B.2 Rotation of Mercury

I. Introduction

Because Mercury is a relatively small planet with very few large surface features and since it is always near the Sun in the sky, it is very difficult to determine the rotation rate of Mercury by direct optical observation. In recent years though, astronomers have used radar techniques to measure the rotation rate of both Mercury and Venus.

The basic principle is to send a pulse of radar at Mercury. Depending on its position relative to the Earth it will take approximately 10 minutes to a half hour to reach Mercury, reflect off the surface, and return to Earth. Because the pulse spreads out as it travels (just as the waves from a pebble dropped into a pond spread), the pulse hits the whole surface of the planet. The first point to reflect the pulse is called the sub-radar point. See Figure B.4 below. The pulse then continues to travel past the surface of Mercury, reflecting back at very small time intervals (microseconds, or millionths of a second) after the sub-radar point. As we sample the reflected wave with our detector, we can obtain information about different points on the surface of Mercury.

The rotation rate can be discovered due to the Doppler effect. Recall that the Doppler effect is the change in frequency (or wavelength) caused by any relative radial motion between the source and the observer. Motion decreasing the distance between the source and the observer results in a shortening (or bluing) of the wavelength - this is known as a blue shift because the wavelength is getting shorter and blue is the shortest portion of visible light- this is equivalent to an increase in the frequency. Motion increasing the distance between the source and observer causes an increase (or reddening) of the wavelength and this is known as a red shift - this is equivalent to decreasing the frequency. Since one edge is moving towards us, that edge will show a redshift relative to the planet as a whole while the opposite edge shows a blueshift. Again, see Figure B.4.

![Figure B.4: Doppler shift from various portions of a rotating object.](image-url)
As noted above there are two motions which will result in Doppler shifts of our radar pulse - the orbital velocity of the planet and rotational velocity of the planet. The orbital velocity can be determined by looking at the reflection from the sub-radar point. At the sub-radar point, the rotational velocity, \( v_r \), will not cause a shift because it is perpendicular to the line of sight to this point so any shift seen in the reflected pulse from the sub-radar point is due solely to the orbital motion. As additional reflected pulses are examined larger areas of the planet are reflecting pulses which are away from the sub-radar point so that there will be both a redshift and blueshift in the returned signal. See Figure B.5 for an example.

![Frequency of reflected pulse](image)

Figure B.5: Example of a reflected radar pulse signal.

In this lab you will simulate an actual radar observation of Mercury using a computer program. Using the program allows us to accurately simulate the radar observations a modern astronomer would make. The program will accumulate a series of 5 different pulse returns - the sub-radar reflected signal in addition to 4 others spaced shortly after the return of the sub-radar signal. From these signals you will be able to calculate the rotational velocity of Mercury and from that calculate its rotational period. Using the sub-radar signal you will be able to calculate the orbital velocity of Mercury and from that determine its orbital period.

II. Reference
- *The Cosmic Perspective*, p. 167

III. Materials Used
- CLEA Radar Measurement of the Rotation of Mercury program
- calculator

IV. Observations
The observations you are making will be simulated using the CLEA program. You will do the following things in this observation:
- calculate the position of Mercury and point a radio telescope at it
- send a radar pulse at Mercury
• calculate various geometrical patterns necessary to interpret the data
• measure the shift in frequency of a radar pulse reflected off of Mercury and from that calculate Mercury’s rotational velocity and period
• measure the shift in frequency of a radar pulse reflected off of Mercury from the sub-radar point and from that calculate the orbital velocity and period of Mercury.

Taking Data

1. Log in to the program by entering all the group members’ names into the appropriate places after selecting Log In from the File menu.

2. Select Start. Press the Tracking button so that the telescope will track Mercury.

3. Select Ephemeris from the main menu; this will start a process to calculate the position of Mercury. Enter your group’s time and date into the appropriate boxes, then press OK to compute the position of Mercury. Leave the window with the computed position on the screen and select the Set Coordinates button.

4. Once you okay the coordinates to the telescope, it will begin to slew (move rapidly) to those coordinates. Once slewing is complete, you may send a pulse by hitting Send Pulse. A graphical representation will then appear showing the progress of the pulse. The orbital size scale is correct and the pulse will travel at the speed of light relative to the scale of the image on the screen. The planet and Sun sizes are however greatly exaggerated in this view.

5. The return pulse will be spread out over several hundred microseconds due to the curved surface of the planet. You will obtain data for 5 different times. The first will be the sub-radar point reflection followed by one at 120 microseconds and then 3 more at 90 microsecond intervals after this (210, 300, and 390 microseconds after the sub-radar pulse is received respectively).

6. It will take at least 10 minutes (up to 30) for your pulse to arrive at Mercury, be reflected, and return to your telescope. While this is occurring, move on to the next section and begin your calculations.

7. A series of 5 windows shown for each of the times will appear on the screen. You will need to measure the spread by recording the left-most and right-most shoulders of the pulse. To measure the values of the frequency, you need only position the cross-hair in the window and click the left mouse button. The value of the difference in frequency will appear in the window. You should position the cross-hair so that it is at the edge of the shoulder of the pulse, where the pulse begins to fall towards zero. Record your data for the left and right frequency values in Table B.6 for all of the data except the data from the reflection due to the sub-radar point.

8. Finally, measure the central peak of the sub-radar point and record that information in Table B.6. Note that this is just the shift from the original frequency.

Calculations

1. You will need to know some geometric quantities to determine the rotation rate. Since you are observing signals that are reflecting from different portions of the surface. Recall that the a Doppler shift occurs only when there is relative motion between the source and the observer along the line of sight. As seen below in Figure B.6, the Doppler velocity you measure is only a fraction of the total rotational velocity of Mercury. You can find the total velocity by using similar triangles. Note that the triangle which contains $x$, $y$, and $R$ is similar to that which contains $v_o$ and $v_r$. Here $d$ is the extra distance along the line of sight that the radar wave travels, $x$ is the difference between the radius of Mercury, $R$, and $d$, $v_o$ is the velocity responsible for the measured Doppler shift, and $v_r$ is the rotational velocity of Mercury.
2. We can calculate the distance $d$ because we know the rate at which the radar wave is traveling, $c = 3.0 \times 10^8$ m/s, and the time it is traveling extra as compared to the wave reflected from the sub-radar point. The time delay is almost equal to the times that we measure the reflected waves back at Earth. However, those times are the total delays and the wave has to travel actually twice the distance $d$, once ingoing and once upon reflection. We just need to divide the distance we get using our delayed observation times by two, so

$$d = \frac{1}{2}c\Delta t.$$  \hfill (B.4) 

Recall a microsecond is $10^{-6}$ seconds.

3. The distance $x$ is just the difference between the radius of Mercury and the distance $d$. The radius of Mercury is $2.42 \times 10^6$ meters.

$$x = R - d$$  \hfill (B.5) 

4. To calculate $y$, we just use the Pythagorean theorem:

$$y = \sqrt{R^2 - x^2}$$  \hfill (B.6) 

After you have calculate these quantities for the 4 different time delayed signals (120, 210, 300, and 390 microseconds), return to the program and measure the frequency shifts. Once those are measured and your data recorded, come back here to the next step in the calculations.

5. Calculate the total frequency shift due to the rotational velocity alone. Note that the total shift you measure is twice as big as the real shift because one side is rotating towards you while the other is rotating away from you. The total shift then is:

$$\Delta f_{\text{total}} = \frac{1}{2}(\Delta f_{\text{right}} - \Delta f_{\text{left}})$$  \hfill (B.7) 

Record this for each of the delayed signals in Table B.6.

6. Calculate the corrected frequency shift $\Delta f_c$ because this is a reflected pulse. The total frequency shift calculated above is still two times too big because the wave will initially look shifted to the surface of Mercury as it approaches and shifted from that in the reflected pulse as seen from Earth so:

$$\Delta f_c = \frac{\Delta f_{\text{total}}}{2}$$  \hfill (B.8) 

Record this for each of the delayed signals in Table B.6.
7. Calculate the velocity from the Doppler shift, \( v_o \). Note that \( f \) is the frequency of the initial pulse which is displayed at the lower left corner of the main window. To match units make sure that you use \( f \) in Hz, not MHz (10^6 Hz).

\[
v_o = c \left( \frac{\Delta f_r}{f} \right)
\]  
(B.9)

Record this for each of the delayed signals in Table B.6.

8. Finally, calculate the rotational velocity, \( v_r \). This can be done using similar triangles:

\[
\frac{v_r}{v_o} = \frac{R}{y},
\]  
(B.10)

\[
v_r = v_o \left( \frac{R}{y} \right).
\]  
(B.11)

9. You can now calculate the rotational period of Mercury. We know the distance Mercury rotates through and its speed so we can calculate the time:

\[
P_{\text{rot}} = \frac{\text{circumference}}{v_r}
\]  
(B.12)

\[
= \frac{2\pi R}{v_r}
\]  
(B.13)

You can convert this time from seconds into days by dividing by the number of seconds per day, 86,400.

10. Calculate an average value for the period of rotation for Mercury. How does this compare to the accepted value of 59 days? Calculate the percent error in your measurement.

You will now calculate the orbital velocity of Mercury using the information from the sub-radar point.

1. Using the reflected pulse from the sub-radar point, you can calculate the orbital velocity of Mercury. The shift you recorded is just twice the total shift because it is an echo. You can use Eq. B.8 – B.9 making only one change - in Eq. B.9, \( v_o \) will be the orbital velocity. Calculate Mercury’s orbital velocity. Write your number for the orbital velocity on the board and next to it a sketch of the orientation of the Sun, Earth, and Mercury for your chosen observation date (you will reference this data later). Do not quit the program!
Table B.6: Mercury Data Table

<table>
<thead>
<tr>
<th>$\Delta t , (\mu s)$</th>
<th>120</th>
<th>210</th>
<th>300</th>
<th>390</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d , (m)$</td>
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<tr>
<td>$x , (m)$</td>
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<tr>
<td>$y , (m)$</td>
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<td>$f_{\text{left}} , (Hz)$</td>
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<tr>
<td>$f_{\text{right}} , (Hz)$</td>
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<tr>
<td>$\Delta f_{\text{total}} , (Hz)$</td>
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<tr>
<td>$\Delta f_c , (Hz)$</td>
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<tr>
<td>$v_o , (m/s)$</td>
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<tr>
<td>$v_r , (m/s)$</td>
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<td></td>
</tr>
<tr>
<td>$P_{\text{rot}} , (\text{days})$</td>
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</tr>
</tbody>
</table>

V. Questions

1. The ephemeris data you recorded initially gave you the distance to Mercury in terms of the astronomical unit. How big is an astronomical unit in km? You can use your data to find out. Do a unit conversion using your round-trip light travel time to calculate the number of kilometers in 1 AU. The first term is written below along with the units you should end up with.

$$\left( \frac{\# \, \text{km}}{1 \, \text{AU}} \right) = \left[ \frac{\text{time}}{\text{distance (AU)}} \right] \times \quad (B.14)$$
2. Look at the blackboard showing the lab results for the orbital velocity and their associated sketches. Can you explain the results? In particular, why would it be difficult for you, the astronomer, to obtain an accurate measurement of the orbital velocity of a planet (not just Mercury) using Doppler shift?

VI. Credit
To receive credit for this lab you must turn in the data from Table B.6, your calculated values for orbital and rotational velocity and the rotational period, and the answers to the above questions.