

Performance Based Learning and Assessment Task

Polynomial Farm

I. ASSESSMENT TASK OVERVIEW & PURPOSE:	This performance task is planned to give students an opportunity to add, subtract, multiply, and divide polynomials in order to solve real-world problems. It is also planned to give students real-world practice factoring completely first- and second-degree binomials and trinomials in one variable. Lastly, this task is designed to encourage students to make connections and to communicate their mathematical thinking clearly and accurately.
II. UNIT AUTHOR:	Emily O'Rourke, Northside High School, Roanoke, Virginia
III. COURSE:	Algebra I
IV. CONTENT STRAND:	Expressions and Operations
V. OBJECTIVES:	The student will be able to add, subtract, multiple, and divide polynomials. The student will also be able to factor completely first- and second-degree binomials and trinomials in one variable.
VI. REFERENCE/RESOURCE MATERIALS:	Students will need access to class notes, a pencil, a graphing calculator, and a "Polynomial Farm" worksheet.
VII. PRIMARY ASSESSMENT STRATEGIES:	The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. Students will be assessed on their understanding of adding, subtracting, multiplying, dividing, and factoring polynomials as it relates to real-life problems. Students will be evaluated on how clearly and accurately they explain their mathematical process and thinking. Students will also be assessed on the connections between their work and their reflection.
VIII. EVALUATION CRITERIA:	A self-assessment and a teacher assessment are attached below. A benchmark is also included at the end of the document in order to demonstrate the level of quality that is expected from each group of students.
IX. INSTRUCTIONAL TIME:	The performance task should take no longer than one ninety-minute block.

Polynomial Farm

Strand

Expressions and Operations

Mathematical Objective(s)

The mathematical objective for this activity is to give students the opportunity to add, subtract, multiply, divide, and factor polynomials in a real-world context.

Related SOL

The student will perform operations on polynomials, including:

A.2b) adding, subtracting, multiplying, and dividing polynomials

A.2c) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.

NCTM Standards

- Understand meanings of operations and how they relate to one another
- Build new mathematical knowledge through problem solving
- Represent and analyze mathematical situations and structures using algebraic symbols
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

Students will use class notes in order to validate how to add, subtract, multiply, divide, and factor polynomials. The students will use a pencil to record answers on their “Polynomial Farm” worksheet. The student will use a graphing calculator as a tool for factoring and for confirming algebraic factorizations.

Assumption of Prior Knowledge

- Students should understand how to calculate the perimeter and the area of a rectangle. The students should be familiar with adding, subtracting, multiplying, and dividing polynomials. The student should have prior knowledge of how to factor first- and second-degree binomials and trinomials in one variable. Additionally, students must be able to communicate their mathematical processes in an organized and legible way.
- Students have difficulty differentiating when they are able to factor and when they are unable to factor. Students also often forget to look for a greatest common factor. Furthermore, students frequently mix up the following: $x + x = 2x$ and $x \cdot x = x^2$. The teacher may need to prompt students to look for a greatest common factor and provide additional support when the students are factoring. Additionally, teachers may need to provide guidance when recalling $x + x = 2x$ and $x \cdot x = x^2$.

Introduction: Setting Up the Mathematical Task

In this activity, you will investigate the relationship between adding, subtracting, multiplying, dividing, and factoring polynomials in real-world scenarios. The Polynomial Farm Activity should take no longer than one ninety-minute block.

For this activity, you will be adding, subtracting, multiplying, dividing, and factoring in order to help a farmer calculate the perimeter and the area of his produce fields. The goal is to use your knowledge of adding, subtracting, multiplying, dividing, and factoring to answer all of the farmer's questions listed on the "Polynomial Farm" worksheet. Once the worksheet is completed, you will be expected to participate in a classroom reflective discussion about what you learned and also be expected to complete a self-assessment.

A recommendation for the teacher is to begin class with a bell ringer in which you review operations on polynomials and calculating the perimeter and area of a rectangle. After fifteen minutes, the teacher will spend ten minutes calling on students to come up to the board and provide their answers to the bell ringer questions. This will create student awareness of what prior knowledge is expected to successfully complete the task at hand.

Next, the teacher will place students in pairs in order to produce workable, productive groups. The groups will then obtain the "Polynomial Farm" worksheet and have forty-five minutes to complete it with their partner. The teacher will be walking around to answer any and all questions. After the worksheet is completed, ten minutes of class will be spent in a reflective discussion. The students will be asked what challenges they faced, what problem-solving skills they used and developed, and what they enjoyed most about the task. Another question teachers could propose to their students is, "What other real-world scenarios can you think of that may require adding, subtracting, multiplying, dividing, and factoring polynomials?" Within the discussion, students are expected to actively and respectfully participate. Students will also be expected to complete a self-assessment in the last ten minutes of class.

Student Exploration

Student/Teacher Actions:

Students will be collaborating in pairs as determined by the teacher. The teacher will explicitly state the expectations for the day upfront and provide constructive feedback along the way. The teacher will also act as a mentor and coach as students begin answering the more difficult questions about Farmer Bob's fields. Teachers will encourage students to draw on the group's knowledge first, prior to seeking out prompting from the teacher, class notes, and/or the Internet.

The teacher will also be available to facilitate the classroom reflective discussion. Students will communicate their mathematical knowledge by using appropriate vocabulary when presenting and making connections between Farmer Bob's fields and operations on polynomials. The teacher can encourage each group to use technology, specifically GeoGebra, if they would like to change the dimensions of Farmer Bob's produce fields.

Monitoring Student Responses

- Students are to communicate their thinking and their new knowledge by actively participating in the group discussion. Each classmate is expected to provide feedback at least once.
- Students are to communicate with each other actively, respectfully, and supportively.
- Students are encouraged to ask effective questions that do not require a yes or no answer.
- Teachers are to highlight and clarify frequently asked questions to the class as they emerge and provide problem-solving strategies to groups in order to resolve difficult situations.
- Teachers should encourage all students to be engaged within their group and therefore, discourage students from moving forward without their group members. If an entire group is ready to move on, encourage the group to begin brainstorming other ways to use operations on polynomials in the real world.

Again, in order to summarize the Polynomial Farm Activity, the teacher should plan to recap on the strengths and feedback from the classroom discussion. The teacher should also focus on how groups overcame difficult tasks and which problem-solving techniques to carry forward in the classroom. Lastly, the teacher should reflect on the content knowledge that was reviewed and applied.

Assessment List and Benchmarks

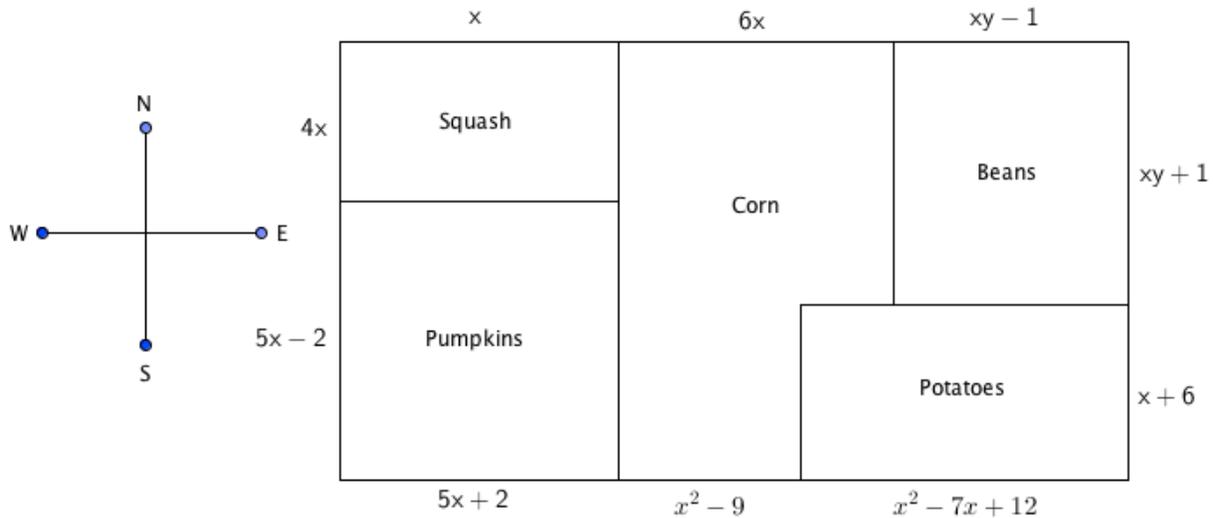
Students will complete the “Polynomial Farm” worksheet with their partner within one ninety-minute block. The teacher will then facilitate a discussion of the challenges, problem-solving strategies, and mathematical content knowledge gained from the Polynomial Farm Activity. Unlike that of the partner worksheet, students will individually complete a self-assessment at the end of the activity. The teacher will use the same rubric to assess each student and will give extra credit for exemplary participation in the classroom discussion and for successfully completing question thirteen on the “Polynomial Farm” worksheet.

Performance Task – Expressions & Operations
A.2bc Polynomial Farm

Names: _____

Date: _____ Block: _____

Directions: Farmer Bob is planting a garden this spring. He wants to plant squash, pumpkins, corn, beans, and potatoes. His plan for the field layout in feet is shown in the figure below. Use the figure and your knowledge of polynomials, perimeter, and area to solve the following:



1. Write an expression that represents the length of the south side of the field.
2. Simplify the polynomial expression that represents the south side of the field.
3. Write a polynomial expression that represents the perimeter of the pumpkin field.
4. Simplify the polynomial expression that represents the perimeter of the pumpkin field. State one reason why the perimeter would be useful to Farmer Bob.
5. Write a polynomial expression that represents the area of the potato field.

6. Simplify the polynomial expression that represents the area of the potato field. State one reason why the calculated area would be useful to Farmer Bob.

7. Write and simplify the polynomial expression that represents the area of the bean field if $x = 3$ and $y = 7$. What unit would the area of Bob's bean field have?

8. The farmer would like his bean plants to grow to a height of $(x + 3)$. Write a polynomial expression to find the volume of the bean plants if they reach a height of $(x + 3)$.

9. Simplify the polynomial expression that represents the volume of the bean plants if they reach a height of $(x + 3)$ feet.

10. Farmer Bob would like to plant three additional fields of produce. Using your graphing calculator, find the dimensions of each field given the area.
 - a. The area of the strawberry field is $16x^2 + 4x$.
 - b. The area of the cucumber field is $x^2 - 4x - 21$.
 - c. The area of the tomato field is $x^2 - 36$.

11. Farmer Bob realized he forgot to include a zucchini field into his field layout. He plans to use half the length and half the width of the squash field in order to plant zucchini. Write a polynomial expression that represents the area of the new zucchini field.

12. Simplify the polynomial expression that represents the area of the newly added zucchini field.

13. Extra Credit: Write and simplify polynomial expressions that represent the perimeter and area of the cornfield.

Performance Assessment Task

A.2bc Polynomial Farm

Self-Assessment

Name: _____

Date: _____ Block: _____

Num	Element	Point Value	Earned Assessment	
			Self	Teacher
1	The expression that represents the length of the south side of the field is provided.	2		
2	The south side of the field polynomial expression is simplified.	2		
3	The polynomial expression that represents the perimeter of the pumpkin field is provided.	2		
4	The polynomial expression that represents the perimeter of the pumpkin field is simplified.	2		
5	The usefulness of the perimeter in terms of Farmer Bob's fields is provided.	2		
6	The polynomial expression that represents the area of the potato field is provided.	2		
7	The polynomial expression that represents the area of the potato field is simplified.	2		
8	The usefulness of the area in terms of Farmer Bob's fields is provided.	2		
9	The polynomial expression that represents the area of the bean field is written.	2		
10	The polynomial expression that represents the area of the bean field is simplified.	2		
11	The unit for the area of the bean field is provided.	2		
12	The polynomial expression to find the volume of the bean plants if they reach a height of $(x+3)$ is provided.	2		
13	The polynomial expression to find the volume of the bean plants if they reach a height of $(x+3)$ is calculated.	2		
14	The dimensions of the strawberry field area provided.	2		
15	The dimensions of the cucumber field are provided.	2		
16	The dimensions of the tomato field are provided.	2		
17	A polynomial expression that represents the area of the new zucchini field is written.	2		
18	A polynomial expression that represents the area of the new zucchini field is simplified.	2		
19	The "Polynomial Farm" worksheet is completed on time.	2		
20	The student actively and respectfully participated in the reflective discussion.	2		
21	All written work is legible.	2		
22	The mathematical responses are well organized.	2		
23	Self-assessment is completed on time.	2		
Total		46		

Performance Assessment Task

A.2bc Polynomial Farm

Teacher Assessment

Name: _____

Date: _____ Block: _____

Num	Element	Point Value	Earned Assessment	
			Self	Teacher
1	The expression that represents the length of the south side of the field is provided.	2		
2	The south side of the field polynomial expression is simplified.	2		
3	The polynomial expression that represents the perimeter of the pumpkin field is provided.	2		
4	The polynomial expression that represents the perimeter of the pumpkin field is simplified.	2		
5	The usefulness of the perimeter in terms of Farmer Bob's fields is provided.	2		
6	The polynomial expression that represents the area of the potato field is provided.	2		
7	The polynomial expression that represents the area of the potato field is simplified.	2		
8	The usefulness of the area in terms of Farmer Bob's fields is provided.	2		
9	The polynomial expression that represents the area of the bean field is written.	2		
10	The polynomial expression that represents the area of the bean field is simplified.	2		
11	The unit for the area of the bean field is provided.	2		
12	The polynomial expression to find the volume of the bean plants if they reach a height of $(x+3)$ is provided.	2		
13	The polynomial expression to find the volume of the bean plants if they reach a height of $(x+3)$ is calculated.	2		
14	The dimensions of the strawberry field area provided.	2		
15	The dimensions of the cucumber field are provided.	2		
16	The dimensions of the tomato field are provided.	2		
17	A polynomial expression that represents the area of the new zucchini field is written.	2		
18	A polynomial expression that represents the area of the new zucchini field is simplified.	2		
19	The "Polynomial Farm" worksheet is completed on time.	2		
20	The student actively and respectfully participated in the reflective discussion.	2		
21	All written work is legible.	2		
22	The mathematical responses are well organized.	2		
23	Self-assessment is completed on time.	2		
Total		46		

Performance Assessment Task

A.2bc Polynomial Farm

Category Descriptions

Name: _____

Date: _____ Block: _____

#	Element	0	1	2
1	The expression that represents the length of the south side of the field is provided.	No length provided	Length is incomplete	Length is provided
2	The south side of the field polynomial expression is simplified.	Not simplified	Simplification incomplete	Simplification provided
3	The polynomial expression that represents the perimeter of the pumpkin field is provided.	No perimeter provided	Perimeter is incomplete	Perimeter is provided
4	The polynomial expression that represents the perimeter of the pumpkin field is simplified.	Not simplified	Simplification incomplete	Simplification provided
5	The usefulness of the perimeter in terms of Farmer Bob's fields is provided.	Perimeter usefulness not provided	Perimeter usefulness incomplete	Perimeter usefulness provided
6	The polynomial expression that represents the area of the potato field is provided.	No area provided	Area is incomplete	Area is provided
7	The polynomial expression that represents the area of the potato field is simplified.	Not simplified	Simplification incomplete	Simplification provided
8	The usefulness of the area in terms of Farmer Bob's fields is provided.	Area usefulness not provided	Area usefulness incomplete	Area usefulness provided
9	The polynomial expression that represents the area of the bean field is written.	No area provided	Area is incomplete	Area is provided
10	The polynomial expression that represents the area of the bean field is simplified.	Not simplified	Simplification incomplete	Simplification provided
11	The unit for the area of the bean field is provided.	Area unit not provided	Area unit incomplete	Area unit is provided
12	The polynomial expression to find the volume of the bean plants if they reach a height of $(x+3)$ is provided.	No volume provided	Volume is incomplete	Volume is provided

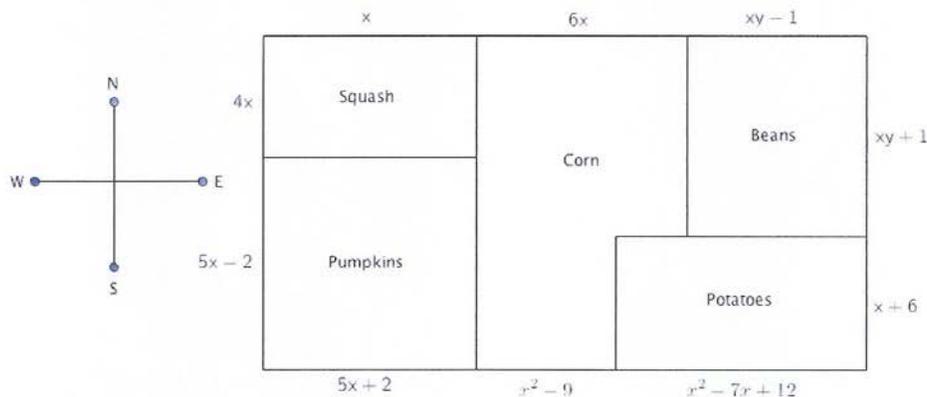
13	The polynomial expression to find the volume of the bean plants if they reach a height of $(x+3)$ is calculated.	Volume not calculated	Calculation incomplete	Volume calculation is provided
14	The dimensions of the strawberry field area provided.	Dimensions are not included	Dimensions are incomplete	Dimensions are provided
15	The dimensions of the cucumber field are provided.	Dimensions are not included	Dimensions are incomplete	Dimensions are provided
16	The dimensions of the tomato field are provided.	Dimensions are not included	Dimensions are incomplete	Dimensions are provided
17	A polynomial expression that represents the area of the new zucchini field is written.	Area not provided	Area incomplete	Area provided
18	A polynomial expression that represents the area of the new zucchini field is simplified.	Not simplified	Simplification incomplete	Simplification provided
19	The "Polynomial Farm" worksheet is completed on time.	No worksheet	Worksheet is incomplete or not provided on time	Worksheet completed on time
20	The student actively and respectfully participated in the reflective discussion.	Does not actively or respectfully participate	Does not fully participate	Actively and respectfully participates
21	All written work is legible.	Written work illegible	Written work partially legible	Written work legible
22	The mathematical responses are well organized.	No evidence of organization	Not fully organized	Well organized
23	Self-assessment is completed on time.	No self-assessment	Self-assessment is incomplete or not provided on time	Self-assessment provided on time

Performance Task – Expressions & Operations
A.2bc Polynomial Farm
Benchmark

Performance Task – Expressions & Operations
A.2bc Polynomial Farm

Names: Benchmark
 Date: 07/11/14 Block: _____

Directions: Farmer Bob is planting a garden this spring. He wants to plant squash, pumpkins, corn, beans, and potatoes. His plan for the field layout in feet is shown in the figure below. Use the figure and your knowledge of polynomials, perimeter, and area to solve the following:



1. Write an expression that represents the length of the south side of the field.

$$(5x+2) + (x^2-9) + (x^2-7x+12)$$

2. Simplify the polynomial expression that represents the south side of the field.

$$\begin{array}{r} 0x^2 + 5x + 2 \\ 1x^2 + 0x - 9 \\ + 1x^2 - 7x + 12 \\ \hline 2x^2 - 2x + 5 \end{array}$$

3. Write a polynomial expression that represents the perimeter of the pumpkin field.

$$(5x-2) + (5x+2) + (5x-2) + (5x+2) \text{ or } 2(5x-2) + 2(5x+2)$$

4. Simplify the polynomial expression that represents the perimeter of the pumpkin field. State one reason why the perimeter would be useful to Farmer Bob.

$$\begin{array}{r} 2(5x-2) + 2(5x+2) \\ 10x - 4 + 10x + 4 = 10x + 10x = 20x \end{array}$$

The perimeter would be useful to Bob if he was installing a fence around the field.

5. Write a polynomial expression that represents the area of the potato field.

$$(x^2 - 7x + 12)(x + 6)$$

Performance Task – Expressions & Operations
A.2bc Polynomial Farm
Benchmark

Benchmark
07/11/14

6. Simplify the polynomial expression that represents the area of the potato field. State one reason why the calculated area would be useful to Farmer Bob.

x^2	$-7x$	12
x^3	$-7x^2$	$12x$
$6x^2$	$-42x$	72

$$x^3 - 7x^2 + 6x^2 + 12x - 42x + 72$$

$$= x^3 - x^2 - 30x + 72$$

The area would be useful to Bob if he needed to decide how many potato seeds to purchase in order to seed the field.

7. Write and simplify the polynomial expression that represents the area of the bean field if $x = 3$ and $y = 7$. What unit would the area of Bob's bean field have?

$$(xy-1)(xy+1) = (3 \cdot 7 - 1)(3 \cdot 7 + 1) = (20)(22) = 440$$

Area = 440 ft²

8. The farmer would like his bean plants to grow to a height of $(x + 3)$. Write a polynomial expression to find the volume of the bean plants if they reach a height of $(x + 3)$.

$$(xy-1)(xy+1)(x+3)$$

9. Simplify the polynomial expression that represents the volume of the bean plants if they reach a height of $(x + 3)$ feet.

xy	-1
x^2y^2	$-xy$
xy	-1

$$= x^2y^2 - 1$$

x^2y^2	-1
x^3y^2	$-x$
$3x^2y^2$	-3

$$= x^3y^2 - x + 3x^2y^2 - 3$$

10. Farmer Bob would like to plant three additional fields of produce. Using your graphing calculator, find the dimensions of each field given the area.

a. The area of the strawberry field is $16x^2 + 4x$. $4x(4x+1)$

b. The area of the cucumber field is $x^2 - 4x - 21$. $(x-7)(x+3)$

c. The area of the tomato field is $x^2 - 36$. $(x+6)(x-6)$

11. Farmer Bob realized he forgot to include a zucchini field into his field layout. He plans to use half the length and half the width of the squash field in order to plant zucchini. Write a polynomial expression that represents the area of the new zucchini field.

$$l = \frac{1}{2}(x) \quad w = \frac{1}{2}(4x) \quad \text{Area} = \left(\frac{x}{2}\right)\left(\frac{4x}{2}\right)$$

12. Simplify the polynomial expression that represents the area of the newly added zucchini field.

$$\text{Area} = \left(\frac{x}{2}\right)\left(\frac{4x}{2}\right) = \frac{4x^2}{4} = x^2$$

13. **Extra Credit:** Write and simplify polynomial expressions that represent the perimeter and area of the cornfield.

Perimeter = $(6x) + (4x) + (5x-2) + (x^2-9) + (x+6) + (6x-(x^2-9)) + (xy+1)$
 $= 22x + xy + 5 \text{ or } 30x - 4$

Area = $6x(xy+1) + (x^2-9)(x+6)$
 $= 6x^2y + 6x + x^3 - 9x + 6x^2 - 54$
 $= x^3 + 6x^2 + 6x^2y - 3x - 54$