

# ***Performance Based Assessment: Building Sandcastles***

## **I. ASSESSMENT TASK OVERVIEW & PURPOSE:**

This task is designed to develop skills in analyzing properties of three-dimensional shapes through making sandcastles.

## **II. UNIT AUTHOR:**

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## **III. COURSE: Geometry**

## **IV. CONTENT STRAND:**

Geometry, Measurement

## **V. OBJECTIVES:**

The student will be able to:

- Develop a two-dimensional design of a sandcastle
- Calculate the surface area and volume of all figures used in design
- Create a three-dimensional model of their sandcastle using a stable medium and nets (a pattern that you can cut and fold to make a model of a solid shape) of geometric figures
- Determine the surface area and volume of the entire sandcastle
- Analyze what happens to a figure's surface area and volume when the dimensions are changed
- Determine the cost of supplying sand to build their sandcastle

## **VI. REFERENCE/RESOURCE MATERIALS:**

Templates of three-dimensional figures

Internet resources for surface area and volume formulas and conversions

Diagrams on pages 24-25 from Microsoft Word and Google Sketchup

## **VII. PRIMARY ASSESSMENT STRATEGIES:**

The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric.

The assessment list is designed to act as a rubric where the student can use as a checklist and then self-assess. The teacher will use this assessment list to assess the student's work and assign grades.

The rubric gives detailed expectations of each task for the student and teacher so they can better assess their work for each task.

**VIII. EVALUATION CRITERIA:**

Assessment List: a checklist, self-assessment and grading tool to evaluate mathematical content, performance, and work habits of students

Task Rubric: Detailed expectations of point values for each given task in the assessment list

Collaboration Rubric: a checklist and self-assessment for students to evaluate their communication skills while working in pairs

Benchmark of exemplary work.

**IX. INSTRUCTIONAL TIME:**

Two (three if needed) 90 minute blocks

# Building Sandcastles

## Strand

Geometry, Measurement

### Mathematical Objective(s)

- Students will evaluate the surface area and volume of three-dimensional figures and apply this knowledge to find the surface area and volume of a composite shape.
- Students will create a sandcastle using their knowledge of three-dimensional shapes and their nets.
- Students will analyze how changing the dimensions of a figure affect surface area and volume.
- Students will evaluate the cost of building a sandcastle.

### Related SOL

- G.13 (The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.)
- G.14 b) (The student will use similar geometric objects in two- or three-dimensions to determine how changes in one or more dimensions of an object affect area and/or volume of the object.)

### NCTM Standards

- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders
- Analyze properties and determine attributes of two- and three-dimensional objects
- Make decisions about units and scales that are appropriate for problem situations involving measurement
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools

### Materials/Resources

- Personal or provided electronic device (access to the internet optional)
- Cardboard/Poster board/Cardstock
- Tape/glue, scissors
- Student Worksheet (pages 6-7)
  - Geometric Figure Details Chart
  - Analysis Questions
- Geometry Figure Formula Sheet (page 8)
- Templates of various three-dimensional figures (pages 9-17, examples from <http://www.math-salamanders.com/geometry-nets.html>)

### Assumption of Prior Knowledge

- Students must have knowledge of vocabulary related to shapes (radius, height, surface area, etc.).
- Students must be comfortable with calculating of surface area and volume of three-dimensional figures.
- Students must have prior knowledge on converting units.
- Students might find it difficult to find a relationship between an original figure and one with the dimensions doubled.
- Students might find it difficult to create a structure that is stable enough to construct.

## Introduction: Setting Up the Mathematical Task

Description of the activity:

You and your friends are excited about entering in the yearly sandcastle competition; however, this year's competition comes with a geometric twist!

You must have at least four different geometric figures that touch and at least one composite figure. The competition also requires you to find the surface area and volume of your figures, approximate the amount and cost of materials you'll need to construct your figure and make predictions about changes to dimension. To enter the contest you must submit a two-dimensional blueprint of your design (complete with a scale, description of medium used for the 3-D figure, and any details for construction) as well as a neat, well-constructed three-dimensional model and the calculations mentioned above. A formula sheet and patterns to aid in forming 3-D solids will be provided to you.

The teacher could introduce the activity by asking "How many people have ever built a sandcastle at the beach?" "Did you know what size sandcastle you were building or what shapes you used in construction?" or "How much sand do you think it took to build that castle?" A discussion around these questions should draw in the students attention to the project.

- Students will be allowed to work with a partner to complete this activity; however, work should be turned in for both students.
- Day 1: Students will be instructed to start their project by drawing a blueprint of what they want their castle to look like. It should include a scale, description of the 3-D medium that will be used and any other details needed for construction.
  - As students complete their blueprint they will be instructed to complete the "Geometric Figure Detail" chart, determining the surface area and volume of each figure (including the model dimensions and the dimensions of the actual sandcastle).
  - The figure detail chart and model blueprint are due at the end of the first 90 minutes for teacher review. Students may begin the analysis questions and turn them in at the end of day 1 in order to get teacher feedback.
- Day 2: Students will utilize nets (patterns that you can cut and fold to make a model of a solid shape) to construct their castles. They should use the medium they proposed and follow the design they created in the blueprint. (Recommendations: cardboard, poster board, etc)
  - As students complete their constructions, they should begin on the "Analysis Questions" and may use this time to re-evaluate any parts of the worksheet they received feedback on from the teacher.
- Day 3: If needed, allow students a third day to complete construction and analysis questions, especially if students have utilized time efficiently.

## Student Exploration

### Small Group Work

- After a description of the project the teacher will hand out the student worksheet and answer any questions students may have about completing the work.
- Teacher may take time before students begin to encourage the use of personal/provided electronic devices to research formulas and conversions.
- The students will begin by creating a blueprint of their castle. While creating this blueprint, students will need to include dimensions. They will need to use their critical thinking skills to ensure that their measurements are reasonable (e.g. they shouldn't have a radius of 1 ft and a height of 90 ft)
- Students will continue to work together on the worksheet that accompanies the task and make sure they show all work individually.
- Students will finish in small groups by creating their three-dimensional model.

### Student/Teacher Actions:

- Teachers should show the class as a whole what the expectations are for each day and what materials are due at the end of each day. Teachers are encouraged to tailor due dates for parts of the project due to time constraints (e.g. 45 min day vs. 90 min day).
- Students should begin by creating a blueprint of their castle. As they create their blueprint they should include dimensions of their actual castle.
- As students create their dimensions for their castle they may need teacher assistance to make sure their dimensions are realistic.
- The teacher should monitor student progress to make sure they remain on task.
- Students may struggle to remember formulas or conversion methods, the teacher may add scaffolding by encouraging use of the provided formula sheet, personal/provided electronic device, provided layouts (nets) to create solid figures, and showing conversion examples or discussion previous examples shown in class.

### Monitoring Student Responses

- The students will have to actively work with a partner to complete this activity. Work should not be all one student's ideas. Students will be given the Collaboration Rubric to help aid in collaboration expectations. Students will submit this rubric at the end of each day to evaluate their partner's collaboration as well as their own participation in the group. It is encouraged that this is not a graded assignment but rather a chance for students to reflect on their collaboration efforts.
- Teacher may choose to evaluate work from Day 1 (and Day 2 if needed) and give feedback for students on an individual basis based on the students work shown.
- All students should complete the "Figure Detail Chart" and "Analysis Questions" individually with the help of their partner.

## Student Materials

### *Building Sandcastles*

Name: \_\_\_\_\_

**You and your friends are excited about entering in the yearly sandcastle competition; however, this year's competition comes with a geometric twist!**

**You must have at least four different geometric figures that touch and at least one composite figure. The competition also requires that you find the surface area and volume of your figures, approximate the amount and cost of materials you'll need to construct your figure, and make predictions about changes to dimension. To enter the contest you must submit a two-dimensional blueprint of your design (complete with a scale, description of medium used for the 3-D figure, and any details for construction) as well as a neat, well-constructed three-dimensional model and the calculations mentioned above. A formula sheet and patterns to aid in forming 3-D solids will be provided to you.**

**Record all your findings on the worksheet below. The two dimensional plan must be attached on a separate sheet of paper and the three-dimensional model must be stable and transported to the contest headquarters with all other documents.**

Geometric Figure Details: Include answers in the table below. All work including formulas used should be shown for full credit. (Note: you are not limited to a total of four figures. Please use a separate sheet of paper to complete the table if more room is needed).

| Geometric Name of Figure | Dimensions of figure<br>(include units) |        | Surface Area<br>(include units) |        | Volume<br>(include units) |        |
|--------------------------|---|--------|---------------------------------|--------|---------------------------|--------|
|                          | Model                                   | Actual | Model                           | Actual | Model                     | Actual |
| 1.                       |   |        |                                 |        |                           |        |
| 2.                       |   |        |                                 |        |                           |        |
| 3.                       |   |        |                                 |        |                           |        |
| 4.                       |   |        |                                 |        |                           |        |



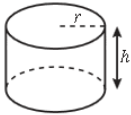
## Geometry Figure Formula Sheet

Please note that this formula sheet lists formulas for possible solids; remember, surface area and volume will need to be found for all figures. Information on unit conversion is included as well.

### VDOE Geometry Formula Sheet Three-Dimensional Figures

#### Abbreviations

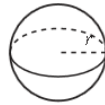
|               |        |
|---------------|--------|
| Area          | $A$    |
| Area of Base  | $B$    |
| Circumference | $C$    |
| Lateral Area  | $L.A.$ |
| Perimeter     | $p$    |
| Surface Area  | $S.A.$ |
| Volume        | $V$    |



$$V = \pi r^2 h$$

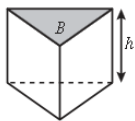
$$L.A. = 2\pi r h$$

$$S.A. = 2\pi r^2 + 2\pi r h$$



$$V = \frac{4}{3} \pi r^3$$

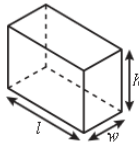
$$S.A. = 4\pi r^2$$



$$V = Bh$$

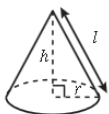
$$L.A. = hp$$

$$S.A. = hp + 2B$$



$$V = lwh$$

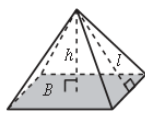
$$S.A. = 2lw + 2lh + 2wh$$



$$V = \frac{1}{3} \pi r^2 h$$

$$L.A. = \pi r l$$

$$S.A. = \pi r^2 + \pi r l$$



$$V = \frac{1}{3} Bh$$

$$L.A. = \frac{1}{2} lp$$

$$S.A. = \frac{1}{2} lp + B$$

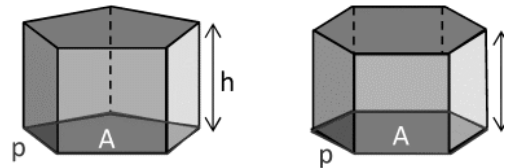
### Other Geometry Formulas for Three Dimensional Figures

#### PRISMS

Volume of any prism =  $Ah$

Surface area of a closed prism =  $2A + (h \times p)$

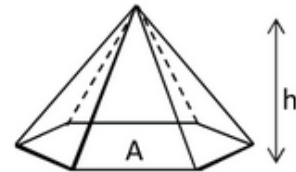
where  $A$  = base area,  $h$  = height,  $p$  = base perimeter



#### PYRAMIDS

Volume of a general pyramid =  $\frac{1}{3} Ah$

where  $A$  = base area and  $h$  = height



#### FRUSTUM OF A CONE

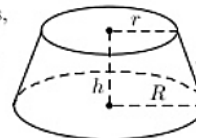
$r$  = top radius,  $R$  = base radius,

$h$  = height,  $s$  = slant height

Volume:  $V = \frac{\pi}{3}(r^2 + rR + R^2)h$

Surface Area:

$S = \pi s(R + r) + \pi r^2 + \pi R^2$



#### Unit Conversions

Volume of one bag sand =  $0.5 \text{ ft}^3$

$1 \text{ in}^3 = 0.000578704 \text{ ft}^3$

#### Resources:

<http://www.slideshare.net/PDF-eBooks-For-Free/geometry-formulas-2d-and-3d-ebook>

<http://www.math-salamanders.com/image-files/high-school-geometry-help-geometry-cheat-sheet-5-3d-shape-formulas.gif>

[http://www.doe.virginia.gov/testing/test\\_administration/ancillary\\_materials/mathematics/2009/2009\\_sol\\_formula\\_sheet\\_geometry.pdf](http://www.doe.virginia.gov/testing/test_administration/ancillary_materials/mathematics/2009/2009_sol_formula_sheet_geometry.pdf)

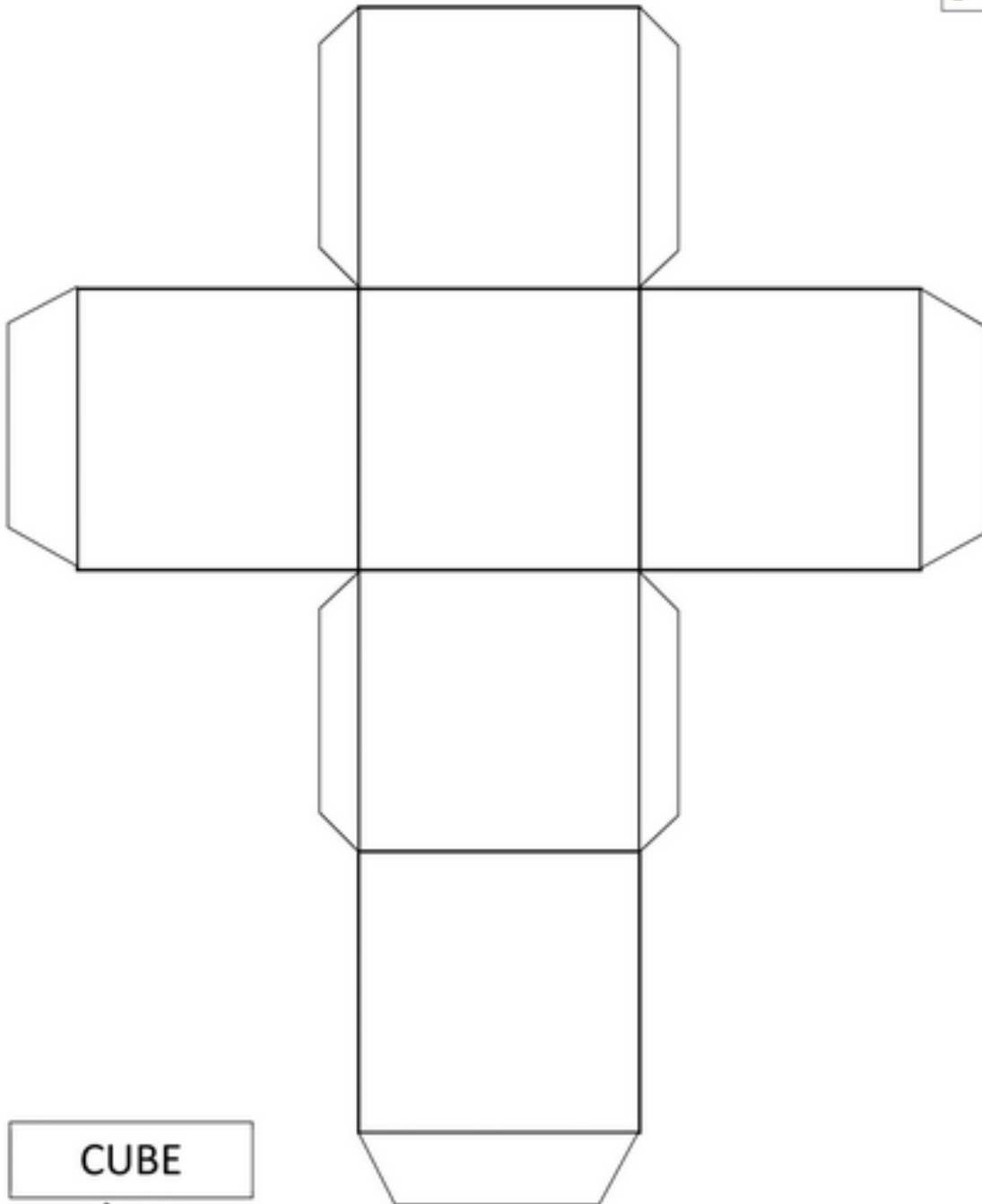
[lowes.com](http://lowes.com)

[metric-conversions.org](http://metric-conversions.org)



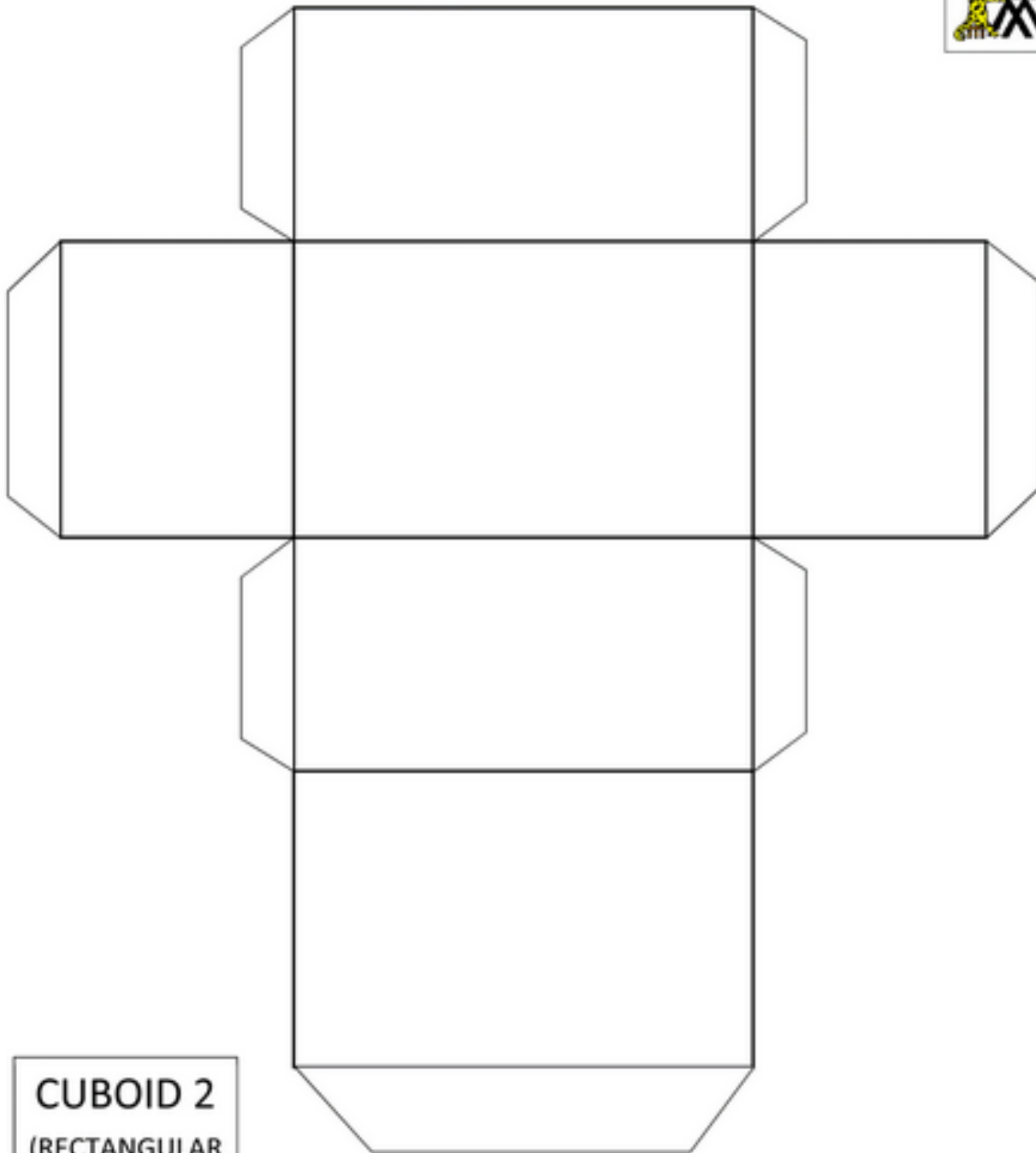
## Geometric Three-Dimensional Figure Nets

These figures are supplied as an example of how students can cut out patterns from their chosen material and fold together sides to construct solids. The supplied figures have tabs to aid in sticking together; however, students may make nets without tabs. Students may use exact shapes for models or copies may be made for enlargement purposes.



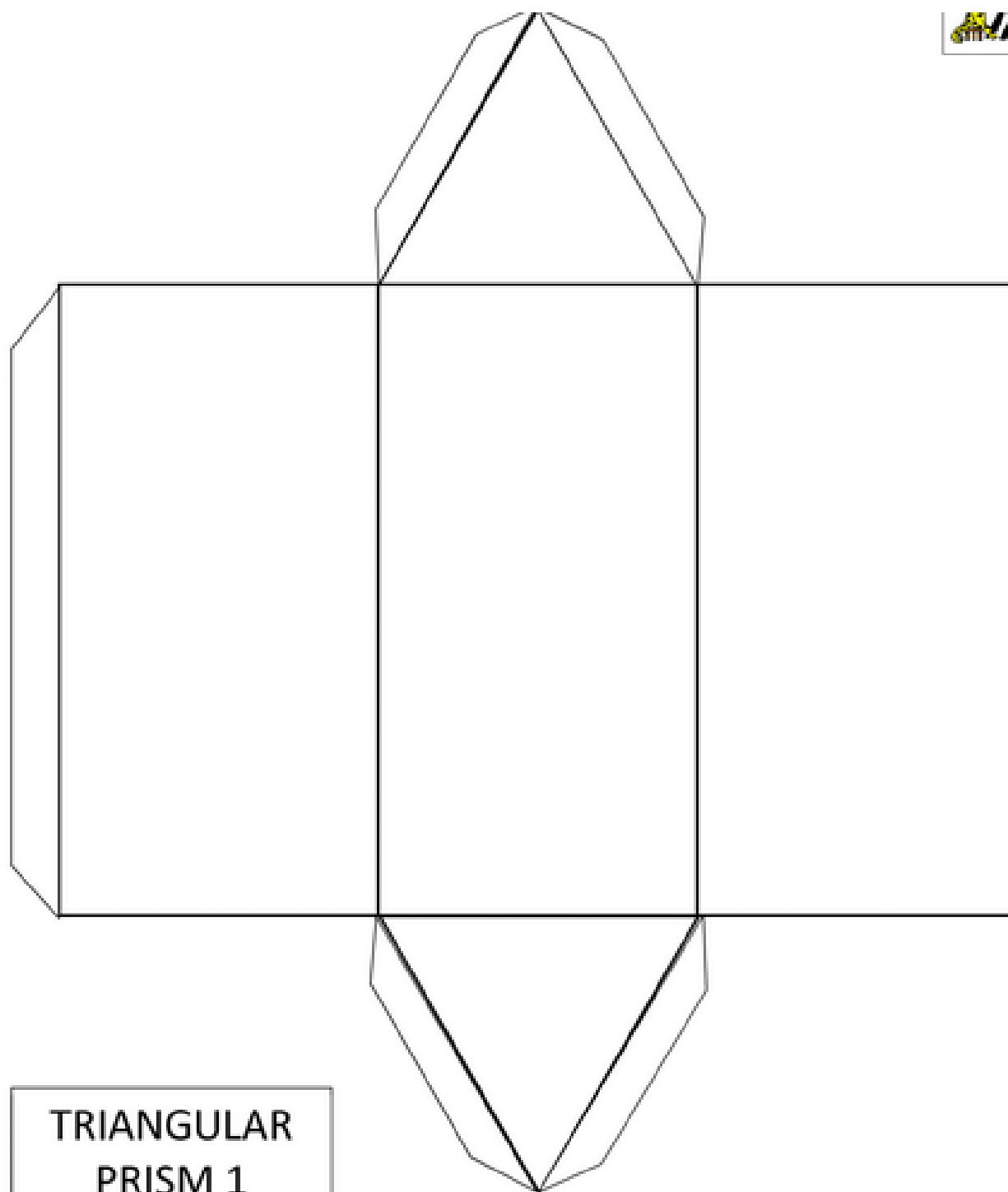
CUBE





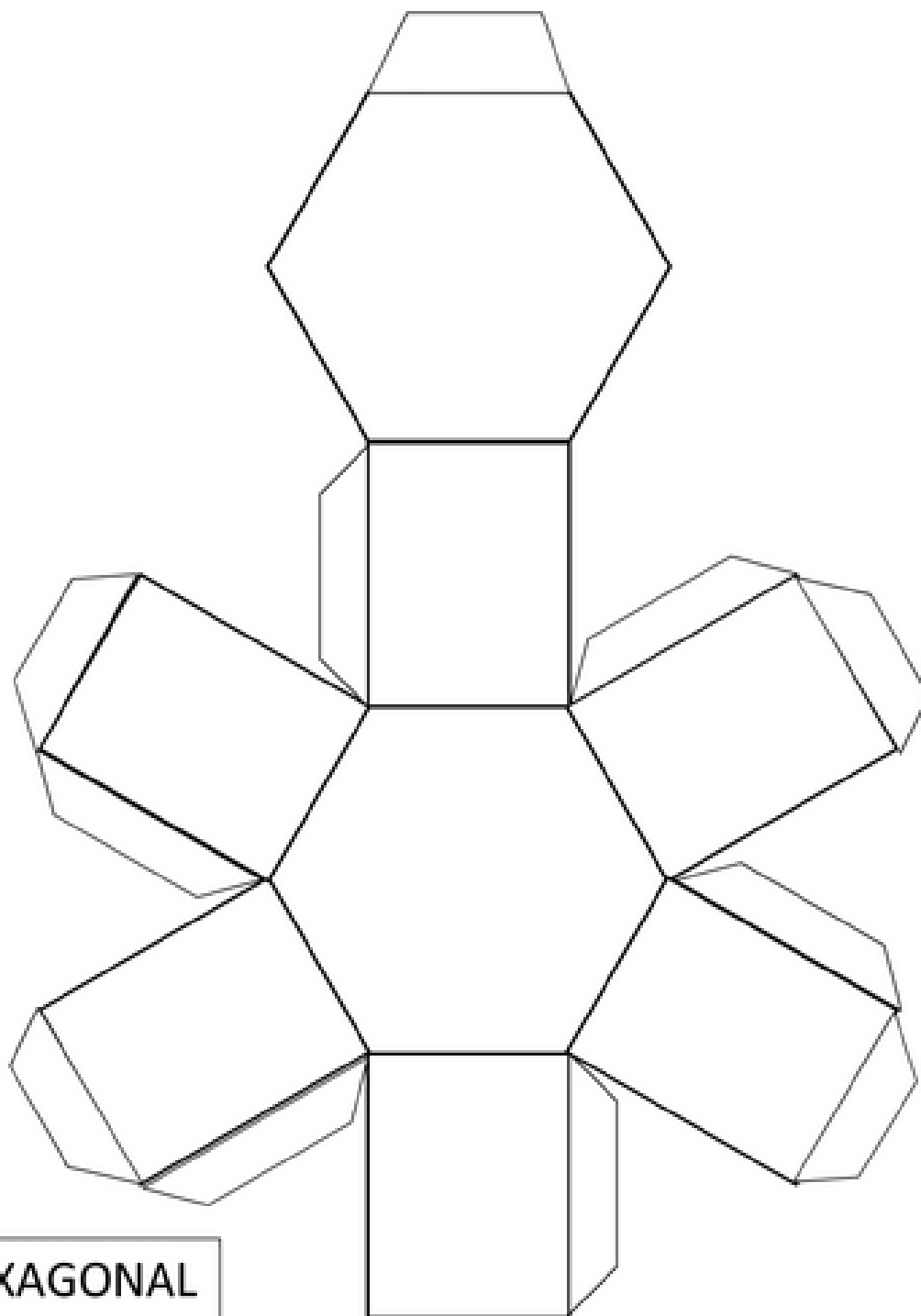
**CUBOID 2**  
(RECTANGULAR  
PRISM)





TRIANGULAR  
PRISM 1



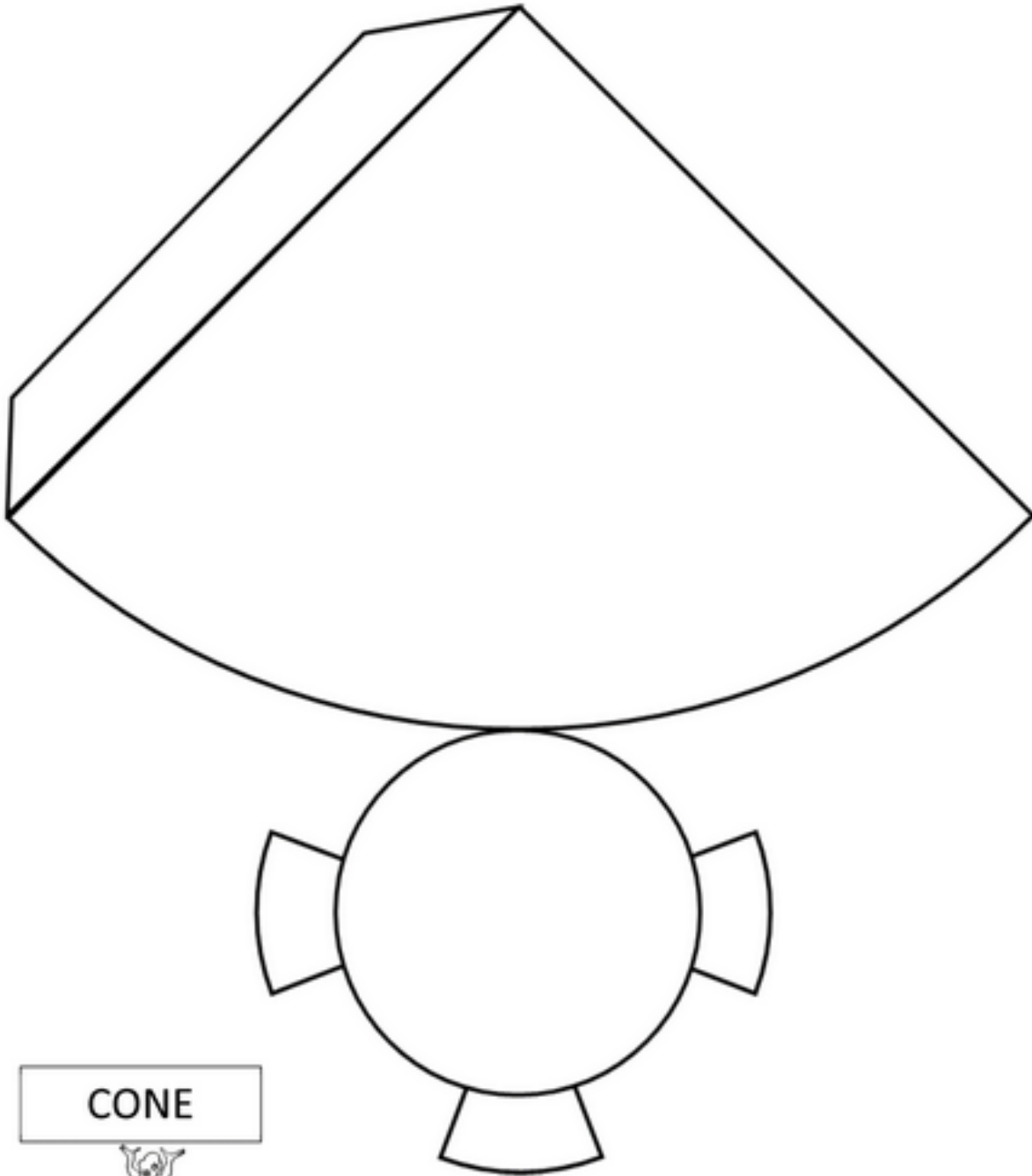


HEXAGONAL  
PRISM



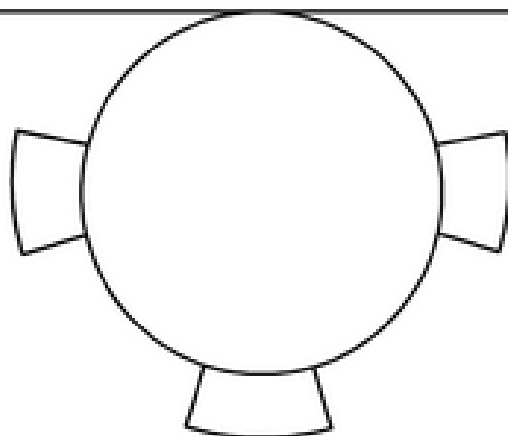
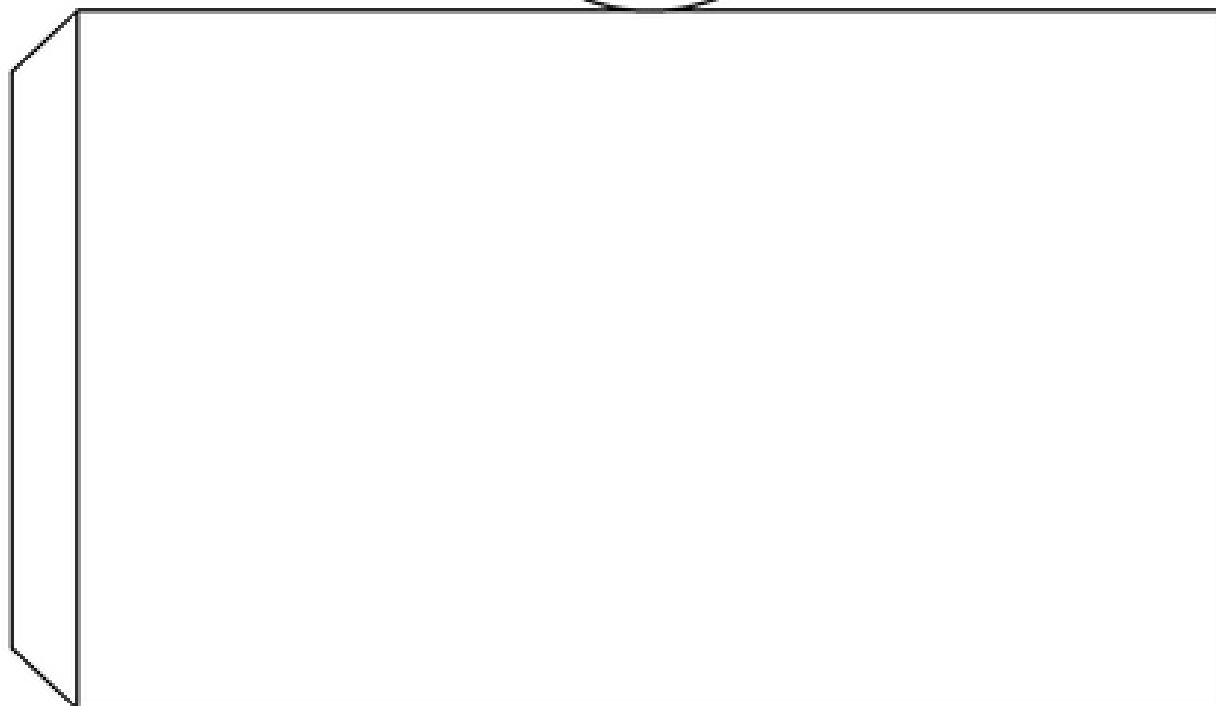
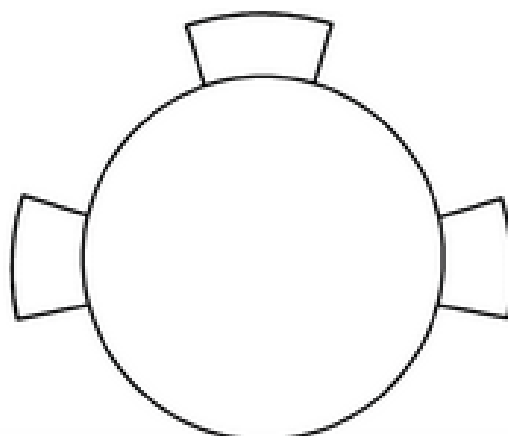
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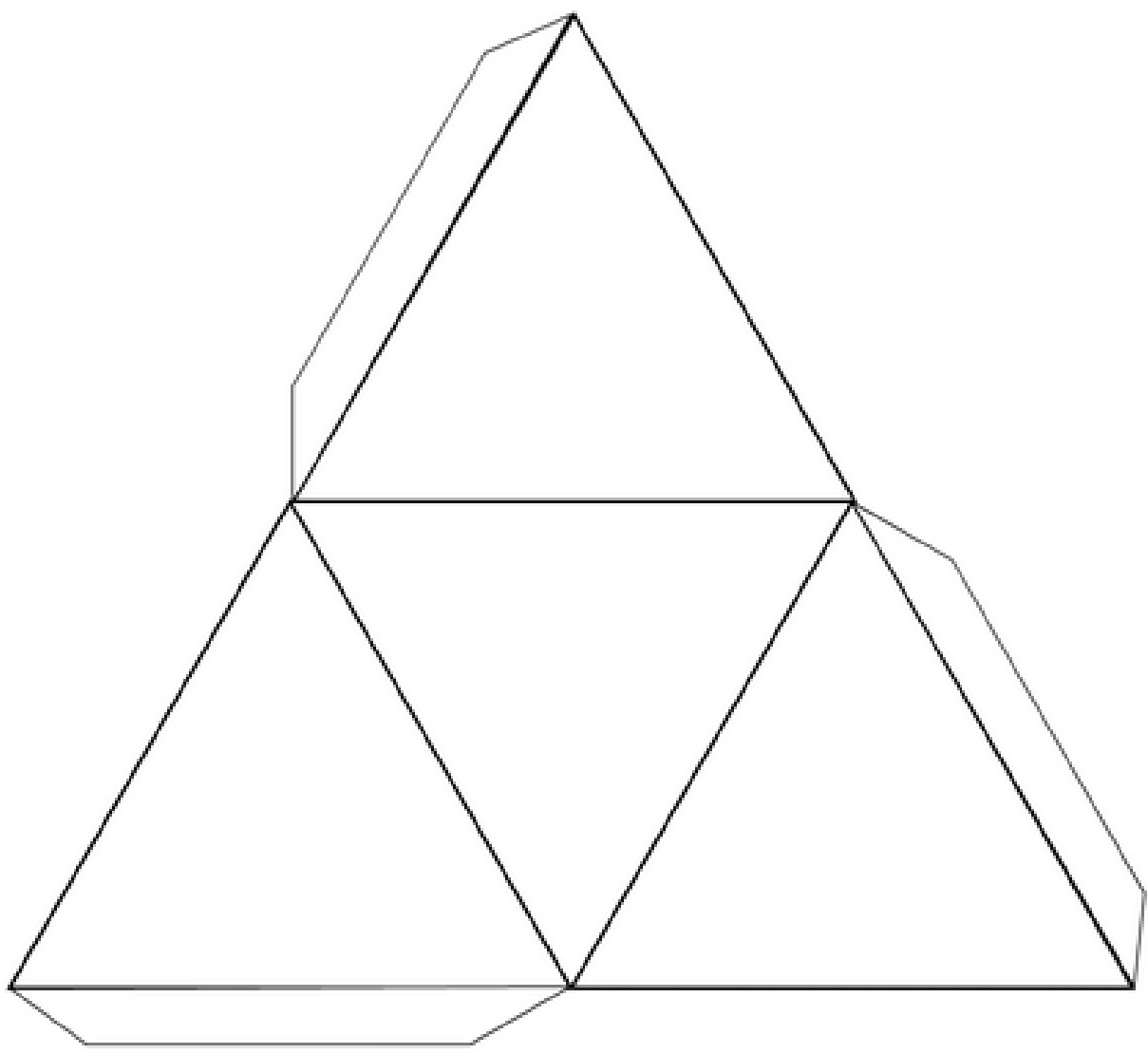
CONE





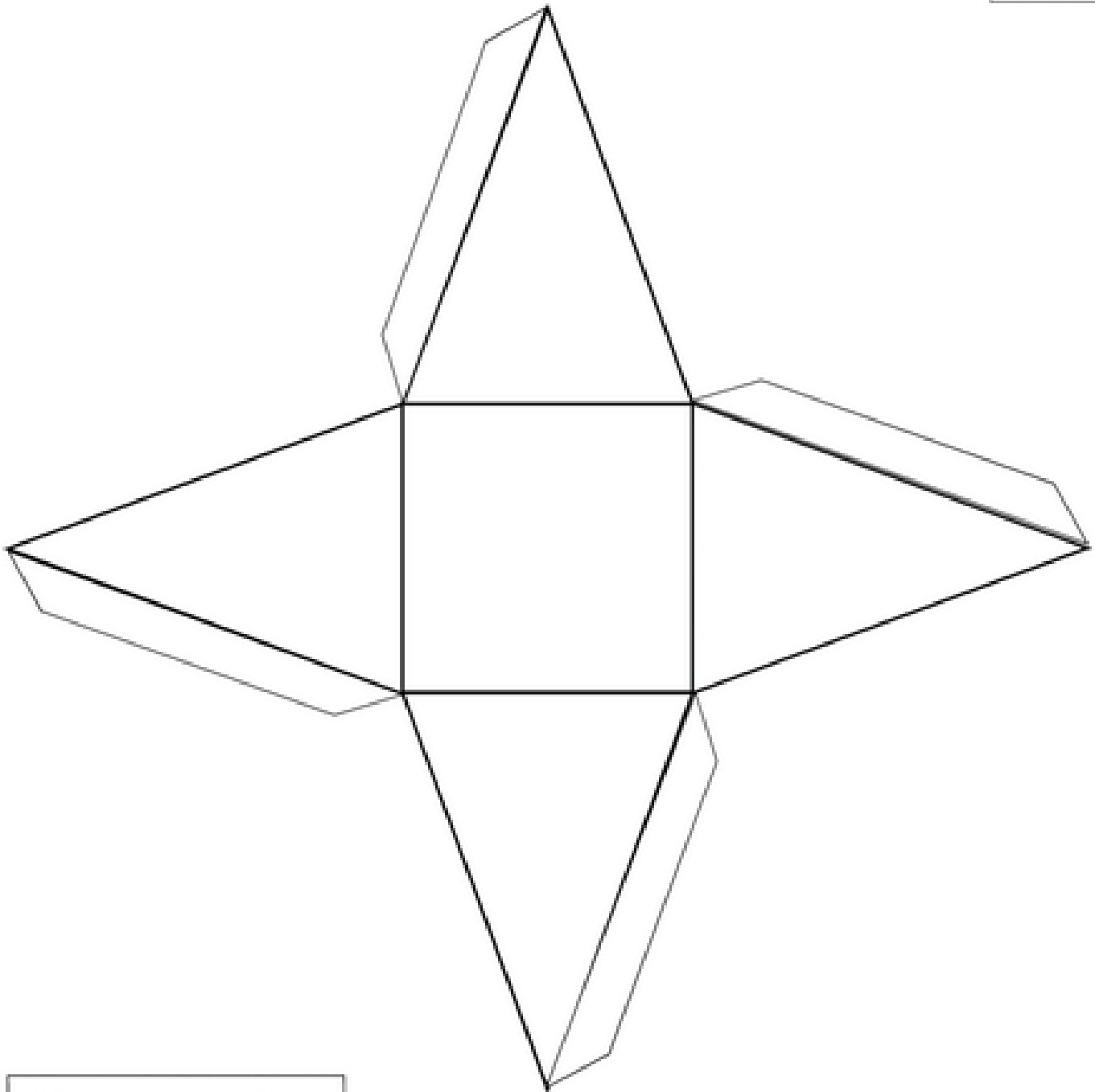
CYLINDER





**TETRAHEDRON**  
(TRIANGULAR  
BASED PYRAMID)

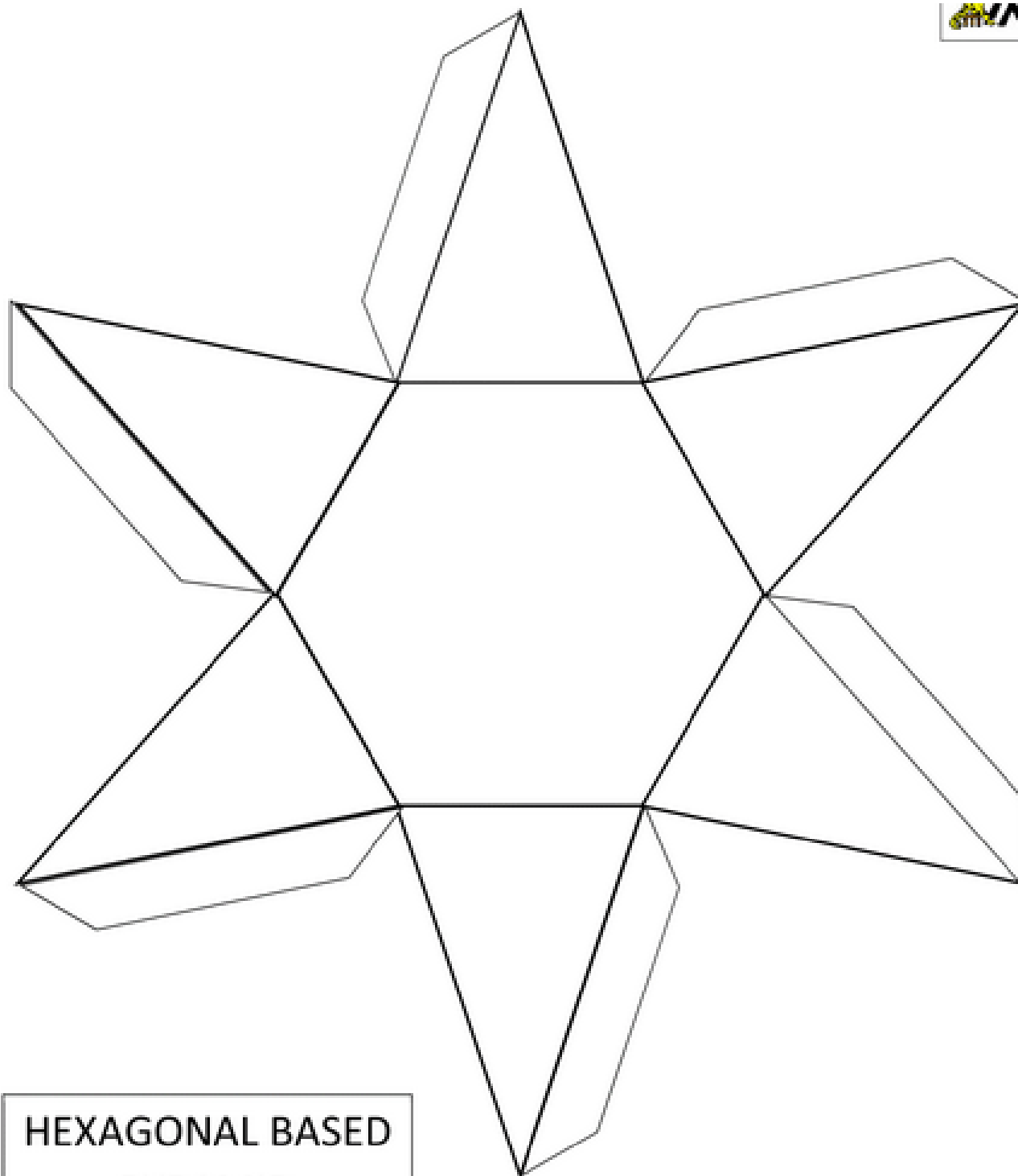




**SQUARE BASED  
PYRAMID 1**







HEXAGONAL BASED  
PYRAMID



## Assessment List and Benchmarks

### Assessment List for Sandcastle Construction and Calculations

|   | Assessment Points |                    |         |
|---|-------------------|--------------------|---------|
|   | Points Possible   | Earned Assessment: |         |
| Element   |                   | Self               | Teacher |
| <b>Part 1: Geometric Figure Details</b>   |                   |                    |         |
| 1. At least four different three-dimensional (3-D) figures are used in creation                                       | 4                 |                    |         |
| 2. Figures are appropriately named and dimensions given for model and actual figure                                   | 4                 |                    |         |
| 3. Surface area formulas and calculations are correct for each figure (model and actual)                              | 4                 |                    |         |
| 4. Surface area solutions and units are correct for each figure (model and actual)                                    | 4                 |                    |         |
| 5. Volume formulas and calculations are correct for each figure (model and actual)                                    | 4                 |                    |         |
| 6. Volume solutions and units are correct for each figure (model and actual)  | 4                 |                    |         |
| 7. Part 1 table is complete, neat and presentable   | 4                 |                    |         |
| <b>Part 1 Total Points</b>  | <b>28</b>         |                    |         |
| <b>Part 2: Two-Dimensional (2-D) Plan</b>   |                   |                    |         |
| 8. 2-D plan includes the placement of at least four unique figures touching and at least one composite figure created | 4                 |                    |         |
| 9. 2-D plan includes a reasonable scale and dimensions for the actual castle, and the medium for sand castle creation | 4                 |                    |         |
| 10. 2-D plan is organized, neat, and detailed   | 4                 |                    |         |

|  |           |  |  |
|--|-----------|--|--|
| <b>Part 2 Total Points</b>   | <b>12</b> |  |  |
| Part 3: Three-Dimensional(3-D) Creation  |           |  |  |
| 11. 3-D model follows 2-D plan and is neat and well-constructed  | 4         |  |  |
| 12. 3-D model includes all requirements, i.e. at least four unique figures and at least one composite figure present | 4         |  |  |
| 13. 3-D model can stand in classroom space and is presentable  | 4         |  |  |
| <b>Part 3 Total Points</b>   | <b>12</b> |  |  |
| Part 4: Analysis Questions   |           |  |  |
| 14. Calculations for total surface area of sand castle is correct  | 4         |  |  |
| 15. Calculations for total volume of sand castle is correct.   | 4         |  |  |
| 16. Appropriate evaluation of change of surface area and volume of one figure when dimensions are doubled            | 4         |  |  |
| 17. Appropriate amount of sand (in pounds) for sand castle construction is determined                                | 4         |  |  |
| 18. Appropriate cost (in dollars) for sand is determined   | 4         |  |  |
| <b>Part 4 Total Points</b>   | <b>20</b> |  |  |

## Task Rubric

| Part 1: Geometric Figure Details  | 4  | 3  | 2   | 1  | 0                  |
|---|--|--|---|--|--------------------|
| 1. At least four different three-dimensional figures are used in creation   | At least four different figures present                            | Three different figures present  | Two different figures present   | One unique figure present  | No evidence shown. |
| 2. Figures are appropriately named and dimensions given for model and actual figure                                   | All figures are named correctly                                    | Most figures named correctly   | Some figures named correctly  | Figures named but none correctly   | No evidence shown. |
| 3. Surface area formulas and calculations are correct for each figure (model and actual)                              | All formulas and work shown are correct.                           | Most formulas and work shown are correct. Few mistakes                 | Some formulas and work shown are correct. Many mistakes.                          | Formulas and work are attempted but none are correct.                              | No evidence shown. |
| 4. Surface area solutions and units are correct for each figure (model and actual)                                    | All solutions and units are correct.                               | Most solutions and units are correct. Few mistakes.                    | Some solutions and units are correct. Many mistakes.                              | Solutions and units are attempted but none are correct.                            | No evidence shown. |
| 5. Volume formulas and calculations are correct for each figure (model and actual)                                    | All formulas and work shown are correct.                           | Most formulas and work shown are correct. Few mistakes                 | Some formulas and work shown are correct. Many mistakes.                          | Formulas and work are attempted but none are correct.                              | No evidence shown. |
| 6. Volume solutions and units are correct for each figure (model and actual)  | All solutions and units are correct.                               | Most solutions and units are correct. Few mistakes.                    | Some solutions and units are correct. Many mistakes.                              | Solutions and units are attempted but none are correct.                            | No evidence shown. |
| 7. Part 1 table is complete, neat and presentable   | Table is complete, neat, and presentable.                          | Table is complete and presentable with few unclear answers.            | Table is not complete with unclear answers and not ready to present.              | Table is not complete and lacks any clarity.                                       | No evidence shown. |
| Part 2: Two-Dimensional (2-D) Plan  | 4  | 3  | 2   | 1  | 0                  |
| 8. 2-D plan includes the placement of at least four unique figures touching and at least one composite figure created | All figures present, including composite figure, all are touching. | All figures present, including composite figure, few are not touching. | Most figures are present, composite figure may be present, some are not touching. | Most figures are not present, including composite figure, no figures are touching. | No evidence shown. |

|   |   |  |   |  |                    |
|---|---|--|---|--|--------------------|
| 9. 2-D plan includes a reasonable scale and dimensions for the actual castle, and the medium for sand castle creation | Scale, dimensions, and medium present and reasonable.                           | Scale, dimensions, and medium are present with few errors.                             | Most of the requirements are present with many errors.                                | Few of the requirements are attempted, no work is correct or reasonable.           | No evidence shown. |
| 10. 2-D plan is organized, neat, and detailed   | Plan is well organized, neat, and detailed.                                     | Plan is organized and clear with some detail.  | Plan is somewhat organized, and most aspects are unclear.                             | Plan exists but is unorganized, messy, and no detail exists.                       | No evidence shown. |
| Part 3: Three-Dimensional (3-D) Creation  | 4   | 3  | 2   | 1  | 0                  |
| 11. 3-D model follows 2-D plan and is neat and well-constructed   | Model follows blueprint plan, is neat and constructed without gaps or overlaps. | Model mostly follows plan, is neat and constructed with few gaps and/or overlaps.      | Model somewhat follows plan and is constructed with many gaps and/or overlaps.        | Model does not follow plan and construction has many gaps and/or overlaps.         | No evidence shown. |
| 12. 3-D model includes all requirements, i.e. at least four unique figures and at least one composite figure present  | All requirements present.   | Most requirements present.   | Some requirements present.  | Model attempted but no requirements met.   | No evidence shown. |
| 13. 3-D model can stand in classroom space and is presentable   | Model is stable and presentable.  | Model is mostly stable and presentable.  | Model is somewhat stable and is somewhat presentable.                                 | Model has no stability or presentation qualities.                                  | No evidence shown. |
| Part 4: Analysis Questions  | 4   | 3  | 2   | 1  | 0                  |
| 14. Calculations for total surface area of sand castle is correct   | All formulas and work shown are correct.  | Most formulas and work shown are correct. Few mistakes                                 | Some formulas and work shown are correct. Many mistakes.                              | Formulas and work are attempted but none are correct.                              | No evidence shown  |
| 15. Calculations for total volume of sand castle is correct.  | All formulas and work shown are correct.  | Most formulas and work shown are correct. Few mistakes                                 | Some formulas and work shown are correct. Many mistakes.                              | Formulas and work are attempted but none are correct.                              | No evidence shown  |
| 16. Appropriate evaluation of change of surface area and volume of one figure when dimensions are doubled             | Correctly evaluates change in dimensions and uses ratios in their description.  | Evaluates a change in dimensions with some error and uses ratios in their description. | Evaluates a change in dimension with many errors with little or no mention of ratios. | Struggles to describe any change in dimension with little or no mention of ratios. | No evidence shown  |

|   |   |  |  |  |                   |
|---|---|--|--|--|-------------------|
| 17. Appropriate amount of sand (in pounds) for sand castle construction is determined | All formulas and work shown are correct: answer is given in pounds. | Most formulas and work shown are correct. Few mistakes: answer is given in pounds. | Some formulas and work shown are correct. Many mistakes: units are not clear or correct. | Formulas and work are attempted but none are correct; units are not mentioned. | No evidence shown |
| 18. Appropriate cost (in dollars) for sand is determined                              | All formulas and work shown are correct.                            | Most formulas and work shown are correct. Few mistakes                             | Some formulas and work shown are correct. Many mistakes.                                 | Formulas and work are attempted but none are correct.                          | No evidence shown |

### Collaboration Rubric

| Area of Focus          | 4  | 3  | 2  | 1  |
|------------------------|--|--|--|--|
| Cooperation            | The student was very open to giving/receiving constructive feedback and provided/received positive feedback as well.   | The student was open to giving/receiving constructive feedback and provided/received positive feedback as well.  | The student was somewhat open to giving/receiving constructive feedback and may not have provided/asked for positive feedback.   | The student was not open to giving/receiving constructive feedback and did not provide/ask for positive feedback.  |
| Member Responsibility  | The student was very clear in communicating during the process. The student was a very active participant during the process. The student was highly respectful of ideas during the process. | The student was clear in communicating during the process. The student was an active participant during the process. The student was respectful of ideas during the process. | The student was somewhat clear in communicating during the process. The student was a somewhat active participant during the process. The student was somewhat respectful of ideas during the process. | The student was not clear in communicating during the process. The student was not an active participant during the process. The student was not respectful of ideas during the process. |
| Mathematical Knowledge | The student demonstrated high general knowledge of the task and was willing to work individually and together to search for understanding and participate in mathematical processes.         | The student demonstrated general knowledge of the task and was willing to work individually and together and participate in mathematical processes.                          | The student demonstrated some knowledge of the task and was somewhat willing to work individually and/or together and somewhat participated in mathematical processes.                                 | The student demonstrated little knowledge of the task and was reluctant to work individually and/or together and did not participate in mathematical processes.                          |

## Benchmark (Student work sample)

### Part 1: Geometric Figure Details

The shapes and equations we use for our sandcastle are as follows:

| Geometric Name of Figure | SA equation   | Volume equation                |
|--------------------------|---|--------------------------------|
| 1. rectangular prism     | $SA = 2lw + 2lh + 2wh$  | $V = lwh$                      |
| 2. semicircular prism    | $SA = \pi r^2 + \pi rh + 2rh$                                 | $V = \pi r^2 h / 2$            |
| 3. hexagonal prism       | $SA = 6sh + 3\sqrt{3}s^2$                                     | $V = \frac{3\sqrt{3}}{2}s^2 h$ |
| 4. hexagonal pyramid     | $SA = \frac{3\sqrt{3}s^2}{2} + 3s\sqrt{h^2 + \frac{3}{4}s^2}$ | $V = \frac{\sqrt{3}}{2}s^2 h$  |

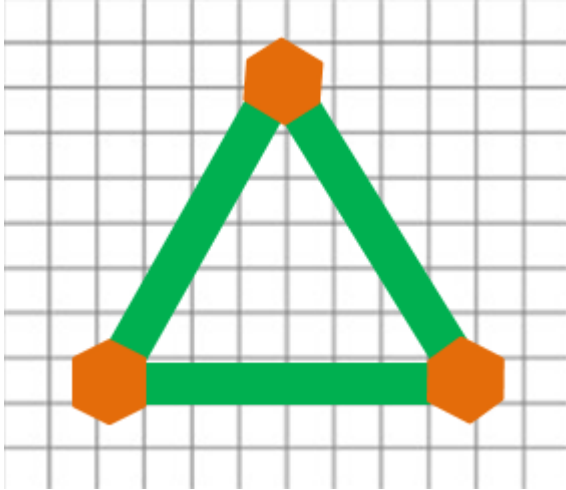
equations from [Google.com](https://www.google.com) and [mathworld.wolfram.com](https://mathworld.wolfram.com)

Areas and volumes, pre-composite, cardboard model listed first (inches), sand sculpture second (feet)

| Geometric Name of Figure | Dimensions of figure (include units for model and actual) | Dimensions for sand        | Surface Area (individual for model and actual)                              | Volume (individual for model and actual)                                   |
|--------------------------|---|----------------------------|---|--|
| 1. rectangular prism     | 2"x6"x12"<br>door: 2"x2"x4"                               | 1'x3'x6'<br>door: 1'x1'x2' | 216in <sup>2</sup> 40in <sup>2</sup><br>54ft <sup>2</sup> 10ft <sup>2</sup> | 144in <sup>3</sup> 16in <sup>3</sup><br>18ft <sup>3</sup> 2ft <sup>3</sup> |
| 2. semicircular prism    | r = 2"<br>h = 2"  | r = 1'<br>h = 1'           | 33.14in <sup>2</sup><br>8.24ft <sup>2</sup>                                 | 12.57in <sup>3</sup><br>1.57ft <sup>3</sup>                                |
| 3. hexagonal prism       | h=12"<br>s=2"   | h=6'<br>s=1'               | 164.78in <sup>2</sup><br>41.2ft <sup>2</sup>                                | 124.71in <sup>3</sup><br>15.59ft <sup>3</sup>                              |
| 4. hexagonal pyramid     | h=4"<br>s=2"  | h=2'<br>s=1'               | 36.55in <sup>2</sup><br>9.14ft <sup>2</sup>                                 | 13.86in <sup>3</sup><br>1.73ft <sup>3</sup>                                |

## Part 2: Two-Dimensional Plan

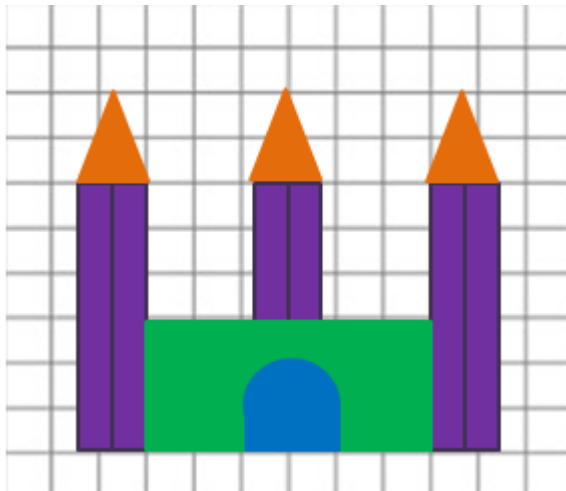
### Top View



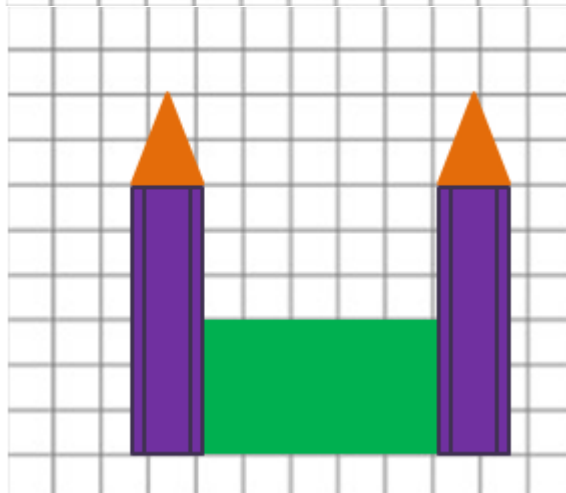
### Legend and Scale

Grid is 2" x 2" for model construction,  
1' x 1' for sand construction  
the scale from our model to actual size is 1 to 6  
Orange: hexagonal pyramid  
Green: rectangular prism  
Purple: hexagonal prism

### Front View



The blue pictured in the front view is the cutaway of the front rectangular prism (entrance), this finished composite shape will resemble an arch



### Side View

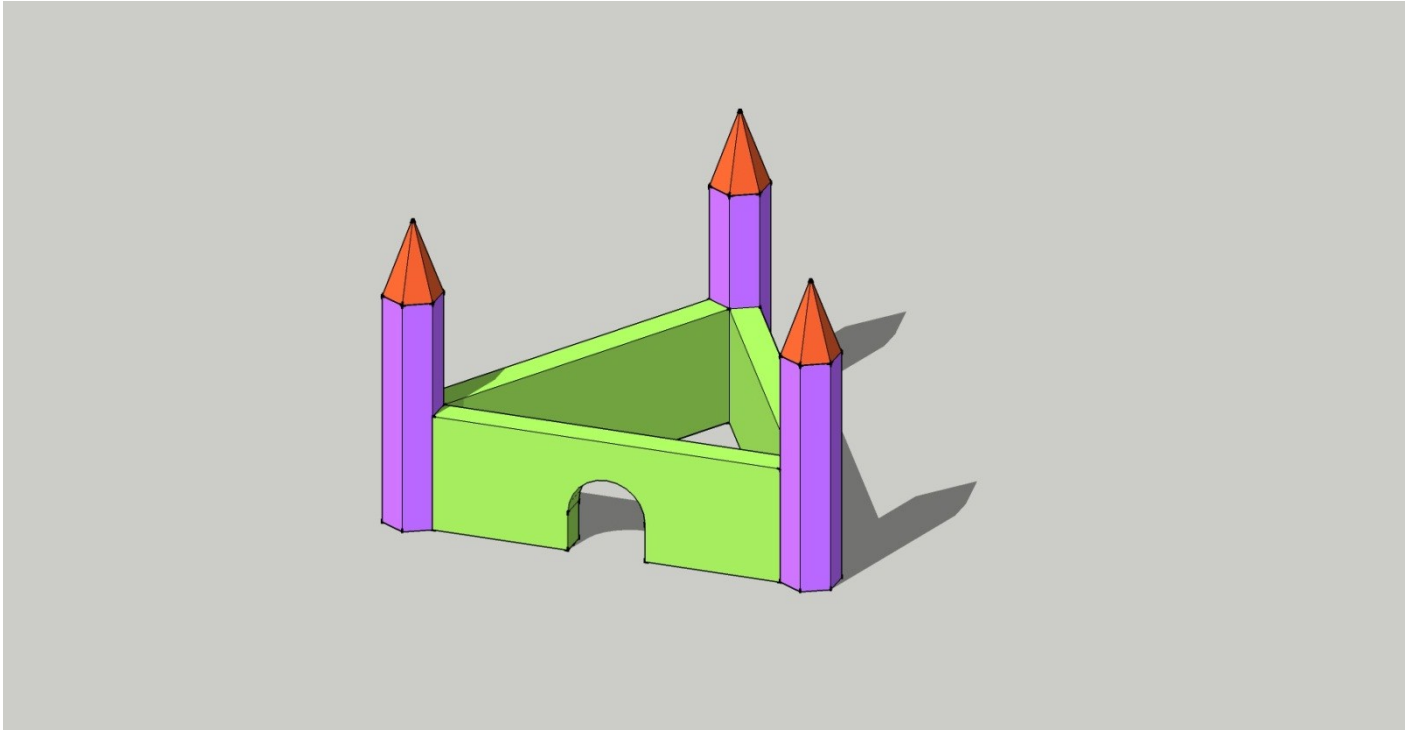
Though the three green rectangular prisms have the same dimensions, the triangular shape of the castle gives this side perspective an oblique view of the rectangular walls. This perspective gives the illusion that this rectangular prism is shorter than the one in the front view.

Medium: a mix of poster board and card stock

(Figures made with Microsoft Word)



Part 3: Three-Dimensional Creation (Figure created using Google Sketchup-computer created model OPTIONAL)



Part 4: Analysis Questions

1) Volume and Surface area of Sandcastle description and calculation

Composite volume calculation summary: add three of each hexagonal shape, three rectangular prisms, subtract one “door” and one semicircular prism

model:  $V = 3*124.71\text{in}^3 + 3*13.86\text{in}^3 + 3*144\text{in}^3 - 16\text{in}^3 - 12.57\text{in}^3$   $V = 819.14\text{in}^3$

sand:  $V = 3*15.59\text{ft}^3 + 3*1.73\text{ft}^3 + 3*18\text{ft}^3 - 2\text{ft}^3 - 1.57\text{ft}^3$   $V = 102.39\text{ft}^3$

Composite surface area calculation summary: no bases (bottom of sand castle need not be painted/constructed)

towers: add lateral areas of three hexagonal prisms, subtract contact area from rectangular prism, add lateral area of three hexagonal pyramids,

walls: add six height(h)\*width(w) and three length(l)\*width(w), subtract two semicircles, subtract two “door” (h)\*(w), add lateral area of semicircle, add two “door” (l)\*(h)

model:

$$\text{towers: } 6 \cdot 3 \cdot 2'' \cdot 12'' - 6 \cdot 2'' \cdot 6'' + 3 \cdot 3 \cdot 2 \sqrt{4^2 + \frac{3}{4} \cdot 2^2} = 438.48 \text{in}^2$$

$$\text{walls: } 6 \cdot 6'' \cdot 12'' + 3 \cdot 2'' \cdot 12'' - 3.14 \cdot 2''^2 - 2 \cdot 2'' \cdot 4'' + 3.14 \cdot 2'' \cdot 2'' + 2 \cdot 2'' \cdot 2'' = 496 \text{in}^2$$

$$\text{total SA} = 934.48 \text{in}^2$$

sand:

$$\text{towers: } 6 \cdot 3 \cdot 1' \cdot 6' - 6 \cdot 1' \cdot 3' + 3 \cdot 3 \cdot 1 \sqrt{2^2 + \frac{3}{4} \cdot 1^2} = 109.62 \text{ft}^2$$

$$\text{walls: } 6 \cdot 3' \cdot 6' + 3 \cdot 1' \cdot 6' - 3.14 \cdot 1'^2 - 2 \cdot 1' \cdot 2' + 3.14 \cdot 1' \cdot 1' + 2 \cdot 1' \cdot 1' = 124 \text{ft}^2$$

$$\text{total SA} = 233.62 \text{ft}^2$$

2) There's some leftover sand up for grabs from your competitors... quick, let's use it! Which shape on your sandcastle makes the most sense to double in size without having to change the other shapes? How does the volume and surface area of this shape change when the dimensions are doubled? Express your answer as a ratio comparing the old and new results (although the total surface area and volume of your sandcastle may change, this question only pertains to one figure).

We chose to double the dimensions of the walls (but not changing the height of the entrance), or more commonly, the rectangular prisms. The new dimensions for our model and sand castle will thus be 4''\*12''\*24'' and 2'\*6'\*12' respectively.

Changes in our calculations: The surface area and volume are both increased with a set ratio, a ratio of the square of our dimension ratio and the cube of the dimension ratio respectively.

|                     | Volume  | Surface Area  |
|---------------------|---|---|
| old value prism     | 144in <sup>3</sup> 18ft <sup>3</sup>  | 216in <sup>3</sup> 54ft <sup>3</sup>  |
| new value prism     | 1152in <sup>3</sup> 72ft <sup>3</sup>   | 864in <sup>3</sup> 432ft <sup>3</sup>   |
| ratio new:old prism | 8:1 (ratio of dimensions is 2:1 thus the ratio of volumes is 2 <sup>3</sup> :1 <sup>3</sup> ) | 4:1 (ratio of dimensions is 2:1 thus the ratio of surface areas is 2 <sup>2</sup> :1 <sup>2</sup> ) |

3. If a 50 pound bag of “play sand” sells for \$3.97 (Lowe’s.com), how much sand will you need to build your actual sandcastle and what will it cost? (Hint: answers should be in pounds and dollars; see Geometry Figure Formula sheet for conversion details).

According to the recommended website, Lowe’s.com, the volume of one bag of play sand is 0.5ft<sup>3</sup>. We would therefore need two bags for every cubic foot of our sand castle’s design. Considering minimum material requirements, we must round up to 103 cubic feet. As each bag costs \$3.97, our calculation for cost of the sand is as follows:

$$\text{Cost} = \$3.97 \cdot 2 \cdot 103 = \$817.82$$