Performance Based Learning and Assessment Task

How High Can You Throw?

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
The students will create an experiment that will allow them to calculate the initial velocity and maximum height of a tennis ball thrown vertically into the air. They will gather data using multiple trials, create a quadratic function that models the height of their tennis ball over time, and create a graph as a visual representation of their function.

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III. COURSE:
Algebra 1

IV. CONTENT STRAND:
Algebra: Functions

V. OBJECTIVES:
The student will be able to:

- represent verbal quantitative situations algebraically
- evaluate an algebraic expression for a given replacement of the variable
- find the zeros and maximum value of a quadratic function
- find the values of a function for elements in its domain
- make connections among multiple representations of a quadratic function
- use data derived from experimentation to calculate numerical averages

VI. REFERENCE/RESOURCE MATERIALS:
Metric tape measure, tennis ball, stopwatch and graph paper

VII. PRIMARY ASSESSMENT STRATEGIES:
Students will perform a self-assessment checklist based on a provided rubric. The teacher will use the same rubric to assess student performance based on correct mathematical computations, correct graphical representation, neat organization of collected data, and an organized presentation of results.

VIII. EVALUATION CRITERIA:
Assessment lists, corresponding rubrics, and a sample benchmark are included.

IX. INSTRUCTIONAL TIME:
This activity will take two 90 minute class periods. The first day will be used to gather the data and complete mathematical computations. The second day is needed for completing the graph, writing a summary of results, and completing the self-assessment checklist.

How High Can You Throw?

Strand
Algebra: Functions

Mathematical Objective(s)
The overall mathematical goal of this activity is for students to create and use the equation for the height of a parabola, given as a function of time. They will perform an experiment in order to gather the following data: the height at which a ball is being released, and the time it takes to fall to the ground when thrown vertically. They will substitute these values into the equation and use their knowledge of the zero of the function to solve for the initial velocity of the ball. With that information, they will make a unique quadratic function representing the height of their ball over time, and use the equation to calculate the maximum height of their ball. They will use investigation and analysis to make connections between multiple representations of functions. Their product will be in the form of gathered data, an equation and mathematical calculations, a written summary and a graphical representation of the height of the ball over time.

Related SOL

- A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.
- A.4f The student will solve multistep linear and quadratic equations in two variables, including solving real-world problems involving equations and systems of equations. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.
- A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including
  - a) determining whether a relation is a function;
  - b) domain and range;
  - c) zeros of a function;
  - d) *x*- and *y*-intercepts;
  - e) finding the values of a function for elements in its domain; and
  - f) making connections between and among multiple representations

NCTM Standards

The student will be able to:
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Use symbolic algebra to represent and explain mathematical relationships

Materials/Resources

Students will need the following materials to conduct their trials:
- Metric tape measure
- Tennis ball
- Stopwatch
- Paper and pencil

The trials must be conducted outside, and it may be preferable to do the trials near a wall with some form of obvious marks (bricks, cinder blocks, etc.) to make measuring the release height easier.

To complete the activity, students will need the following:

- Student activity sheet with equation and directions
- Data gathering and mathematical calculations sheet
- Graph paper to create final graph
- Word processing program for written summary

Assumption of Prior Knowledge

Students should have basic knowledge of conducting experiments and gathering data. They should know how to calculate the mean of a data set containing ten items of data. They must be able to operate a stopwatch and use a tape measure. They should have an understanding of how to substitute known values into an equation, and how to find the vertex of a parabola algebraically, as well as be familiar with function notation. Students should also be able to create a graph with labeled axes, using a reasonable scale on each axis.

Students may find it difficult to accurately measure the height at which the thrower is releasing the ball. It may be helpful to be near a wall so that students may use it for a reference, or even hold their tape measure against the wall. Students may capture the release of the ball on a cell phone camera in order to get more accuracy on the release height. In that case, having something behind the thrower as a reference will be helpful. Part of the goal of the activity is for students to create their own experiment to gather their data, so it may not be desirable to be too specific about how they should go about it. Any groups needing help can be given suggestions. They will complete ten trials, and will be using an average release height in their equation, so it will not be as important if a measurement is slightly off in one trial.

When substituting the initial height and elapsed time into the given equation of the parabola, students will likely struggle to realize that they actually know the height of the ball at the end of the elapsed time. h(t) must be zero, because at the end of the experiment, the ball is on the ground with a height of zero. The teacher should not be too quick to provide this information. Students should be given time to come up with it because it is a significant piece of information that relates to projectile motion. Hints may be given as needed, such as: How many seconds was your ball in the air? Do you know the height of the ball at that time? Where was the ball after that many seconds? (on the ground) So its height is…?

In performing the calculations for finding the maximum height of the ball, students will use $x = -\frac{b}{2a}$ to find the x-coordinate of the vertex (time), and then substitute it into the equation to get the y-coordinate (height of the ball). The teacher should be sure to show students how to use this method to find the vertex of a parabola before the activity. Students will need to know that the vertex of a parabola represents the maximum or minimum point of the quadratic function, depending on which way it opens.

Introduction: Setting Up the Mathematical Task

In this activity, students will determine the initial velocity and maximum height of a tennis ball thrown vertically. Students will create an experiment in which they can gather data to determine the height at which the ball was released when thrown into the air (h), and the time it took for the ball to fall to the ground (t). They will use this information and the equation for the height of a parabola as a function of time (provided) to calculate the initial velocity (v) of the ball. With this new information, they will be able to make a unique equation of a function representing the height of *their* tennis ball over time. Using this equation, they will calculate the maximum height of their tennis ball.

Prior to the day of the activity, students will need to be taught how to find the vertex of a parabola from the quadratic form by using $x = -\frac{b}{2a}$ to find the x-coordinate of the vertex, and substituting it into the equation to get the y-coordinate. They should be taught that the maximum or minimum value of a quadratic function will occur at the vertex and they should understand the meaning of an x-intercept, or zero, of the graph, and that the x-intercept always has a y-value of zero. This will be important in the calculations, as they must know the height of the tennis ball is zero at the end of its travel time (the time-intercept).

The activity will take approximately two 90 minute class periods, in which students will perform their trials with the tennis ball and record their data, then do their mathematical calculations and draw a graph. The second day is needed in order to have adequate time to summarize the activity and complete the self-assessment. Students will be organized into groups of four and told to choose a thrower, observer, timer, and recorder for the group. The activity sheet guides students through performing the trials and calculating an average release height (h) and average travel time (t). The second page of the activity sheet guides them through the mathematical calculations leading to the determination of the initial velocity of the ball (v) and its maximum height. After the activity sheet is completed, students will make a graph of the height of their ball over time, and summarize how they conducted their experiment and what their results were.

Student Exploration

Student/Teacher Actions:

Each student will be given an activity sheet. In addition, each group will be given a metric tape measure, tennis ball and stopwatch. Before going outside, they should decide who will perform which task in the group. The recorder will fill in the data table, the thrower will throw the tennis ball, the timer will time the flight of the ball, and the observer will note and measure the release height of the ball for each trial so that it can be recorded as accurately as possible.

During the trials, the teacher should monitor student progress and be available with suggestions for how to get a fairly accurate measurement of the release height of the ball for the trials.

After the trials are complete and the data charts are filled in, the class may move back indoors so groups may complete their mathematical calculations. Students should be encouraged to check each other's calculations for accuracy. Students may struggle with the calculation of the initial velocity of the ball because they do not realize that at the end of the time when the ball is on the ground, it has a height of 0, so h(t)=0. The teacher may have to remind students that in their initial equation, they have 4 unknowns. In order to find the initial velocity, v, they must already know t, h, and h(t).

The teacher should remind students that the ultimate goal is to come up with a function h(t) to calculate the height of their tennis ball at a given time, t. The function must find the height of the ball h(t) at any given time t. They may struggle to remember that the equation must only have two variables: t and h(t). Once they have the equation of the function, if students have been taught how to find the vertex of a parabola prior to this activity, they will be able to find the maximum height of their ball without difficulty. The teacher should encourage students to make a chart of several required (t,h(t)) pairs, including the vertex, so they may make an accurate graph of their function. The teacher should encourage students to label their axes and use a reasonable scale on each axis. They may use a calculator to determine y-values of their points and to make calculations.

Monitoring Student Responses
Each student in the group should be able to describe how they conducted their experiment, and explain the mathematical steps performed in order to calculate the initial velocity of the ball, determine the unique equation of their function, and find the maximum height of their ball. The teacher should question individual students in each group as they are completing their graphs to determine if everyone is coming away with the desired understanding. The group will write a summary of the process they used to gather their data, and tell the maximum height of their ball. They will have to agree on how to present their mathematical calculations in addition to making a graph. The teacher should monitor the groups to ensure that the tasks and workload are being shared by all members of the group. They should be communicating about the mathematics and staying on task.

The teacher will determine if an entire 90 minute class period is needed to complete the graphs, summaries, and self-assessments. A class discussion should be held after the activity has been completed to discuss what students learned about the use of the equation of a quadratic function. A point of discussion should include a question about whether the functions can be used to predict the height of a ball thrown by someone else in the class, and how to go about finding the time the ball hit the ground. The discussion time can also be used to allow students to tell what they found to be the most challenging part of the activity and what they liked the most. They will also want to compare their heights to see which group had the highest throw, but they should also discuss which group had the most unique way of measuring the release height, which group had the nicest graph, and other items of interest to the students. If desired, students could be asked to make a display board of their experiment and results. This would entail additional class time, however.

Assessment List and Benchmarks
Students will complete an activity sheet which includes their data gathering and mathematical analysis. They will create a graph of their function, explain the procedures they used to gather their data, and produce a written summary of their results. Students will also self-assess their work using the rubric provided. The teacher will use the same rubric to assess the group’s performance.

Possible Accommodations

- Assistance with directions to check for understanding
- Use of a calculator
- Use of a spell checker (for written report)
- Frequent checks for understanding and accuracy

How High Can You Throw?

The study of projectile motion in physics is directly related to Algebra. If you throw a tennis ball vertically into the air, you can predict the velocity and height of the ball with just a tape measure, tennis ball, stopwatch…and your knowledge of Algebra!

The height of a ball thrown vertically into the air, h(t), can be determined with the following equation:

$$h(t) = -4.9t^2 + vt + h$$

where:

v = the initial velocity of the ball (in m/sec)

t = the time the ball is in the air (in sec)

h = the height at which the ball is released when thrown (in meters)

h(t) = the height of the ball at any given time t (in meters)

-4.9 comes from the gravitational constant for acceleration (-9.8 m/sec^2)

Your Goals:

Determine:

1. The initial velocity of the ball as you throw it into the air
2. The equation of the function h(t) for your particular situation
3. The maximum height of your ball

Getting Organized:

In your group of four, decide who will perform each of the following tasks:

1. Thrower: This person will throw the ball vertically into the air.
2. Timer: This person will time how long the ball is in the air.
3. Observer: This person will observe when the ball leaves the thrower's hand and make the measurements of the height at which the ball is released.
4. Recorder: This person will write down the heights and times in the provided table.

Your Task:

1. Decide as a group how you want to gather your data. You may design your experiment any way you like using a tape measure (metric), tennis ball, and stopwatch. On your own paper, provide a description of how you will gather your data. Be specific about your procedure.

2. Gather data about the height at which the ball is released and the time the ball is in the air. Make sure you throw the ball *straight up* into the air with as close to the same force as you can. Observe very closely where the thrower's hand is when the ball is released. Note the amount of time it takes until the ball hits the ground. You must perform ten trials and write your data in the provided table.

3. Use the equation for the height of a ball at any given time and your knowledge of Algebra to accomplish the three listed goals (above).

Your Product:

- A table of your data, including ten trials showing the release height of the ball (h) and the travel time of the ball (t). (form provided)
- The average (mean) of your release height and travel time. (form provided)
- The mathematical computations used to accomplish each of your three goals listed above. (form provided)
- A graph of your function representing the height of your ball over time. The graph must include points representing the time and height when the ball was released, the maximum height of the ball, the time and height when the ball hit the ground, and 2 additional distinct points. Label all points. Be sure to use a reasonable scale on each axis, and to label your axes and title your graph.
- A typed report to include:

  Procedures you used to complete your experiment (include clear descriptions so that someone else could duplicate your experiment).

  A summary of what you learned in the activity, telling in detail how you completed all of your calculations, and what they meant. Be sure to include your final results.

  Include in your report the answer to this question: Now that you have an equation, can it be used to predict the height of a ball thrown by anyone in the class? Why or why not?

Data Gathering and Mathematical Analysis

Data Gathering:

Trial	1	2	3	4	5	6	7	8	9	10
h										
t										

Average release height (h): ___________

Average travel time (t): ___________

Mathematical Calculations:

$$h(t) = -4.9t^2 + vt + h$$

1. Calculate the initial velocity of the ball (v). Round final answer to 3 decimal places.

2. Write your function h(t) representing the height of your ball at any given time t.

3. Use your function to find the maximum height of your ball. Round to 3 decimal places.

Rubrics

Data Gathering				
Number	**Element**	**0**	**1**	**2**
1	**Group participation** All group members contributed equally to achieving the goal	Some members were not involved in the process	Some members did more work than others	All members contributed equally
2	**Trials** Ten trials were completed and recorded	No trials were competed	Completed and recorded some trials, but less than ten trials	Completed and recorded all ten trials
3	**Accuracy** Time and height measurements appear to be accurate	Unreasonable height and/or time measurements	Trials have a lot of variation in number	Reasonable degree of variation in height and time measurements

Mathematical Calculations				
Number	**Element**	**0**	**1**	**2**
1	**Correct h and t averages** Average height and average time for the trials is accurate	Average height and average time are both calculated incorrectly	Average height OR average time is correct, based on collected data	BOTH average height and average time are correct, based on collected data
2	**Correct v** Initial velocity of the ball is accurate	Initial velocity was not found at all, or the equation was used incorrectly	The equation was used correctly to calculate initial velocity, but the value of v is incorrect (credit will be given here if h and t were incorrect, but were correctly used in finding v)	The equation was used correctly and the initial velocity is correct. (credit will be given here if h and t were incorrect, but were correctly used in finding v)
3	**Correct equation of function h(t)** The equation of the function for calculating the height of a ball at a given time is accurate	The equation was not found at all	The equation is correct but in not in correct function notation (credit will be given here if the value of v found was not correct, but was correctly used to form the equation of the function)	The function equation is correct and in the proper form (credit will be given here if the value of v found was not correct, but was correctly used to form the equation of the function)
4	**Correct maximum height** The maximum height of the ball for the given function is accurate	The maximum height was not found	The calculations for the x-value (time) of the vertex are correct, but were not correctly plugged into the function to find the y-value (height). (Credit will be given here if the equation of the function is not correct, but was correctly used to find the vertex)	The calculations for the x and y value of the vertex are correct, and the maximum height is accurate. (Credit will be given here if the equation of the function is not correct, but was correctly used to find the vertex)

Final Product				
Number	**Element**	**0**	**1**	**2**
1	**Description of procedure** The procedure used to perform the trials is described clearly	Someone would not be able to follow the procedure to duplicate the experiment	The directions could be followed, but they are not presented in an organized manner	The directions are clear and easy to follow
2	**Graph** An accurate and detailed graph of the function is provided	The graph is not accurate or is not legible	The graph is accurate, but does not have all points labeled, or does not have a reasonable scale on the axes with axes labeled, or is missing a title	The graph is accurate, has all points and axes labeled, has reasonable scales on each axis, and has a title.
3	**Summary** A written summary of what was learned in this experiment, including final results	There is no written summary	The summary does not include final results, or is not well written, or does not use good grammar and sentence structure	The summary is well-written, uses good grammar and sentence structure, and includes all results
4	**Materials turned in** All materials were turned in, to include description of procedure, data gathering and mathematical calculation sheet, written summary, and graph.	No materials turned in	Some materials turned in	All materials turned in

Assessment List (to be used by each group as well as the teacher)

Data Gathering				
Number	**Element**	**Point Value**	**Self**	**Teacher**
1	**Group participation**	**2**		
2	**Trials**	**2**		
3	**Accuracy**	**2**		
	Total this section	**6**		
Mathematical Calculations				
Number	**Element**	**Point Value**	**Self**	**Teacher**
1	**Correct h and t averages**	**2**		
2	**Correct v**	**2**		
3	**Correct equation of function h(t)**	**2**		
4	**Correct maximum height**	**2**		
	Total this section	**8**		
Final Product				
Number	**Element**	**Point value**	**Self**	**Teacher**
1	**Description of procedure**	**2**		
2	**Graph**	**2**		
3	**Summary**	**2**		
4	**Materials turned in**	**2**		
	Total this section	**8**		
	Total of all three sections	**22**		

Benchmark (Hypothetical student example)

Data Gathering and Mathematical Analysis

Data Gathering:

Trial	1	2	3	4	5	6	7	8	9	10
h	1.3	1.3	1.4	1.4	1.5	1.4	1.5	1.3	1.6	1.5
t	1.91	1.88	2.15	2.18	2.0	2.14	2.21	1.89	2.25	2.08

Average release height (h): <u>1.42 meters</u>

Average travel time (t): <u>2.069 seconds</u>

Mathematical Calculations:

$$h(t) = -4.9t^2 + vt + h$$

1. Calculate the initial velocity of the ball (v). Round final answer to 3 decimal places.

$$\begin{aligned}
&h(t) = -4.9t^2 + vt + h \\
&0 = -4.9(2.069)^2 + v(2.069) + 1.42 \\
&0 = -20.9757289 + 2.069v + 1.42 \\
&0 = -19.5557289 + 2.069v \\
&19.5557289 = 2.069v \\
&v = 9.451778105
\end{aligned}$$

The initial velocity of the ball is 9.452 meters per second.

2. Write your function h(t) representing the height of your ball at any given time t.

The equation of the function is $h(t) = -4.9t^2 + 9.452t + 1.42$

3. Use your function to find the maximum height of your ball. Round to 3 decimal places.

$t = -\frac{9.452}{-9.8} = .964$ seconds $\qquad h(.964) = -4.9(.964)^2 + 9.452(.964) + 1.42 = 5.978$ meters

The maximum height of the ball was 5.978 meters.

Procedure

Savannah stood in front of the cinder block wall and threw the ball straight up in the air. Aubrey started the timer at the exact moment that the ball left Savannah's hand, and stopped it when it hit the ground. Colton watched to see exactly where Savannah's hand was when she released the ball, using the lines on the cinder block wall as a guide so we had some lines to use as a reference to where her hand was. Then he measured the height in meters and Kristen wrote it down in the chart. We did this ten times and then calculated the average of our time and the average of our release height.

Summary

The maximum height of our ball was 5.978 meters, and it occurred .964 seconds after Savannah released the ball. We learned that we could use the amount of time the ball was in the air and the height when the ball was released to find the initial velocity of the ball. When we put 2.069 (our average time) in for t and 1.42 (our average release height) in for h, we also had to put 0 in for h(t) because the height of the ball when it hit the ground was 0. So the only thing missing in our equation at this point was v. Our initial velocity was 9.452 meters per second, which we all thought was pretty fast. Next, we put our release height and initial velocity into the equation and made it a function in function notation, with t for the time and h(t) for the height of the ball. The equation of our function was $h(t) = -4.9t^2 + 9.452t + 1.42$. We can use this function to find the height of our ball at any given time. But our equation won't work to find the height of anyone else's ball because the equation only fits our data, so it is unique. We can't even use it to find the height of a ball thrown by someone else in our group; only Savannah. We then used the method we learned in class for finding the maximum value of the function, which occurs at the vertex. We found the x-value, which is really the t-value, and plugged it into the function to find the y-value. The y-value is the maximum height of our ball.

Graph

t	H(t)	
0	1.42	The ball was released
.5	4.921	
.964	5.978	The maximum height
1.5	4.573	
2.069	0	The ball hit the ground