Performance Based Learning and Assessment Task

Triangle Inequality Task

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:
Students will discover the Triangle Inequality Theorem through a performance task

II. UNIT AUTHOR:
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III. COURSE:
Geometry

IV. CONTENT STRAND:
Geometry, Measurement

V. OBJECTIVES:
Students will: 1)Discover that the sum of the lengths of any two sides of a triangle is greater than the length of the third side and identify this as the Triangle Inequality Theorem, 2)Determine whether three given side lengths will form a triangle and explain why it will or will not work, 3)Develop a method for finding all possible side lengths for the third side of a triangle when two side lengths are given

VI. REFERENCE/RESOURCE MATERIALS:
"Triangle Inequality Exploration" handout, pipe cleaners, "Yes/No" cards, "Triangle Inequality Performance Based Task" handout, VA map, ruler, Smart Board

VII. PRIMARY ASSESSMENT STRATEGIES:
Student understanding will be assessed through the use of the Performance-Based Task. This task requires students to apply their knowledge of the Triangle Inequality Theorem in the context of a real-world application (distances between cities on a map).

VIII. EVALUATION CRITERIA:
Students will be assessed using a performance based task rubric on the accuracy and neatness of their work

IX. INSTRUCTIONAL TIME:
Two forty-five-minute periods or one ninety-minute block

Activity/Task 1 Title

Strand
Geometry/Measurement

Mathematical Objectives
Students will discover that the sum of the lengths of any two sides of a triangle is greater than the length of the third side and identify this as the Triangle Inequality Theorem. Using the theorem, they will be able to determine whether three given side lengths will form a triangle and explain why it will or will not work. Through the Performance-Based Task, students will also be asked to come up with a method for finding all possible side lengths for the third side of a triangle when two side lengths are given.

Related SOL
G.5c (determine whether a triangle exists)
G.5d (determine the range in which the length of the third side must lie)

NCTM Standards
- Analyze properties and determine attributes of two- and three-dimensional objects.
- Make decisions about units and scales that are appropriate for problem situations involving measurement.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Materials/Resources
Students will use: "Triangle Inequality Exploration" handout, pipe cleaners of various lengths, "Yes" and "No" cards, "Triangle Inequality Performance-Based Task" handout, map of Virginia, rulers, Smart Board

Assumption of Prior Knowledge
Students will know that a triangle is a polygon with three sides. After completing the Triangle Inequality Exploration, students will begin to express the idea that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. Students might not understand immediately that a triangle cannot be formed if the sum of the lengths of two sides is equal to the length of the third side. When given the lengths of two sides of a triangle (as in Question #4 on the Performance-Based Task), students may understand how to find the upper bound for the length of the third side, but finding the lower bound may not be clear at first.

Introduction: Setting Up the Mathematical Task

In this activity, students will investigate the relationship between the lengths of the sides of a triangle. The teacher will start by asking students, "If we are given any three side lengths, can we always form a triangle with them?" If students think the answer is "yes," the teacher will have them try to sketch three segments on a sheet of paper that do not form a triangle. Students will then be grouped in pairs and given a set of pipe cleaners of various lengths (2, 3, 4, 5, 7, and 8 inches). With their partner, students will complete the "Triangle Inequality Exploration," in which they will try to form triangles with different combinations of pipe cleaners and record their results. (Note: An alternate exploration for more advanced students is provided.)

Student Exploration

Student/Teacher Actions

During the "Triangle Inequality Exploration," students will record the results they come up with in the chart and begin answering the questions. In the process, they should create a rule that will allow them to determine if any set of measurements will form a triangle. The teacher will circulate around the room during this activity, stopping in on each group to make sure students understand the task and to pose questions to guide student thinking. Students will most likely be able to create a similar statement for the measurements that did not form a triangle; however, they may have trouble with the cases in which the length of the small plus medium sides is equal to the length of the large side. When working with these measurements, students may mistakenly believe that they will form a triangle, when in fact they do not. The teacher can help students by showing them, for example, that the 2-inch and 3-inch pipe cleaners will "collapse" onto the 5-inch pipe cleaner since they are the same length. To challenge more advanced students, the teacher can pose the following question: "Only one combination of measurements will form a right triangle. Which one is it? Explain your reasoning."

Throughout the exploration, students will communicate with their partner as they determine which side lengths will form triangles and develop a rule. When all the groups are finished, the teacher will lead a whole-class discussion that will give each pair a chance to share what they discovered and give the teacher an opportunity to clarify any misconceptions that still exist. A formal statement of the Triangle Inequality Theorem will be developed. Students will then move back to their seats. A series of measurements will be displayed on the Smart Board, and each student will hold up a "yes" or "no" card to indicate whether or not each set of measurements will form a triangle. If there are any discrepancies, the teacher can have a student on each side explain their thinking so that the whole class can come to an agreement. After this activity, students will be asked to summarize the rule with a partner and a pair will be called on to share with the class.

Assessment List and Benchmarks

The task and the corresponding assessment list are included in this document. A map of Virginia, which is required for Questions 2 and 3, is included as a separate file. A rubric containing evaluation criteria is included, along with a benchmark of exemplary student work.

Triangle Inequality Exploration

1. With your partner, try to form triangles with each of the given measurements. Complete the table for each set of measurements.

Small	Medium	Large	Does it form a triangle? Y/N
2 in.	3 in.	4 in.	
2 in.	3 in.	5 in.	
2 in.	3 in.	7 in.	
2 in.	3 in.	8 in.	
2 in.	4 in.	5 in.	
2 in.	4 in.	7 in.	
2 in.	4 in.	8 in.	
2 in.	5 in.	7 in.	
2 in.	5 in.	8 in.	
2 in.	7 in.	8 in.	
3 in.	4 in.	5 in.	
3 in.	4 in.	7 in.	
3 in.	4 in.	8 in.	
3 in.	5 in.	7 in.	
3 in.	5 in.	8 in.	
3 in.	7 in.	8 in.	
4 in.	5 in.	7 in.	
4 in.	5 in.	8 in.	
4 in.	7 in.	8 in.	
5 in.	7 in.	8 in.	

2. Look at the measurements that **did** form a triangle. What do you notice?

3. Look at the measurements that **did not** form a triangle. What do you notice?

4. Write a rule that would allow you to determine if any set of measurements will form a triangle.

5. Is it possible to have a triangle such that the sum of the measures of the small and medium sides is equal to the measure of the large side? Explain.

Triangle Inequality Exploration
(Alternate Activity for Advanced Students)

1. With your partner, come up with all possible combinations of measurements using your pipe cleaners. Try to form triangles with each set of measurements. Complete the table as you go.

Small	Medium	Large	Does it form a triangle? Y/N

2. Look at the measurements that **did** form a triangle. What do you notice?

3. Look at the measurements that **did not** form a triangle. What do you notice?

4. Write a rule that would allow you to determine if any set of measurements will form a triangle.

5. Is it possible to have a triangle such that the sum of the measures of the small and medium sides is equal to the measure of the large side? Explain.

Triangle Inequality Exploration – Sample Solution

1. With your partner, try to form triangles with each of the given measurements. Complete the table for each set of measurements.

Small	Medium	Large	Does it form a triangle? Y/N
2 in.	3 in.	4 in.	**Y**
2 in.	3 in.	5 in.	**N**
2 in.	3 in.	7 in.	**N**
2 in.	3 in.	8 in.	**N**
2 in.	4 in.	5 in.	**Y**
2 in.	4 in.	7 in.	**N**
2 in.	4 in.	8 in.	**N**
2 in.	5 in.	7 in.	**N**
2 in.	5 in.	8 in.	**N**
2 in.	7 in.	8 in.	**Y**
3 in.	4 in.	5 in.	**Y**
3 in.	4 in.	7 in.	**N**
3 in.	4 in.	8 in.	**N**
3 in.	5 in.	7 in.	**Y**
3 in.	5 in.	8 in.	**N**
3 in.	7 in.	8 in.	**Y**
4 in.	5 in.	7 in.	**Y**
4 in.	5 in.	8 in.	**Y**
4 in.	7 in.	8 in.	**Y**
5 in.	7 in.	8 in.	**Y**

2. Look at the measurements that **did** form a triangle. What do you notice?
The sum of the "small" and "medium" measurements is greater than the "large" measurement.

3. Look at the measurements that **did not** form a triangle. What do you notice?
The sum of the "small" and "medium" measurements is less than or equal to the "large" measurement.

4. Write a rule that would allow you to determine if any set of measurements will form a triangle.
If the length of the small plus medium sides is greater than the length of the large side, it will form a triangle. If the length of the small plus medium sides is less than or equal to the length of the large side, it will not form a triangle.

5. Is it possible to have a triangle such that the sum of the measures of the small and medium sides is equal to the measure of the large side? Explain.
No, it is not possible. The following sets of measurements illustrate this:
2, 3, 5 2, 5, 7 3, 4, 7 3, 5, 8
Visually, the small and medium sides will "collapse" onto the large side.