r^2 + \pi r^2 + 2\pi rh$

$S.A. = 2\pi r^2 + 2\pi rh$

5. Using the ruler, measure the dimensions needed of the provided net to calculate the surface area.

$h = 4$ inches

$r \approx 1.3125$

6. Use the surface area formula from question 4 to calculator the surface area of the net. Then tape the net together.

$S.A. = 2\pi(1.3125)^2 + 2\pi(1.3125)(4)$

$S.A. \approx 43.81\ in^2$

7. If volume is calculated by multiplying the area of the base of the figure times the height, what is the formula for the volume of a cylinder? Explain.

$V = \pi r^2 h$

The base of a cylinder is a circle and the area of a circle is πr^2.

8. Use the measurements found in question 5 to find the volume of the net when it is made into a cylinder.

$V = \pi(1.3125)^2(4)$

$V = 21.65\ in^3$

Activity Sheet 2: Calculating Cost using Surface Area and Volume

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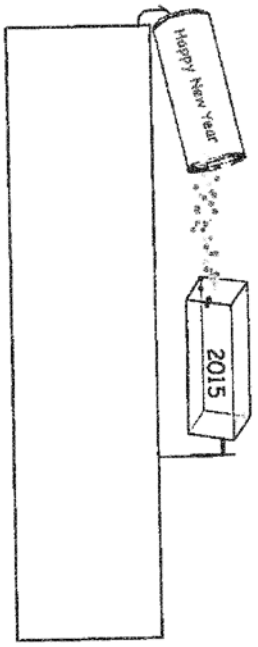


1. What are the dimensions of the containers in inches?

cylinder

$d = 7ft \cdot \frac{12in}{1ft} = \frac{84in}{2} =$ 42 in

$h = 14ft \cdot \frac{12in}{1ft} = 168in$

Prism

$h = 168in$

$w = 84in$

$l = 84in$

2. What is the surface area of the two containers? Round answers to the nearest tenth and include unit of measurements.

S.A cylinder $= 2\pi(42)^2 + 2\pi(42)(168)$

$= 55,417.69\ in^2$

S.A prism $= 4(84)(168) + 84(84)$

$= 63,504\ in^2$

3. Which container has the largest volume? Include all calculations as part of your solution. Is the bottom container large enough for the project? Explain.

Cylinder volume $= \pi(42)^2(168) = 931,017.27\ in^3$

Prism volume $= (168)(84)(84) = 1,185,408\ in^3$

The rectangular has a larger volume, therefore it is large enough for the project because it can hold all the confetti that is in the cylinder.

4. Using your answers from questions 2 and 3, how much will the total cost be to make the containers and fill the can with confetti?

$.008(55,417.69 + 63,504) + .002(931,017.27)$

Total Amount $= \$2,813.41$

5. If the bottom container was a prism in which the bases were equilateral triangles and the side lengths were 7 feet, which container would have the greatest surface area and which would have the greatest volume? Include all calculations as part of your solution.

$7ft = 84in$

Prism S.A $= 3(84)(84) + \left(\frac{1}{2}(84)\sqrt{84^2 - 42^2}\right)$

$\approx 24,223.34\ in^2$

Prism Volume $= \frac{1}{2}\left(84\sqrt{84^2 - 42^2}\right)(84)$

$= (3,055.34)(84) = 256648.36\ in^2$

The rectangular prism has the greatest volume and surface area.

Cylinder Surface Area and Volume Assessment:

For the following questions all answers of measurement should be in inches. Round results to the nearest hundredths. Make sure you include units of measurements with your solutions. Show all work that leads to your calculations.

The City of New York asked a can company to create two mega containers to set up in Time Square. A can will be set up high in Time Square to leak confetti into a rectangular prism below all day of the last day of the year to count down to the new year. At 12:00 am, the rectangular prism will throw all the confetti that is in it into the air to bring in the New Year. The city originally wanted the can dimensions to have a diameter of 7 feet and a height of 14 feet. The rectangular prism would have a height of 14 feet with a width and length of 7 feet. The can would need to be a closed can to keep the confetti from coming out to soon and the rectangular prism would not need a top. The company charges $.008 per square inch to create each container. To fill the can with confetti it will cost $.002 per cubic inch.

1. **The city decided to invest between $3,500 and $4,000 for this project.**

a. **Create new dimensions for the can ~~and the rectangular prism in inches~~ that would be larger than the original containers that stay within the budget. Include units of measurement.**

Your creation must include the following:

I. The dimensions of the cylinder and rectangular prism

The Cylinder would have a radius of 48 in and a height of 174 in.

The prism would have a height of 174 in with length and width of 96 in.

II. Surface area of the cylinder and rectangular prism

Cylinder S.A. $= 2(48)^2 + 2\pi(48)(174) = 66,953.62\ in^2$

Prism S.A $= 4(96)(174) + (96)(96)$

$= 76,032\ in^2$

III. The volume of the container that is to be filled with confetti.

Cylinder Volume $= \pi(48)^2(174)$

$= 1,259,451.93\ cm^3$

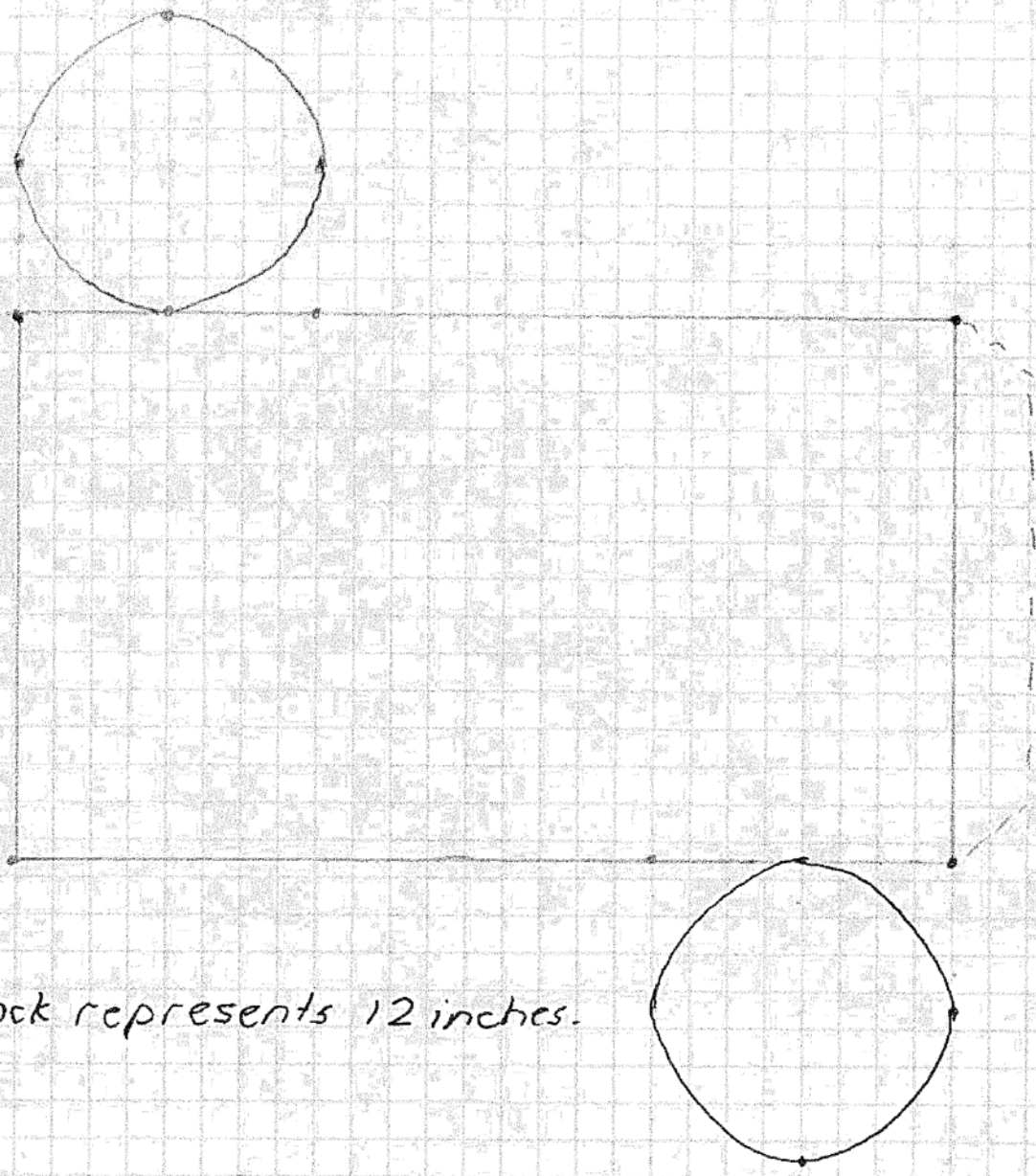
IV. The total amount it would cost the city for the project.

$.008(66,953.62 + 76,032) + .002(1,259,451.93)$

Total Amount is $\$3,662.79$ which falls in the allowed budget.

b. **On the graph paper provided, draw the net of the can.**

radius ≈48 in
diameter ≈96
height ≈174
lenght ≈ 302



Each block represents 12 inches.

2. If the city wanted to max out their spending on the project at $4,000 and they wanted the rectangular prism to have a height of 14.5 feet and the length and width of 8.5 feet. What would be dimensions of the largest cylinder that could be created to not exceed $4,000? Include all calculations as part of your solution. Would the rectangular prism given be large enough to hold the confetti of that cylinder? Explain.

$14.5\,ft \cdot \frac{12\,in}{1\,ft} = 174\,in$

$8.5\,ft \cdot \frac{12\,in}{1\,ft} = 102\,in$

Prism S.A =

$= 4(174)(102) + (102)(102)$

$= 81,396\,in^2$

Prism Cost $= .008(81,396)$

$= \$651.17$

$4,000 - 651.17 = \$3348.83$ left to make cylinder and fill it with confetti:

Trial 1: if the cylinder radius is 50 in w/height of 176 in.

$S.A = 2\pi(50)^2 + 2\pi(50)(176) = 70999.99\,in^2$

$V = \pi(50)^2(176) = 1382300.77\,in^3$

$Cost = .008(70999.99) + .002(1382300.77) = \$3332.61$

Trial 2: if the cylinder radius is 51 in w/height of 176 in

$S.A = 2\pi(51)^2 + 2\pi(51)(176) = 72740.44\,in^2$

$V = \pi(51)^2(176) = 1438145.72\,in^3$

$Cost = .008(72740.44) + .002(1438145.72) = \$3,458.21$

To Much

Trial 3: if the cylinder radius is 50 in w/height of 177 in

$S.A = 2\pi(50)^2 + 2\pi(50)(177) = 71314.15\,in^2$

$V = \pi(50)^2(177) = 1390154.75\,in^3$

$Cost = .008(71314.15) + .002(1390154.75) =$

$\$3350.82$ To Much

The largest cylinder would have to have a radius of 50 in^2 and height of 176 in^2. If either of these measurements are increased the cost for the project would exceed $ 4000.

Prism volume = 174(102)(102)
= 1,810,296 in^3

Cylinder Volume = 1382300.77 in^3

Since the prism's volume is greater than the largest cylinder's volume the prism would be large enough to hold the confetti.