Performance Based Learning and Assessment Task

Pizza Sector Task

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:
Students will be introduced to the concept of a sector of a circle and will learn the formula used to calculate the area of a sector.

II. UNIT AUTHOR:
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III. COURSE:
Geometry

IV. CONTENT STRAND:
Measurement

V. OBJECTIVES:
Students will:
1)Explore sectors of circles
2)Understand how the formula for the area of a sector of a circle is derived
3)Use the formula to find sector areas, angle measures, and radii/diameters of circles in the context of real-world applications

VI. REFERENCE/RESOURCE MATERIALS:
Students will use: Smart Board, compass, ruler, individual whiteboard, markers, "Sector Area Performance-Based Task

VII. PRIMARY ASSESSMENT STRATEGIES:
Student understanding will be assessed through the use of the Performance-Based Task. This task requires students to apply their knowledge of sector area in the context of a real-world application (slices of pizza).

VIII. EVALUATION CRITERIA:
Students will be assessed on accuracy, neatness, and completion of task (see scoring rubric)

IX. INSTRUCTIONAL TIME:
Two forty-five minute periods or one ninety-minute period.

Pizza Sector Task

Strand
Measurement

Mathematical Objectives:
Students will explore sectors of circles. They will understand how the formula for the area of a sector of a circle is derived from the formula for the area of a circle. Students will be able to use the formula to find sector areas, angle measures, and radii/diameters of circles in the context of real-world applications.

Related SOL:
G.11c (find arc lengths and areas of sectors in circles)

NCTM Standards:
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Materials/Resources:
Smart Board, compass, ruler, individual whiteboard, markers, "Sector Area Performance-Based Task"

Assumption of Prior Knowledge:
In order to be successful with this lesson, students should already know the definitions of a circle, a radius, a diameter, a central angle, and an arc. They should know to find the area of a circle. Students should also know that a circle has 360 degrees. During the lesson, students will begin to formulate a connection between a sector of a circle and the entire circle and how the sector area formula is related to the circle area formula. Since many students struggle with fractions, they may struggle with the concept of fractional area. They may also have difficulty working backwards from the formula when asked to find the angle measure or the radius.

Introduction:Setting Up the Mathematical Task
In this lesson, students will investigate the concept of the sector of a circle and discover the formula for the area of a sector. As an introduction, students will be given a compass and ruler and instructed to draw a circle of any size on a piece of paper. They will be asked to measure the radius of their circle and calculate the area.

Student Exploration
Student/Teacher Actions
Students will be instructed to divide their circle in half and find the area of just one half, then divide the circle into fourths and find the area of just one fourth. The teacher will ask a student for his or her results to show to the whole class on the board. Even though students' answers

will vary, we will discuss how they all probably used the same process to come up with the areas. A formal definition of a sector of a circle will be presented (A sector of a circle is a region bounded by an arc of the circle and the two radii to the arc's endpoints.) The teacher will get students thinking about angle measure by asking the following questions:

"How many degrees are in a whole circle?" (360)
"When you divided your circle in half, what was the degree measure of the angles you formed?" (180)
"What fraction of the total degree measure is 180 degrees?" (1/2)
"How does this relate to the areas you found?" (The area of half the circle is exactly half the total area.)
"When you divided your circle in fourths, what was the degree measure of the angles you formed?" (90)
"What fraction of the total degree measure is 90 degrees?" (1/4)
"How does this relate to the areas you found?" (The area of a fourth of the circle is exactly one-fourth of the total area.)

The teacher will then draw a sector formed by a 50 degree angle in a circle of radius 5 inches and ask students, "What if we wanted to find the area of this sector?"

Hopefully, students will make the connection that the area of the sector would be $\frac{50}{360} \cdot \pi(5)^2$, or about 10.9 square inches. If not, the teacher can guide the students by writing out explicit formulas for the previous examples and helping them discover the pattern. The formal sector area formula ($A = \frac{x}{360} \cdot \pi r^2$, where *x* is the measure of the central angle and *r* is the radius of the circle) can then be established.

Small Group Work
Students will be paired up and each pair will be given a whiteboard and a marker. As pictures of circles with shaded sectors are displayed on the board, students will work with their partner to find the area of each sector and write their answer on the whiteboard. This will give the teacher immediate feedback as to which students understand the concept and will allow him or her to work with those students who are still struggling. Student pairs will be called on to share their reasoning with the class. A few problems will also be presented which require students to find the measure of the angle or the length of the radius/diameter. Since many students may struggle with "reversing" the formula in this way, it may be necessary for the teacher to work through examples of each type with the class before asking them to try these on their own.

When it is clear that students have grasped the concepts, they will be asked to summarize with their partner the definition of a sector of a circle and the procedure they would use to find a sector area, the measure of a central angle, or the radius/diameter of the circle. Pairs will be called on to share their ideas with the class.

Assessment List and Benchmarks

Included are: the performance task handout with instructions and questions, performance-based assessment rubric, and benchmark

Pizza Sector Task

1. A 12-inch pizza is cut into 8 slices. You eat one slice of pizza. How many square inches of pizza did you eat? Show your work. Round your answer to the nearest tenth.

2. A pizza shop sells pizza by the slice. They make two different sizes of pizza – a 14-inch pizza which they cut into 60-degree slices and a 16-inch pizza which they cut into 45-degree slices. They sell a slice of each one for the same price. Which is a better deal? Show your work.

3. You eat 2 slices of a 14-inch pizza which total 34 square inches. At what angle were the pizza slices cut? Show your work. Round your answer to the nearest whole number.

4. You are making a pizza for 10 friends. You want each friend to have a 25-square-inch slice of pizza. How big of a pizza do you need to make? Show your work. Round your answer to the nearest whole number.

<u>Extension</u>

A pizza shop specializes in deep-dish pizzas. They sell a 14-inch pizza that is 3 inches deep and a 16-inch pizza that is 2 inches deep. The 14-inch pizza is cut into 8 slices and the 16-inch pizza is cut into 7 slices, and each slice is sold for the same price. If you are purchasing one slice of pizza, which size should you choose to get the most food?

Rubric

			Earned Assessment	
Number	Element	Point Value	Self	Teacher
1	Question 1 – The correct answer is provided.	2		
2	Question 1 – Work is shown.	2		
3	Question 2 – The correct answer is provided.	2		
4	Question 2 – Work is shown and an explanation is provided.	2		
5	Question 3 – The correct answer is provided.	2		
6	Question 3 – Work is shown.	2		
7	Question 4 – The correct answer is provided.	2		
8	Question 4 – Work is shown.	2		
9	The work is neat and presentable.	2		

Rubric

Number	Element	0	1	2
1	Question 1 – The correct answer is provided.	No answer is provided.	An incorrect answer is provided.	The correct answer is provided.
2	Question 1 – Work is shown.	No work is shown.	Work is incomplete or incorrect.	Work is complete and correct.
3	Question 2 – The correct answer is provided.	No answer is provided.	An incorrect answer is provided.	The correct answer is provided.
4	Question 2 – Work is shown.	No work is shown.	Work is incomplete or incorrect.	Work is complete and correct.
5	Question 3 – The correct answer is provided.	No answer is provided.	An incorrect answer is provided.	The correct answer is provided.
6	Question 3 – Work is shown.	No work is shown.	Work is incomplete or incorrect.	Work is complete and correct.
7	Question 4 – The correct answer is provided.	No answer is provided.	An incorrect answer is provided.	The correct answer is provided.
8	Question 4 – Work is shown.	No work is shown.	Work is incomplete or incorrect.	Work is complete and correct.
9	The work is neat and presentable.	The work lacks neatness.	The work needs improvement.	The work is neat and presentable.

Benchmark

1. A 12-inch pizza is cut into 8 slices. You eat one slice of pizza. How many square inches of pizza did you eat? Show your work. Round your answer to the nearest tenth.

$$A = \frac{1}{8} \cdot \pi(6)^2$$
$$A \approx 14.1$$

You ate about 14.1 square inches of pizza.

2. A pizza shop sells pizza by the slice. They make two different sizes of pizza – a 14-inch pizza which they cut into 60-degree slices and a 16-inch pizza which they cut into 45-degree slices. They sell a slice of each one for the same price. Which is a better deal? Show your work.

14-inch pizza:

$$A = \frac{60}{360} \cdot \pi(7)^2$$
$$A \approx 25.7$$

16-inch pizza:

$$A = \frac{45}{360} \cdot \pi(8)^2$$
$$A \approx 25.1$$

Buying a slice of the 14-inch pizza is a better deal than buying a slice of the 16-inch pizza.

3. You eat 2 slices of a 14-inch pizza which total 34 square inches. At what angle were the pizza slices cut? Show your work. Round your answer to the nearest whole number.

$$34 = \frac{x}{360} \cdot \pi(7)^2$$
$$\frac{34}{49\pi} = \frac{x}{360}$$
$$\frac{34}{49\pi} \cdot 360 = x$$
$$79.51251034 = x$$

$$\frac{79.51251034}{2} \approx 40$$

The pizza slices were cut at approximately a 40-degree angle.

4. You are making a pizza for 10 friends. You want each friend to have a 25-square-inch slice of pizza. How big of a pizza do you need to make? Show your work. Round your answer to the nearest whole number.

$$25 = \frac{1}{10} \cdot \pi r^2$$
$$250 = \pi r^2$$
$$\frac{250}{\pi} = r^2$$
$$8.920620581 = r$$

$$d = 2(8.920620581) \approx 18$$

You need to make an 18-inch pizza.

Extension

A pizza shop specializes in deep-dish pizzas. They sell a 14-inch pizza that is 3 inches deep and a 16-inch pizza that is 2 inches deep. The 14-inch pizza is cut into 8 slices and the 16-inch pizza is cut into 7 slices, and each slice is sold for the same price. If you are purchasing one slice of pizza, which size should you choose to get the most food?

14-inch pizza:

$$\boldsymbol{V = \frac{1}{8} \cdot \pi(7)^2(3)}$$
$$\boldsymbol{V \approx 57.7}$$

16-inch pizza:

$$\boldsymbol{V = \frac{1}{7} \cdot \pi(8)^2(2)}$$
$$\boldsymbol{V \approx 57.4}$$

You should choose a slice of the 14-inch pizza to get the most food.